Strategy Extraction in Reachability Games

Niklas Een, Alexander Legg, Nina Narodytska and Leonid Ryzhyk

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

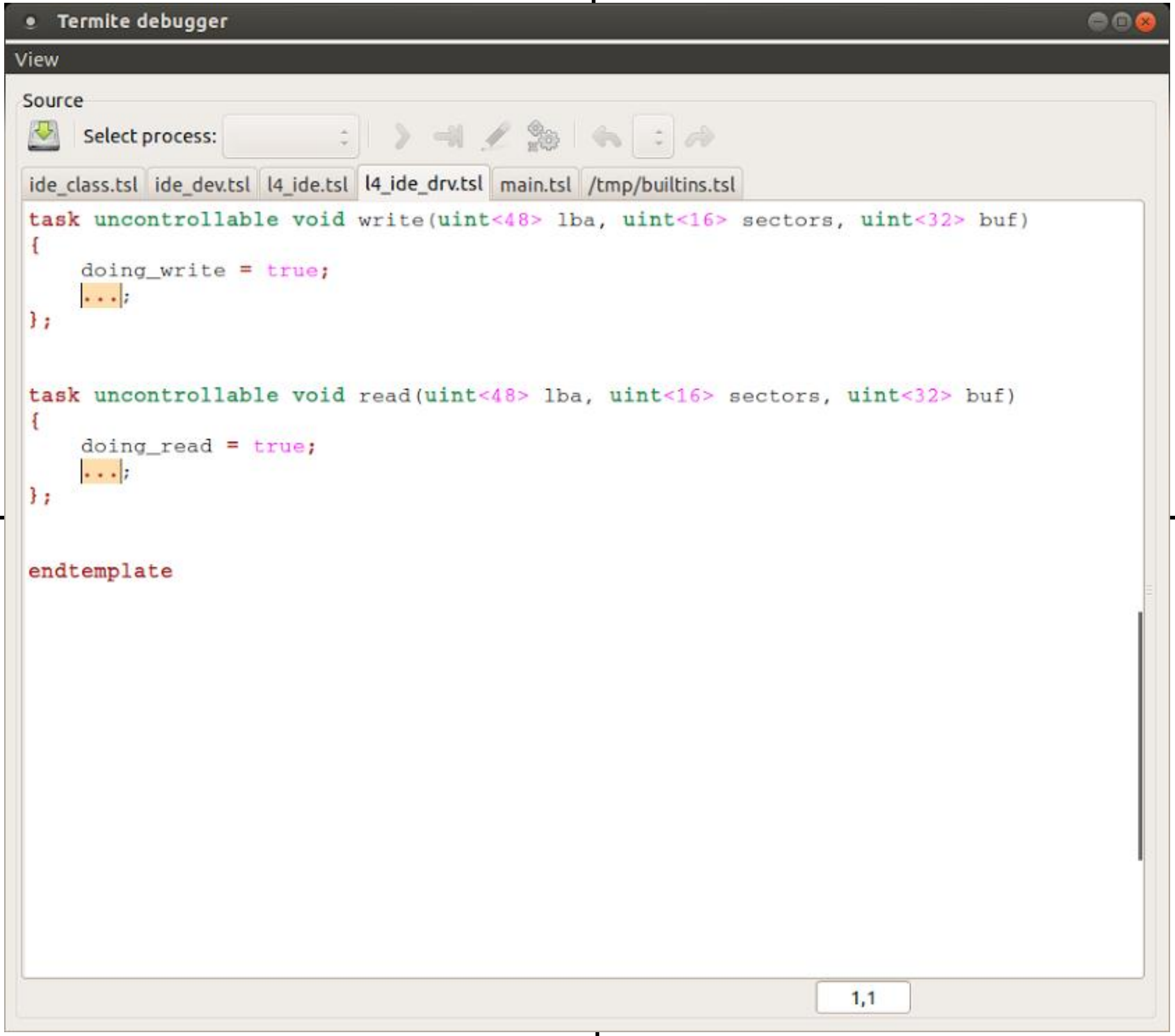
Outline

- Motivation
- Introduction into reachability games
- CEGAR-based algorithm (EvaSolver)
- Strategy extraction
- Conclusion

Automatic Driver Synthesis Project
(funded by Intel)

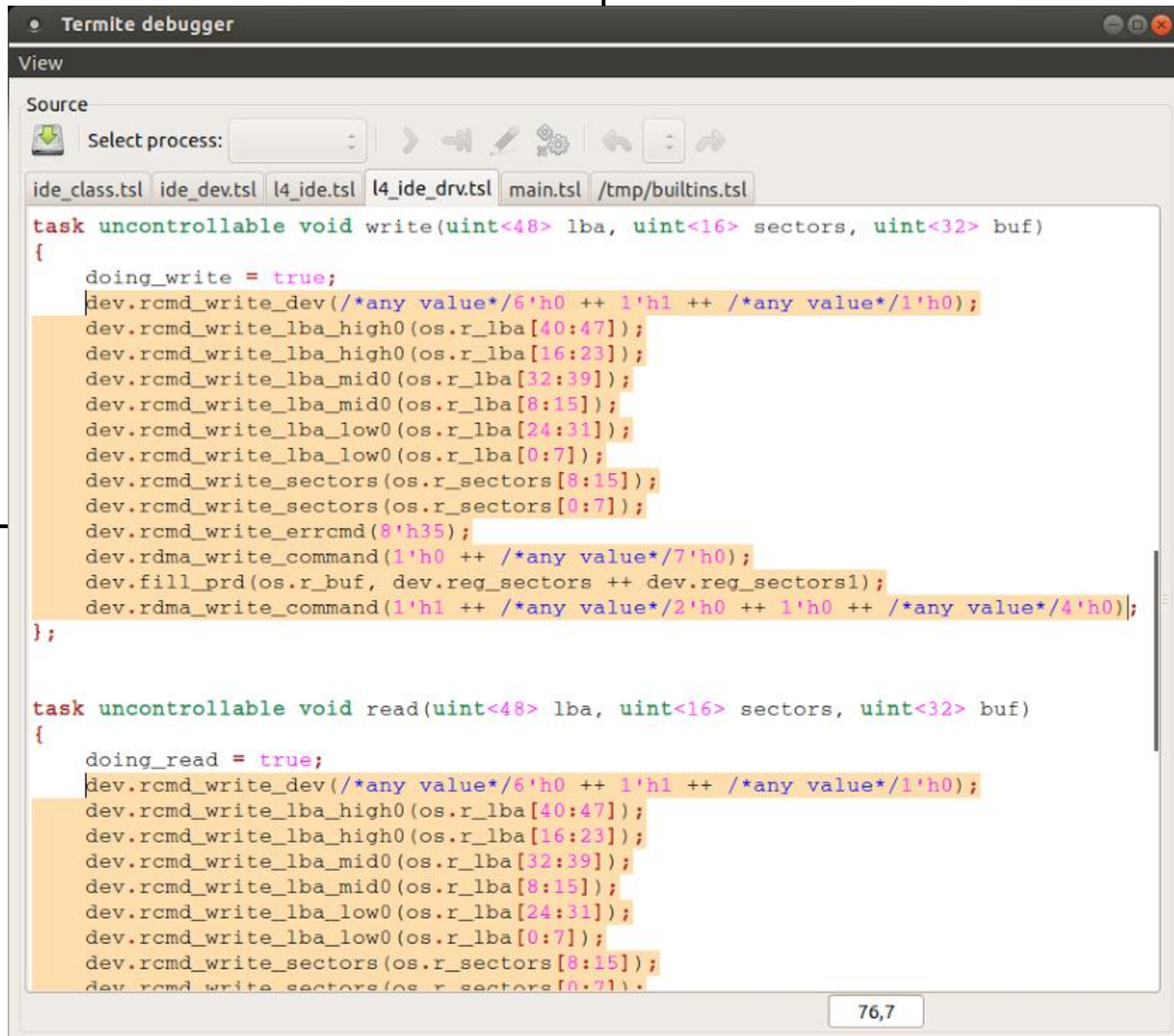
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Practical Device Driver Synthesis (OSDI'14)



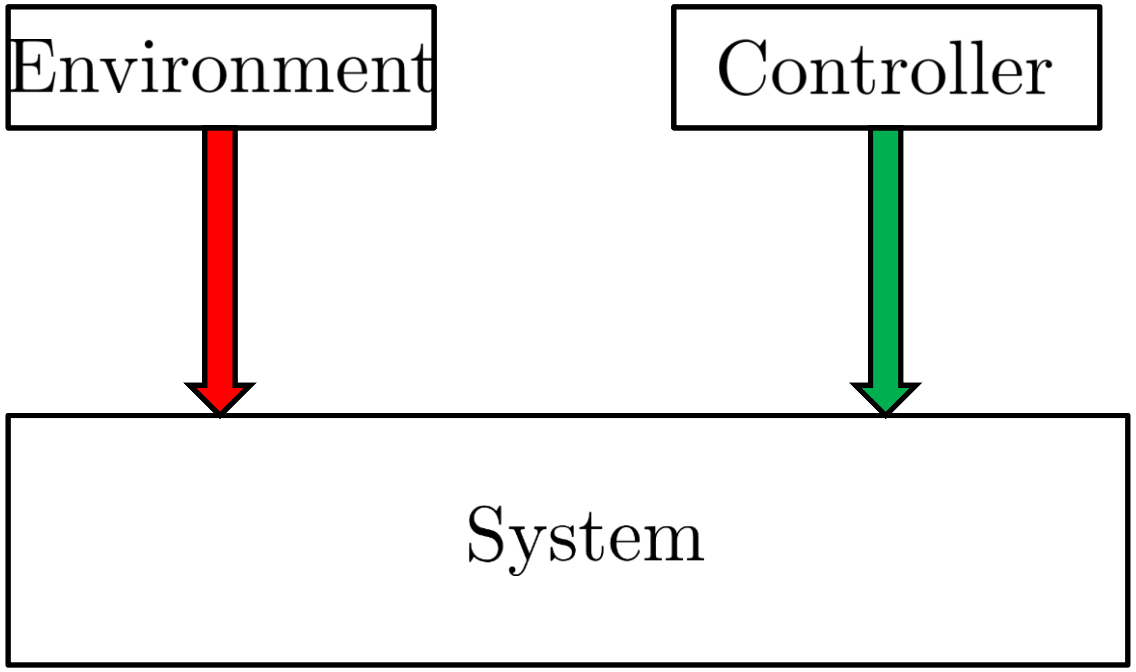
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Practical Device Driver Synthesis (OSDI'14)



Introduction into reachability games

Reachability Games



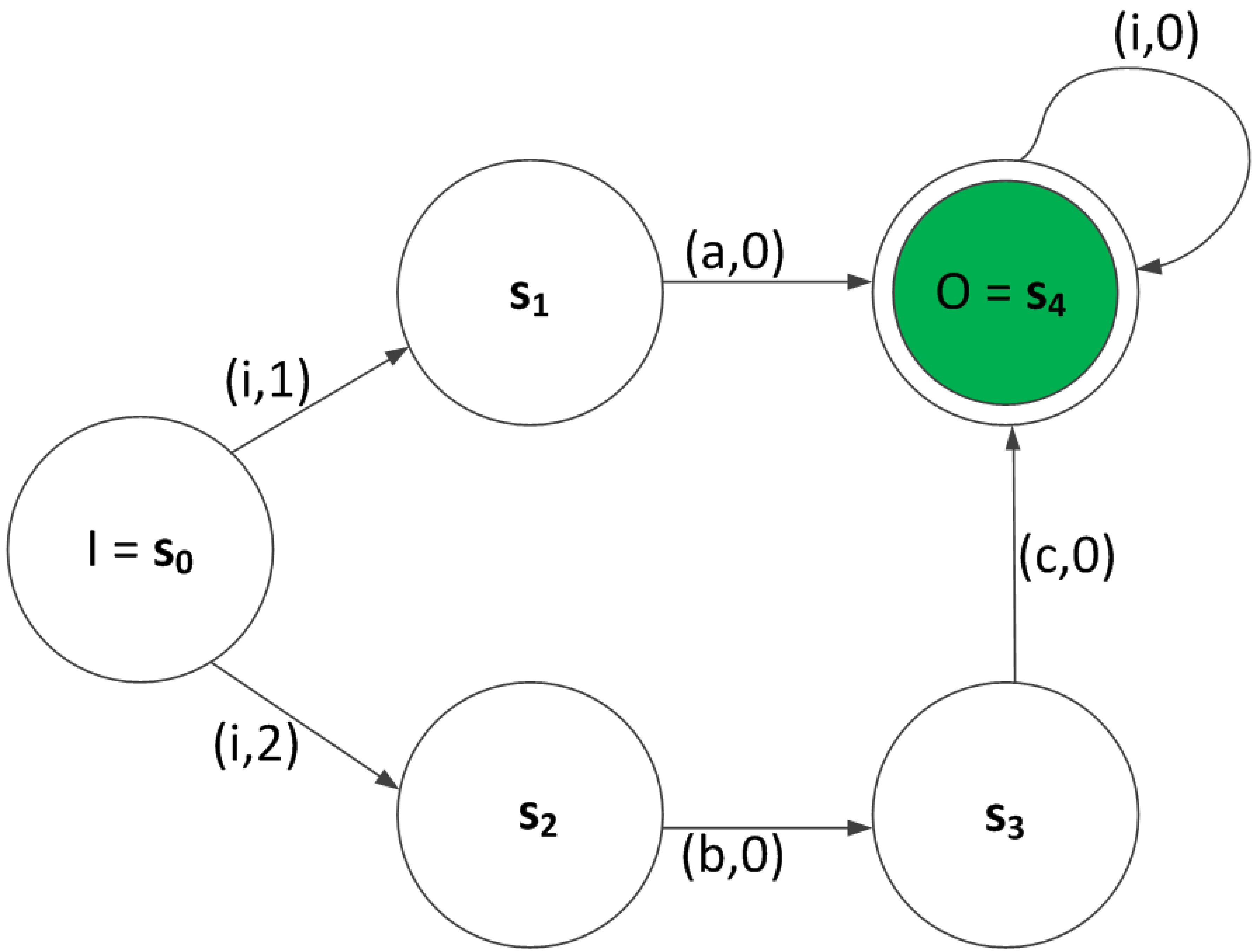
Can the system be controlled to satisfy ϕ?

Reachability games

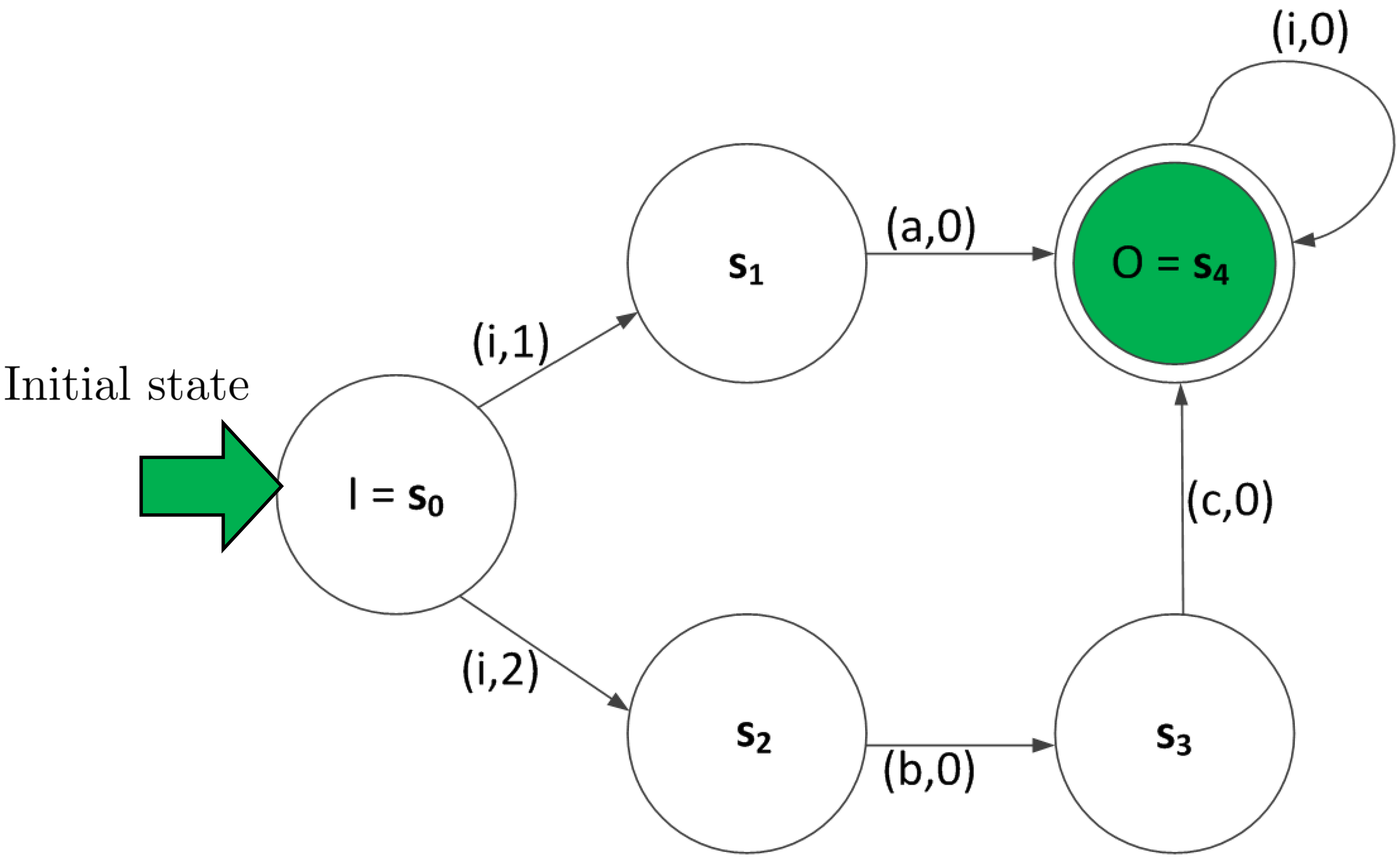
A reachability game $G = \langle S, L_c, L_u, I, O, \delta \rangle$ consists of

- a set of states S,
- controllable actions L_c,
- uncontrollable actions L_u,
- initial state $I \in S$,
- a set $O \in 2^S$ of goal states, and
- a transition function $\delta : (S, L_c, L_u) \to S$.

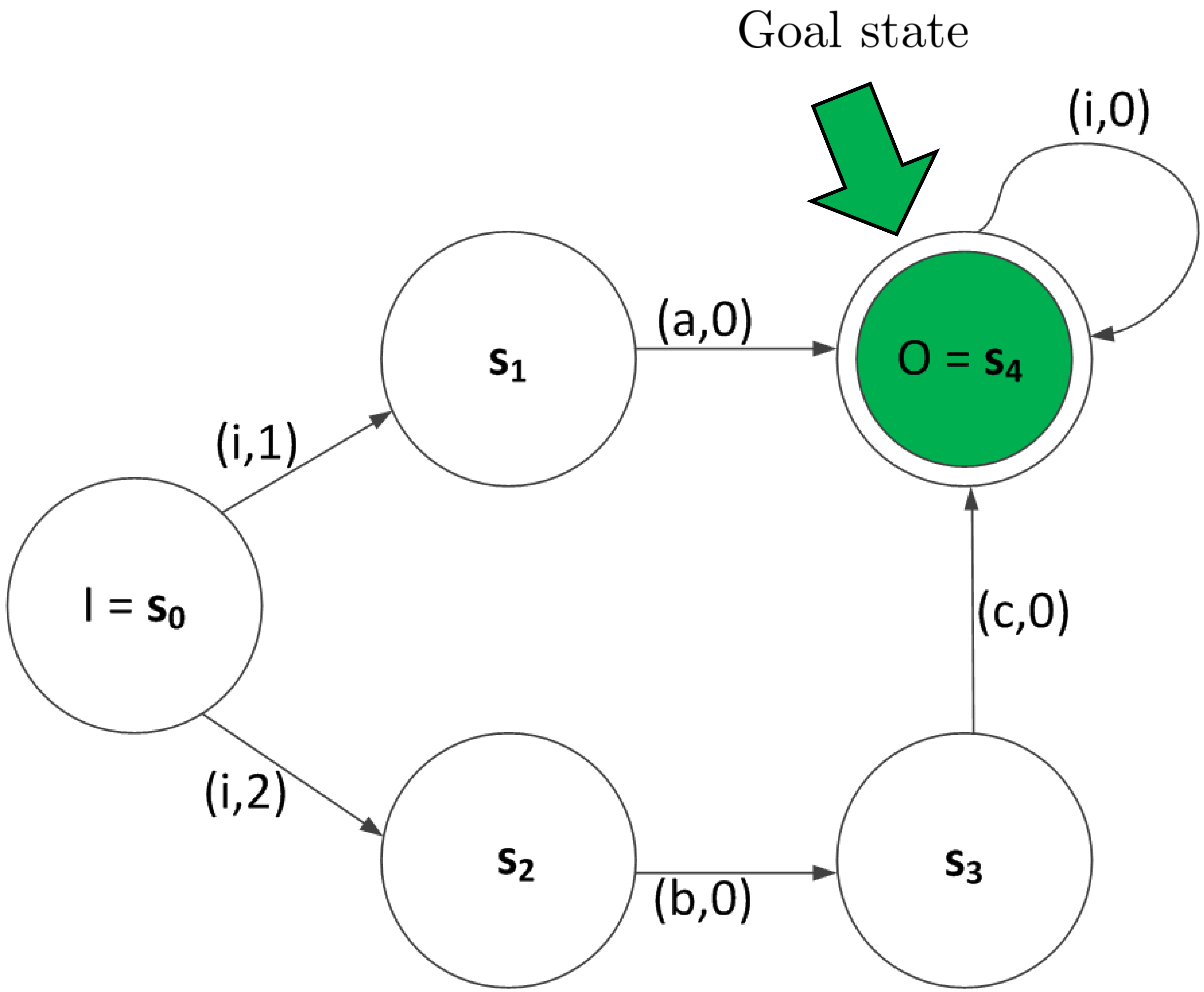
Reachability games



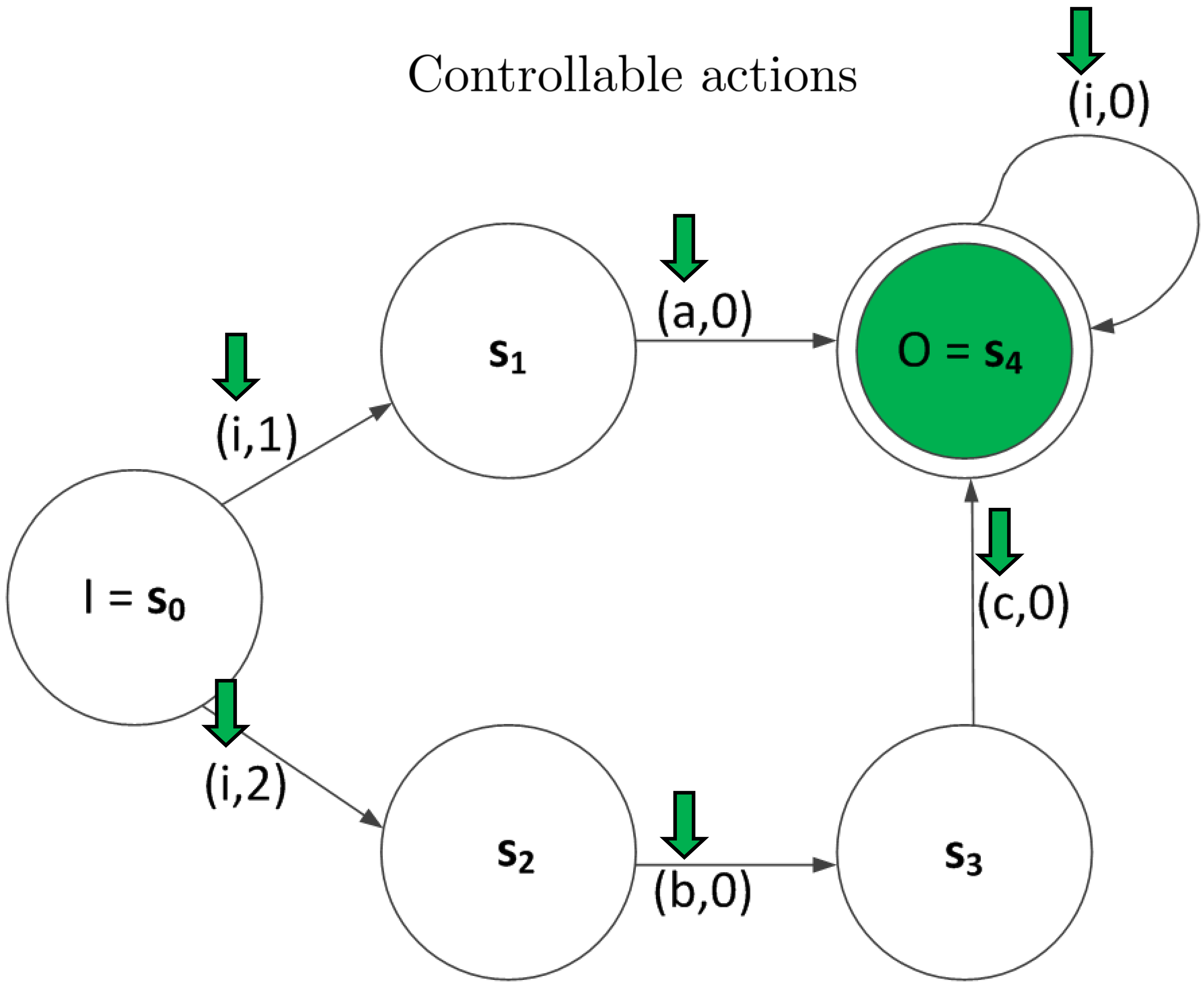
Reachability games



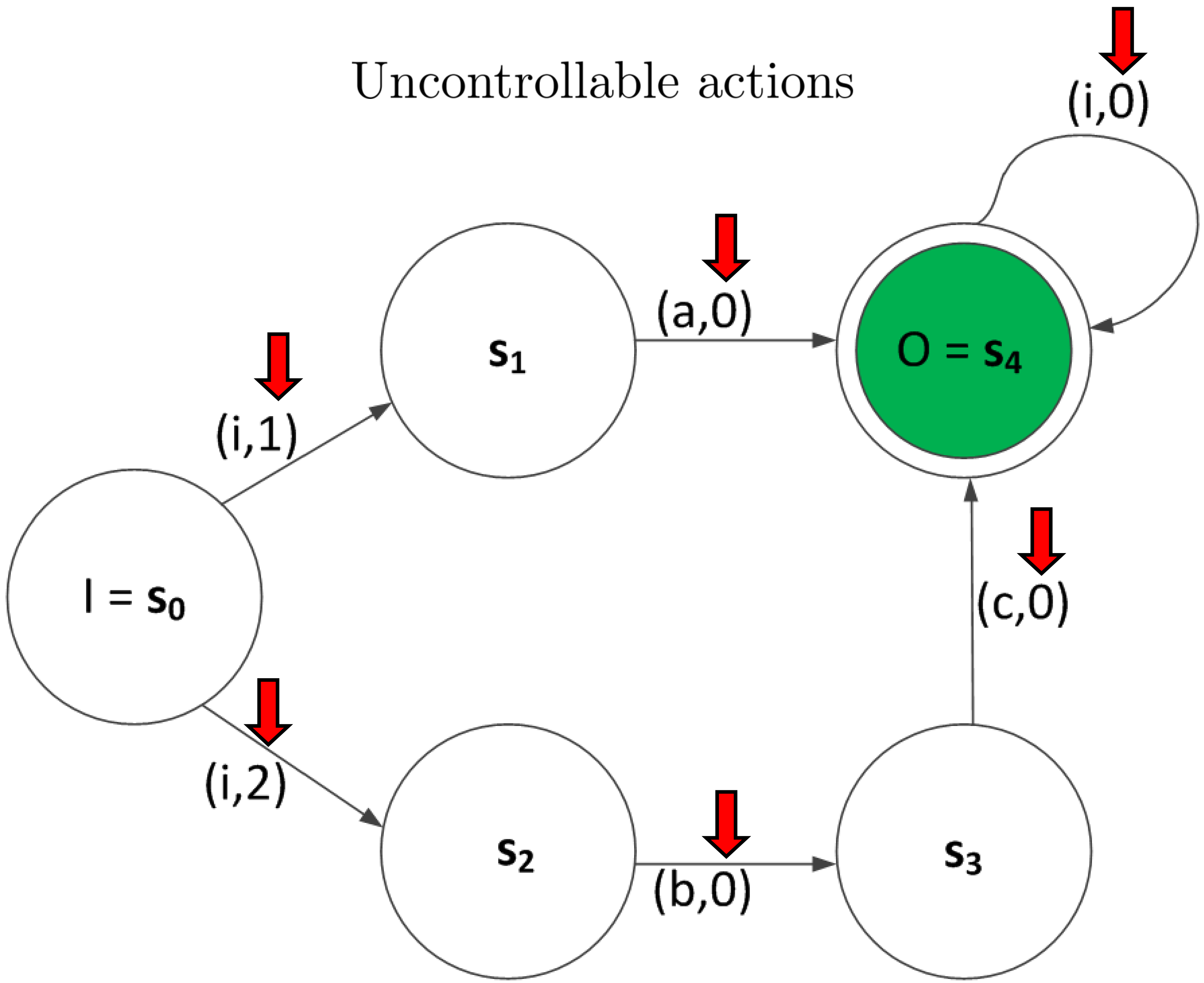
Reachability games



Reachability games



Reachability games



Reachability games

The goal of the controller is to reach the goal state regardless of the behaviour of the environment.

Reachability games

The goal of the controller is to reach the goal state
in n steps
regardless of the behaviour of the environment.

Reachability games

A *controller strategy* $\pi : S \to L_c$
associates with every state a controllable action to play in this state.

Reachability games

Given a bound n, π is a *winning* strategy in state $\mathbf{s}$ at round $i \leq n$ if any sequence $(\mathbf{s}_i, \mathbf{u}_i, \mathbf{s}_{i+1}, \mathbf{u}_{i+1}, ..., \mathbf{s}_n)$, such that $\mathbf{s}_i = \mathbf{s}$ and $\mathbf{s}_{k+1} = \delta(\mathbf{s}_k, \pi(\mathbf{s}_k), \mathbf{u})$, visits the goal set: $\exists j \in [i, n].\mathbf{s}_j \in O$.

Winning strategy: Existence vs Extraction

Reachability games solver

Reachability games solver

No

Reachability games solver

No

Yes, here is a strategy.

Reachability games solver

No

Yes, here is a strategy.

1. SAT-Based Synthesis Methods for Safety Specs, VMCAI'14
Roderick Bloem,Robert Knighofer, Martina Seidl

2. Solving Games Using Incremental Induction, IFM'13
Andreas Morgenstern, Manuel Gesell, Klaus Schneider

Reachability games solver

No

~~Yes, here is a strategy.~~

1. SAT-Based Synthesis Methods for Safety Specs, VMCAI'14
Roderick Bloem,Robert Knighofer, Martina Seidl

2. Solving Games Using Incremental Induction, IFM'13
Andreas Morgenstern, Manuel Gesell, Klaus Schneider

Reachability games solver

No

~~Yes, here is a strategy.~~
Yes, there is a strategy.

1. SAT-Based Synthesis Methods for Safety Specs, VMCAI'14
Roderick Bloem,Robert Knighofer, Martina Seidl

2. Solving Games Using Incremental Induction, IFM'13
Andreas Morgenstern, Manuel Gesell, Klaus Schneider

EvaSolver
[Narodytska at el., CAV'14]

No

~~Yes, here is a strategy.~~

Yes, there is a strategy.

EvaSolver
[Narodytska at el., CAV'14]

No

~~Yes, here is a strategy.~~

~~Yes, there is a strategy.~~

Yes, here is a strategy certificate.

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Game tree

Game tree

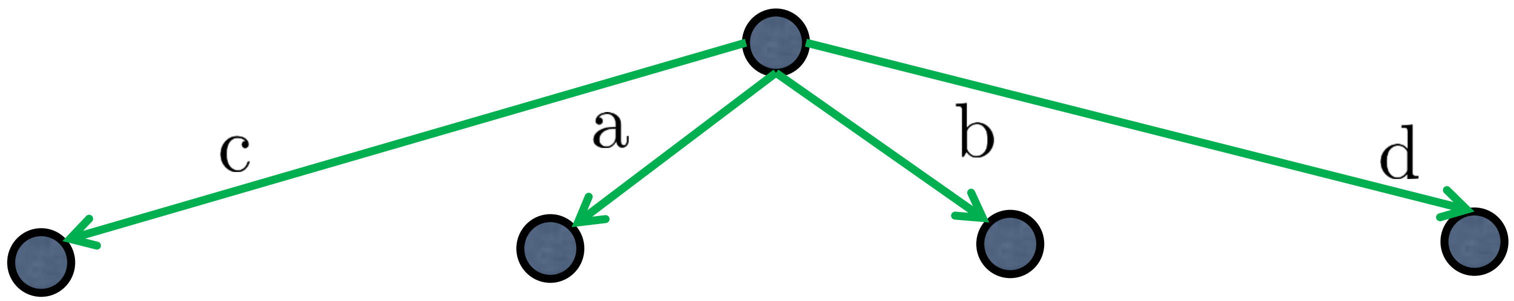
controllable move

uncontrollable move

Simplifying assumptions:

- consider reachability games
- bound the number of rounds in the game
- players strictly alternate

Game tree



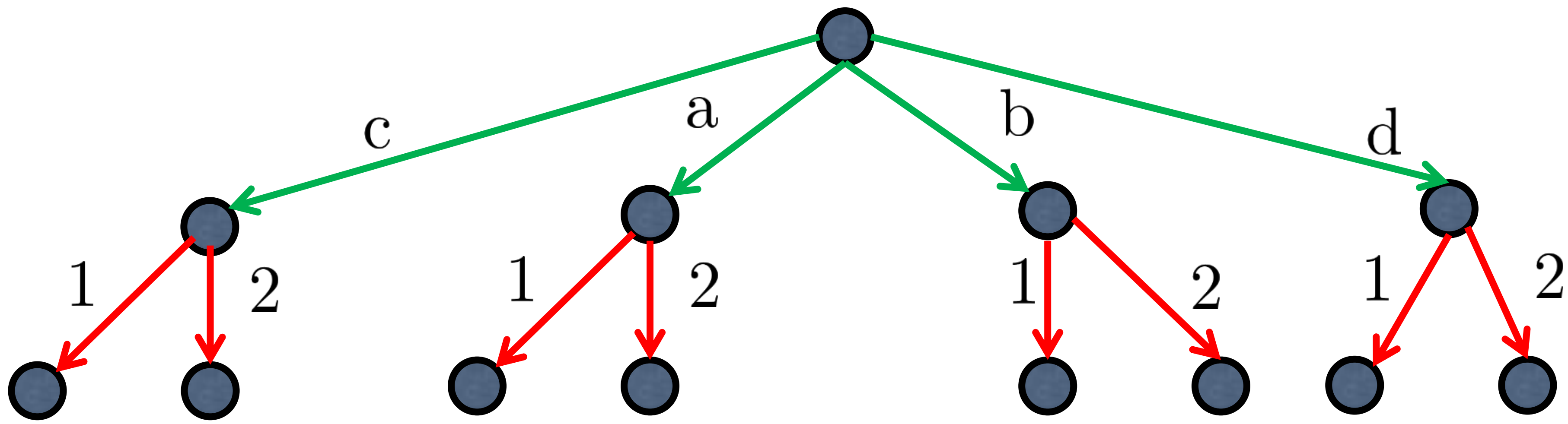
controllable move

uncontrollable move

Simplifying assumptions:

- consider reachability games
- bound the number of rounds in the game
- players strictly alternate

Game tree



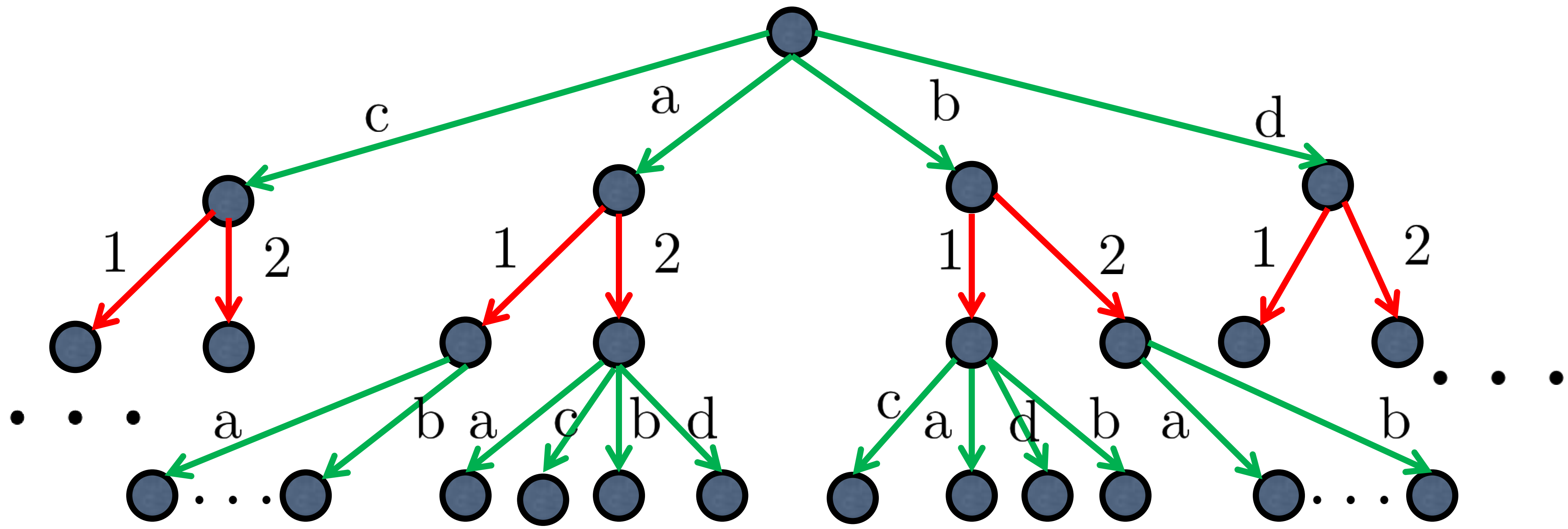
controllable move

uncontrollable move

Simplifying assumptions:

- consider reachability games
- bound the number of rounds in the game
- players strictly alternate

Game tree



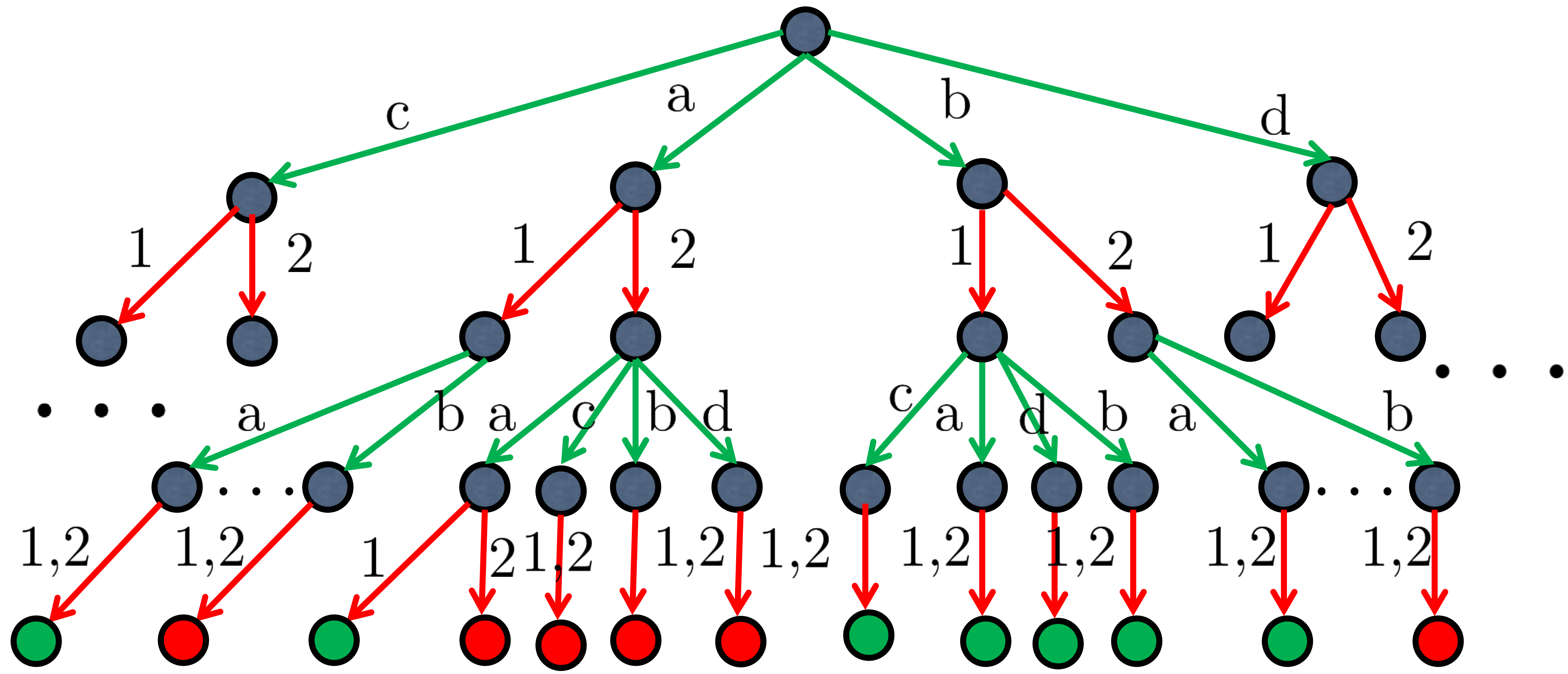
controllable move

uncontrollable move

Simplifying assumptions:

- consider reachability games
- bound the number of rounds in the game
- players strictly alternate

Game tree



controllable move

uncontrollable move

winning state

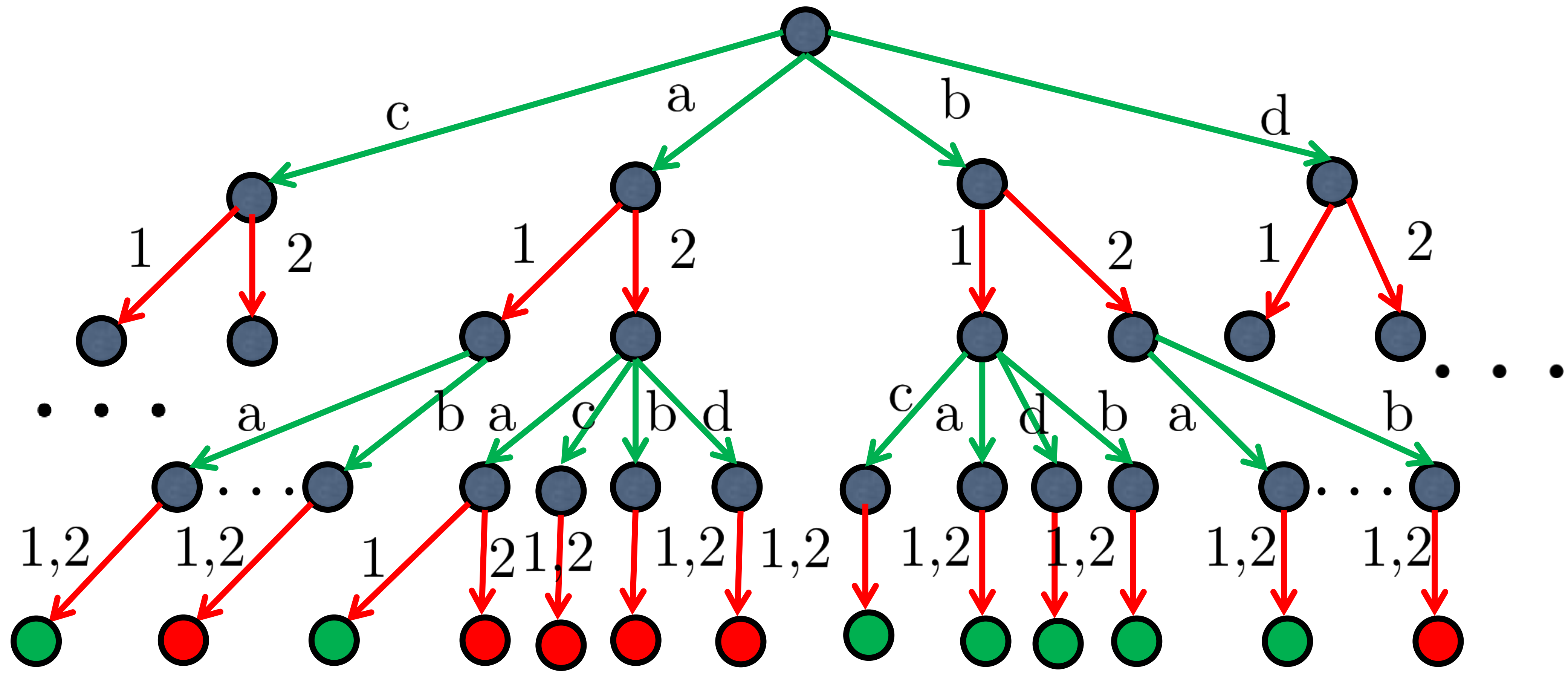
losing state

Simplifying assumptions:

- consider reachability games
- bound the number of rounds in the game
- players strictly alternate

Winning strategy certificate

Game tree



controllable move

uncontrollable move

winning state

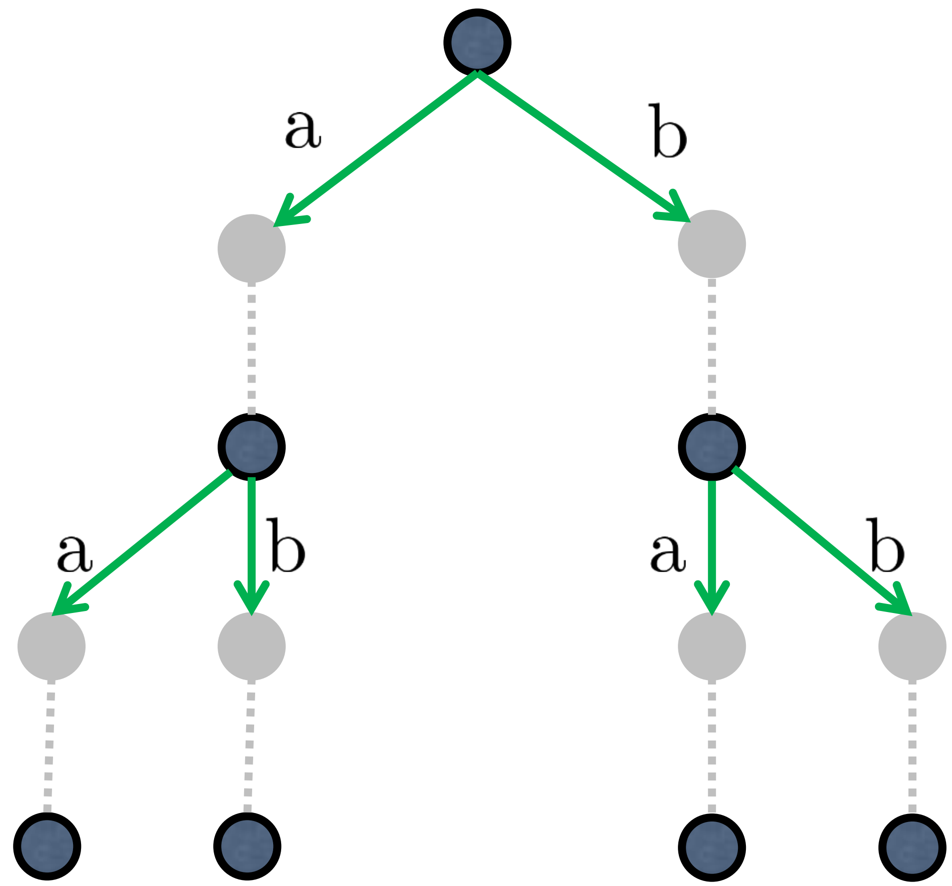
losing state

Simplifying assumptions:

- consider reachability games
- bound the number of rounds in the game
- players strictly alternate

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Abstract Game tree



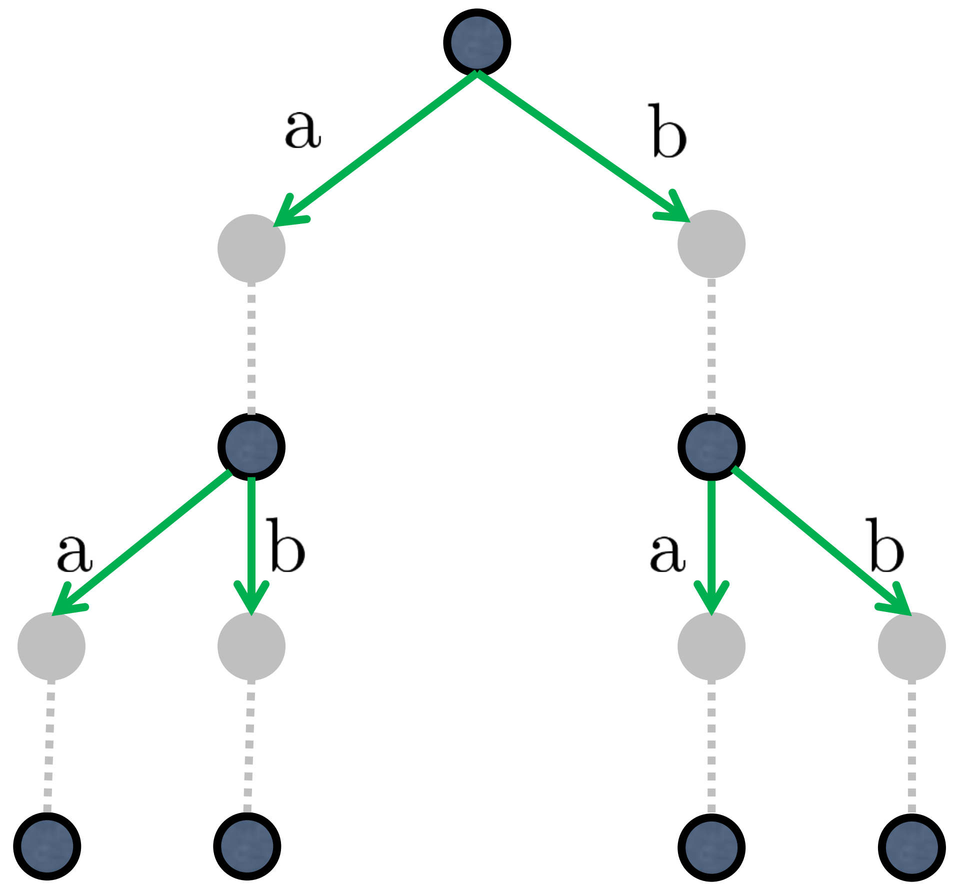
controllable move

some uncontrollable move

restricts controllable player moves

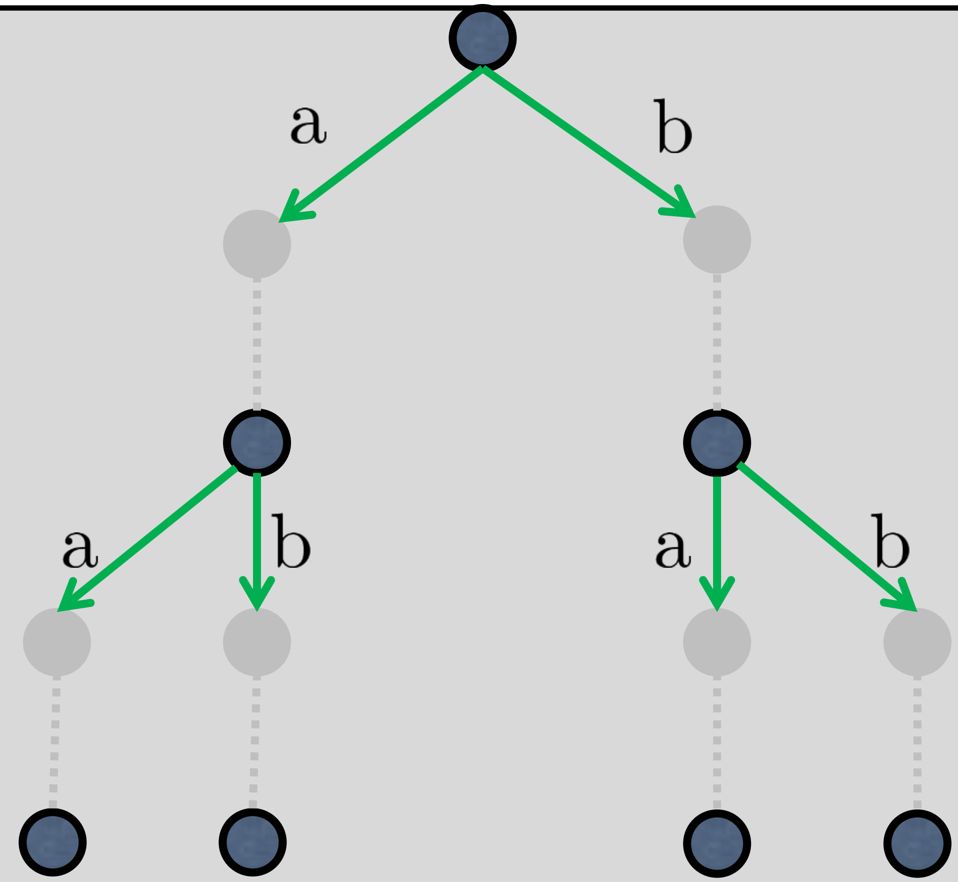
Verifying a certificate

Abstract Game tree verification



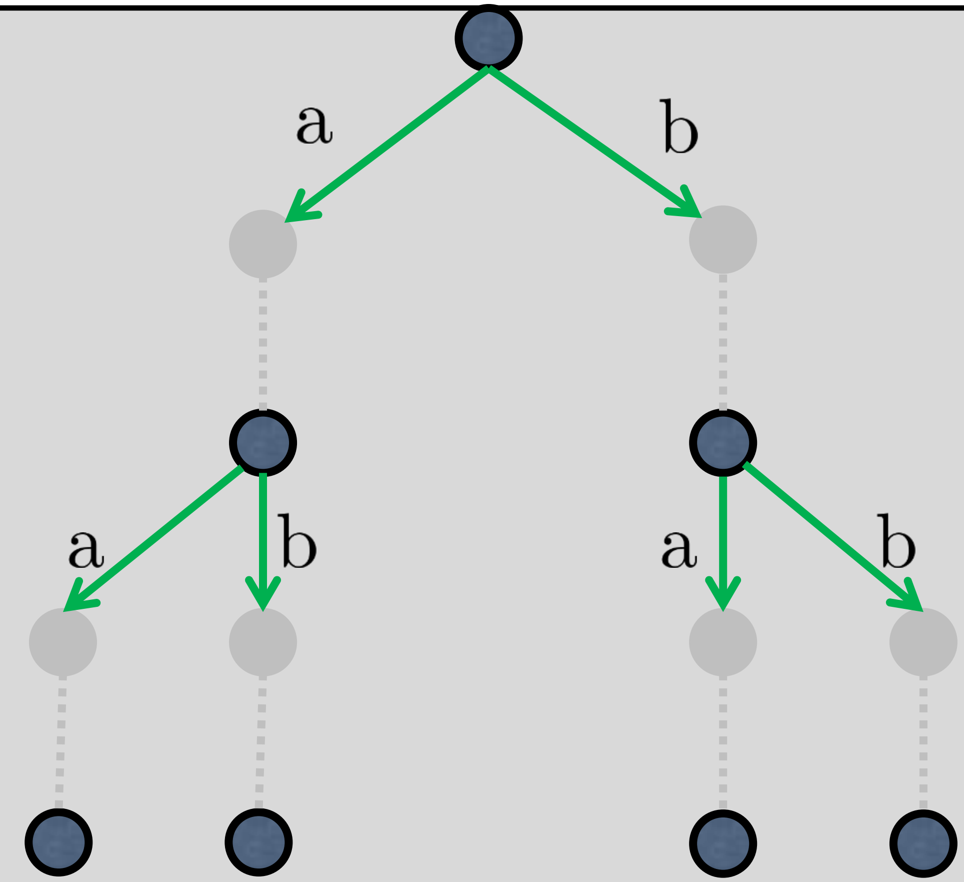
Can the enviroment win a safety game if the controller moves are restricted to the ones in the tree?

Abstract Game tree verification



Can the enviroment win a safety game if the controller moves are restricted to the ones in the tree?

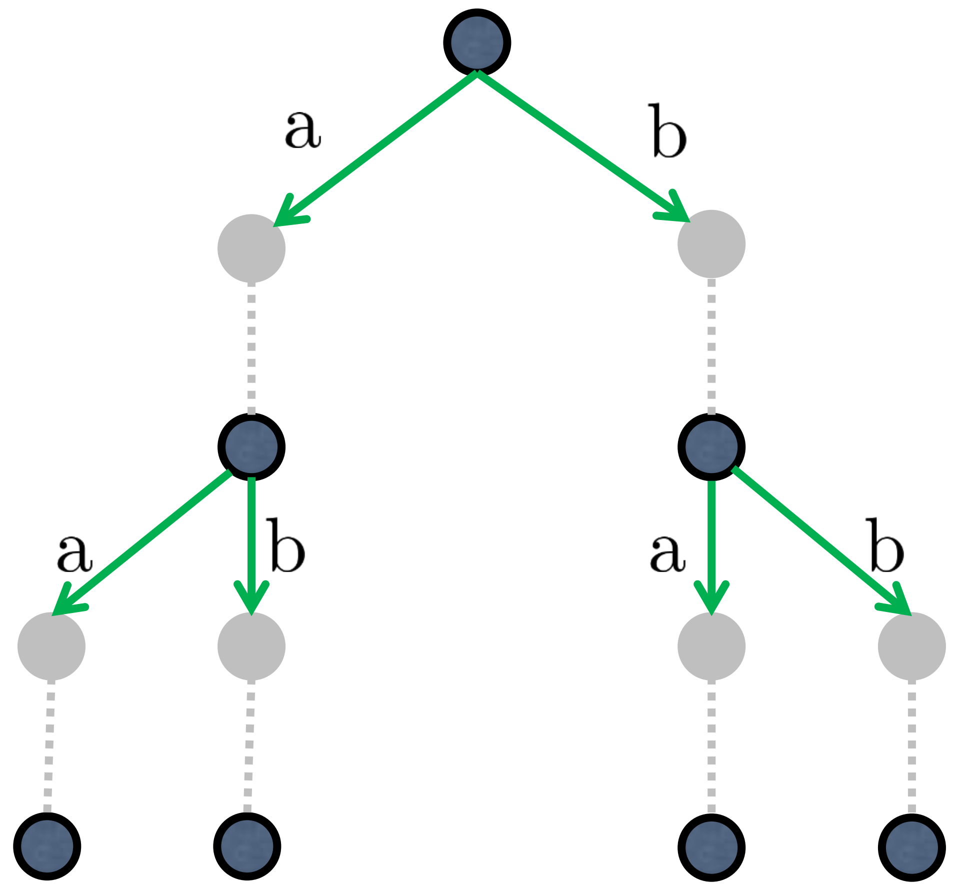
Abstract Game tree verification



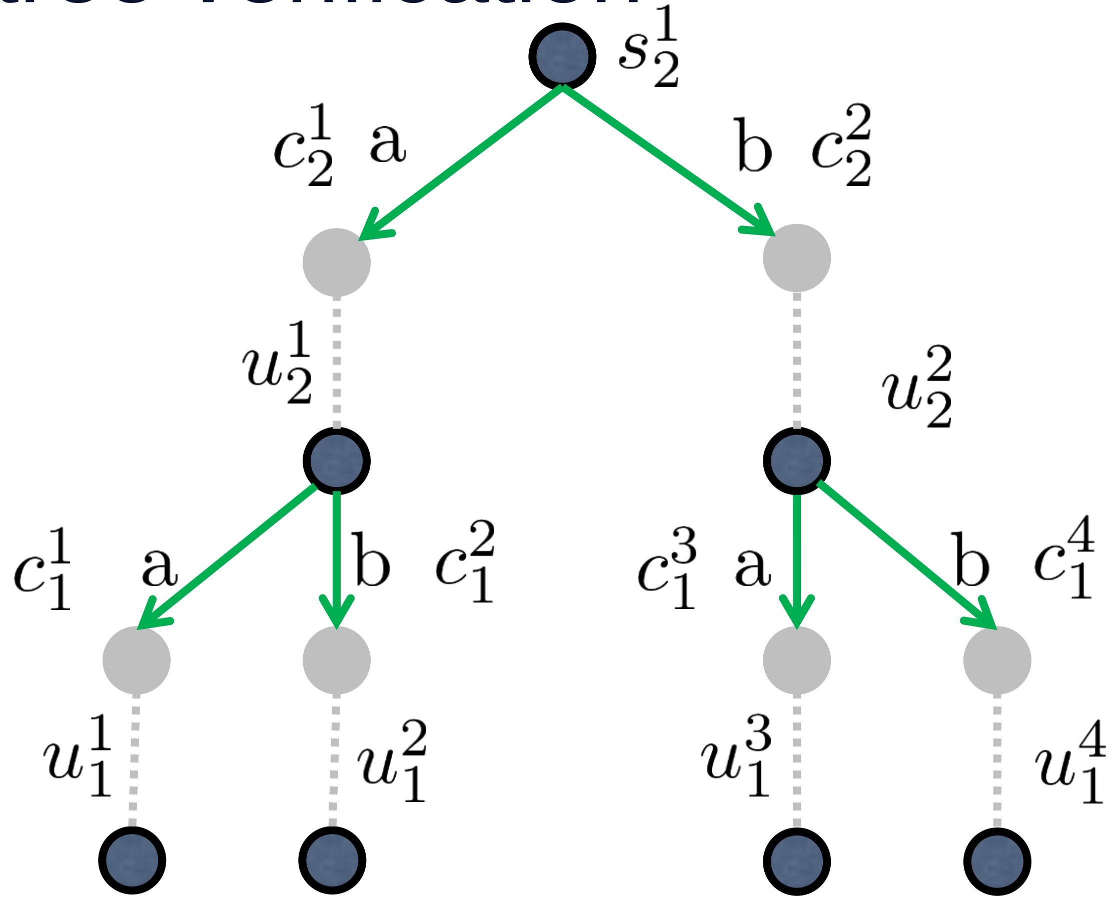
Can the enviroment win a safety game if the controller moves are restricted to the ones in the tree?

Encode into a SAT formula.

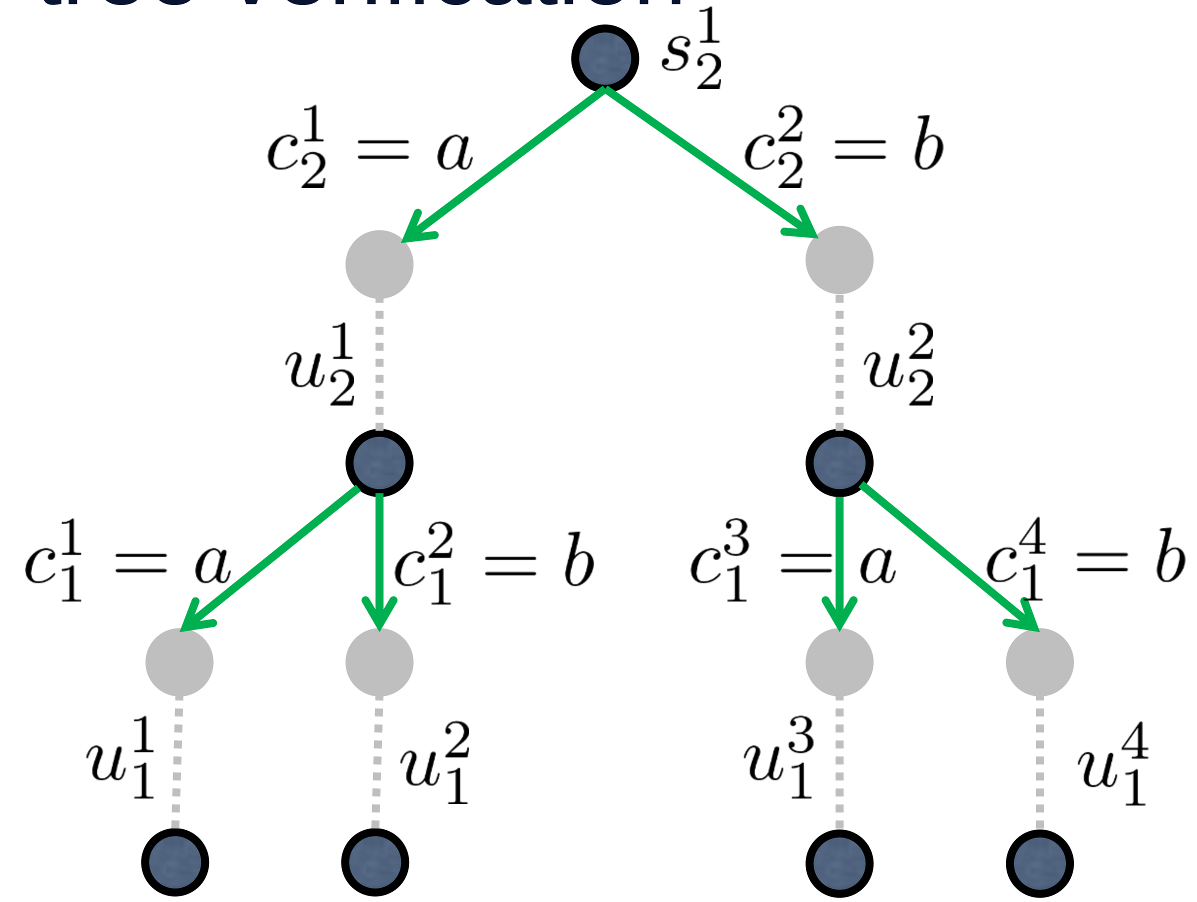
Abstract Game tree verification



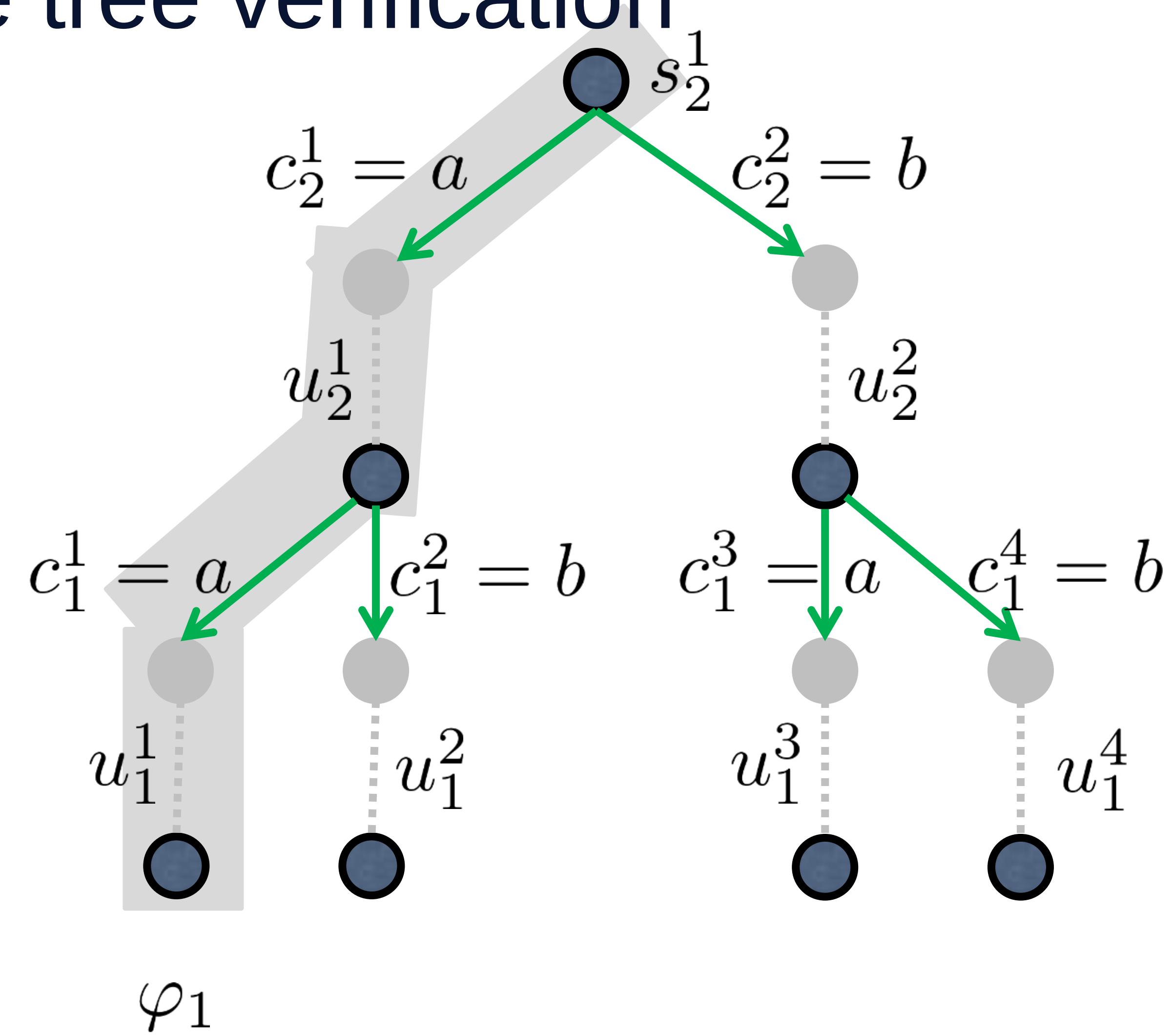
Abstract Game tree verification



Abstract Game tree verification

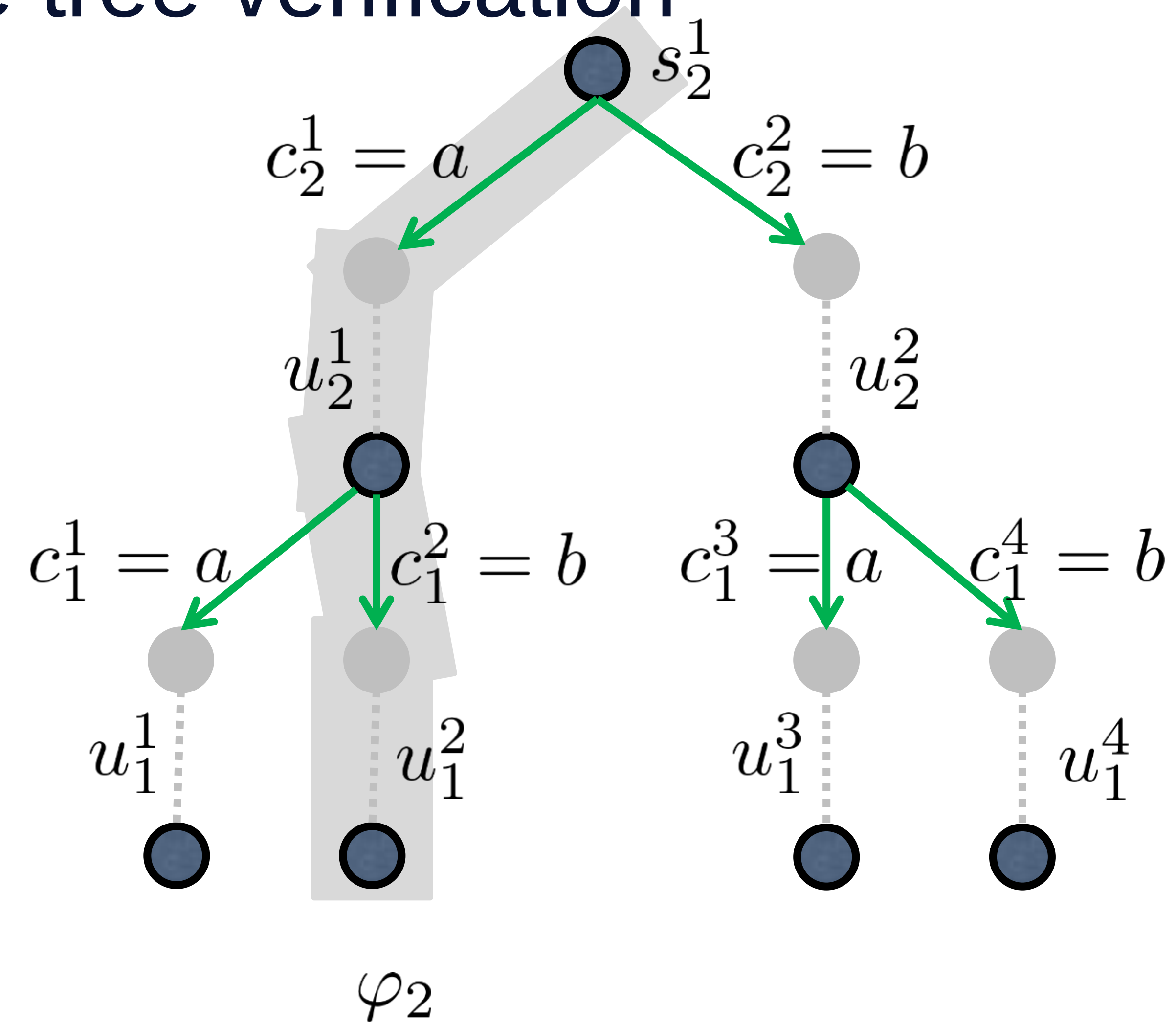


Abstract Game tree verification

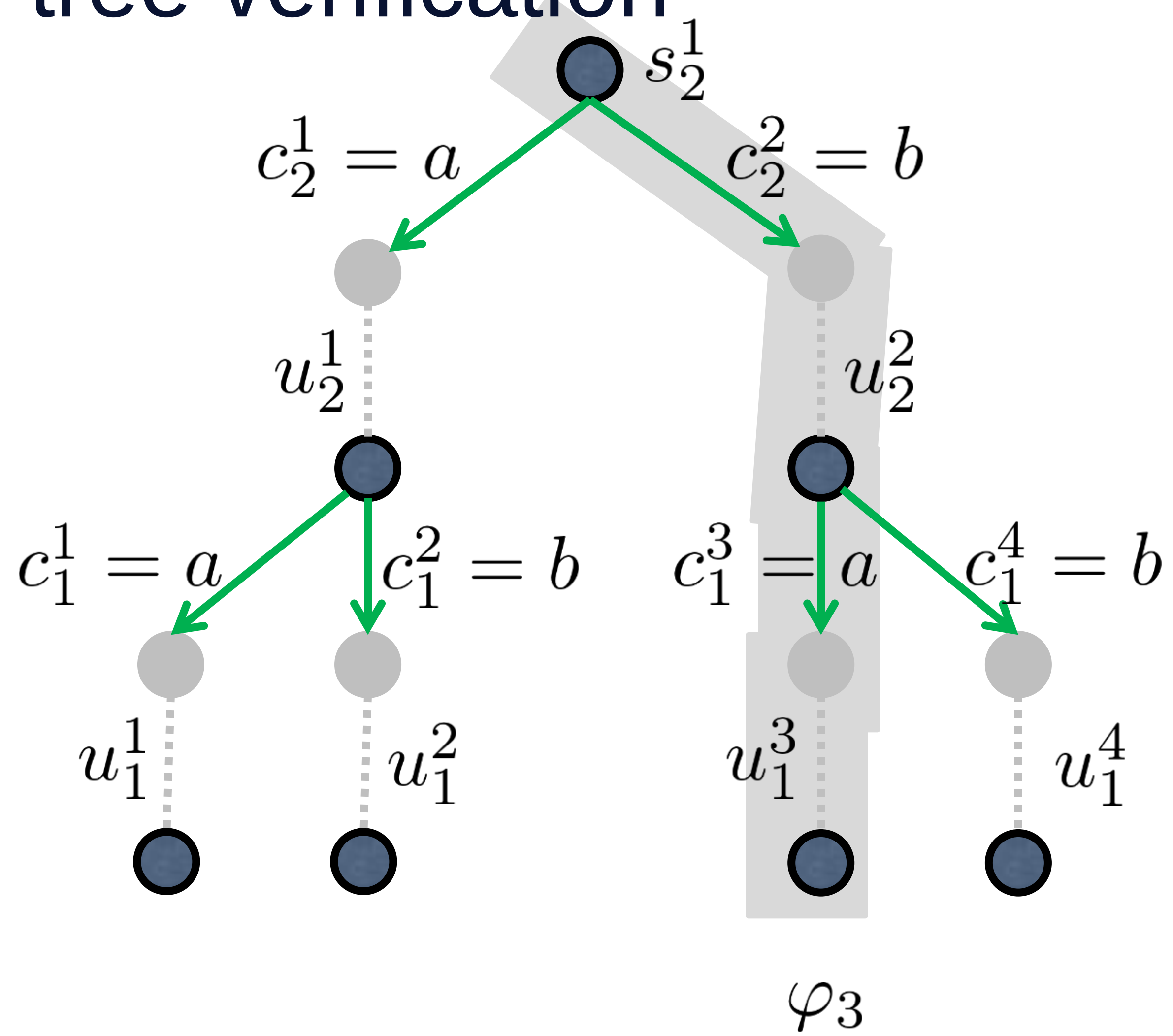


UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

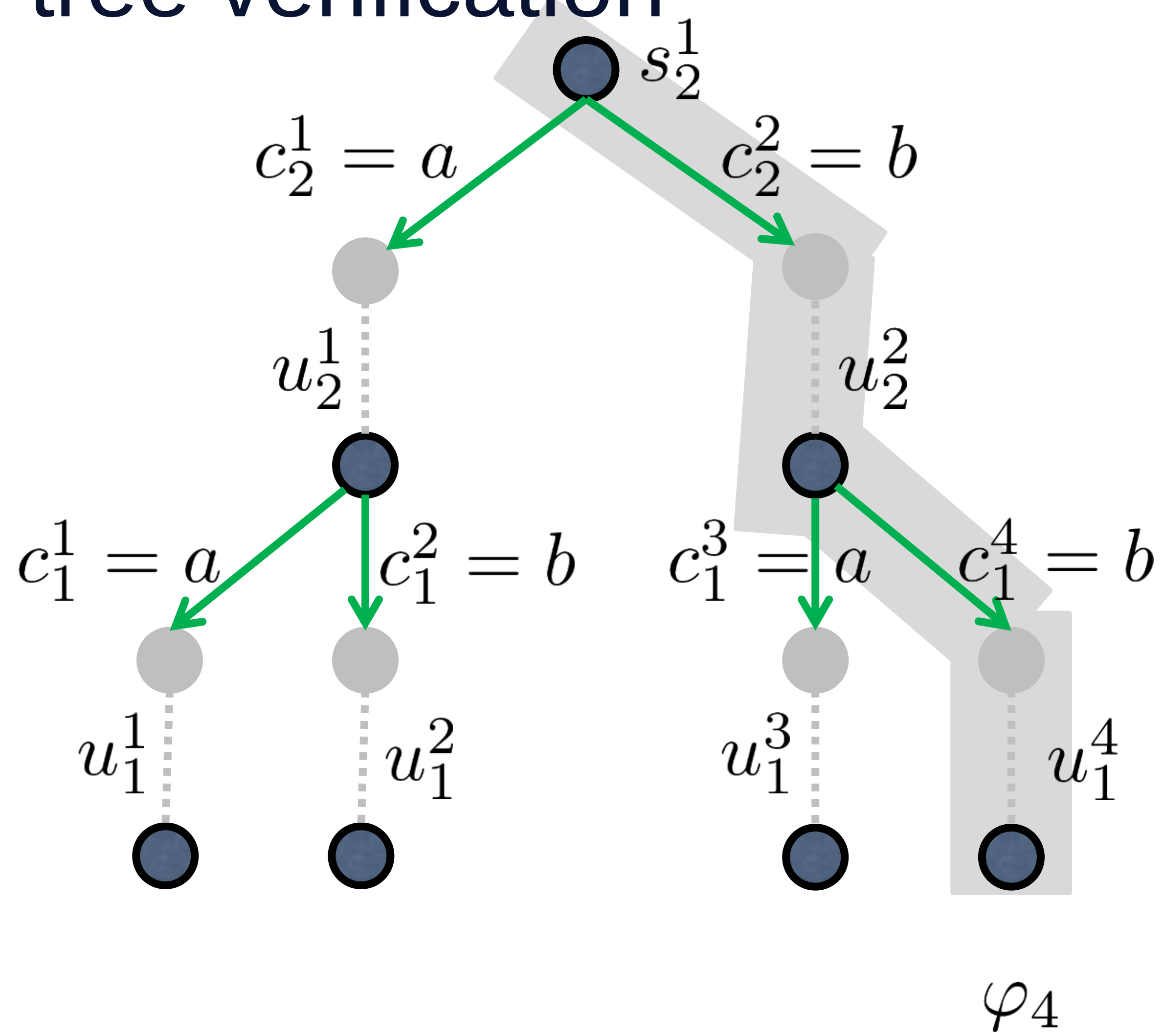
Abstract Game tree verification



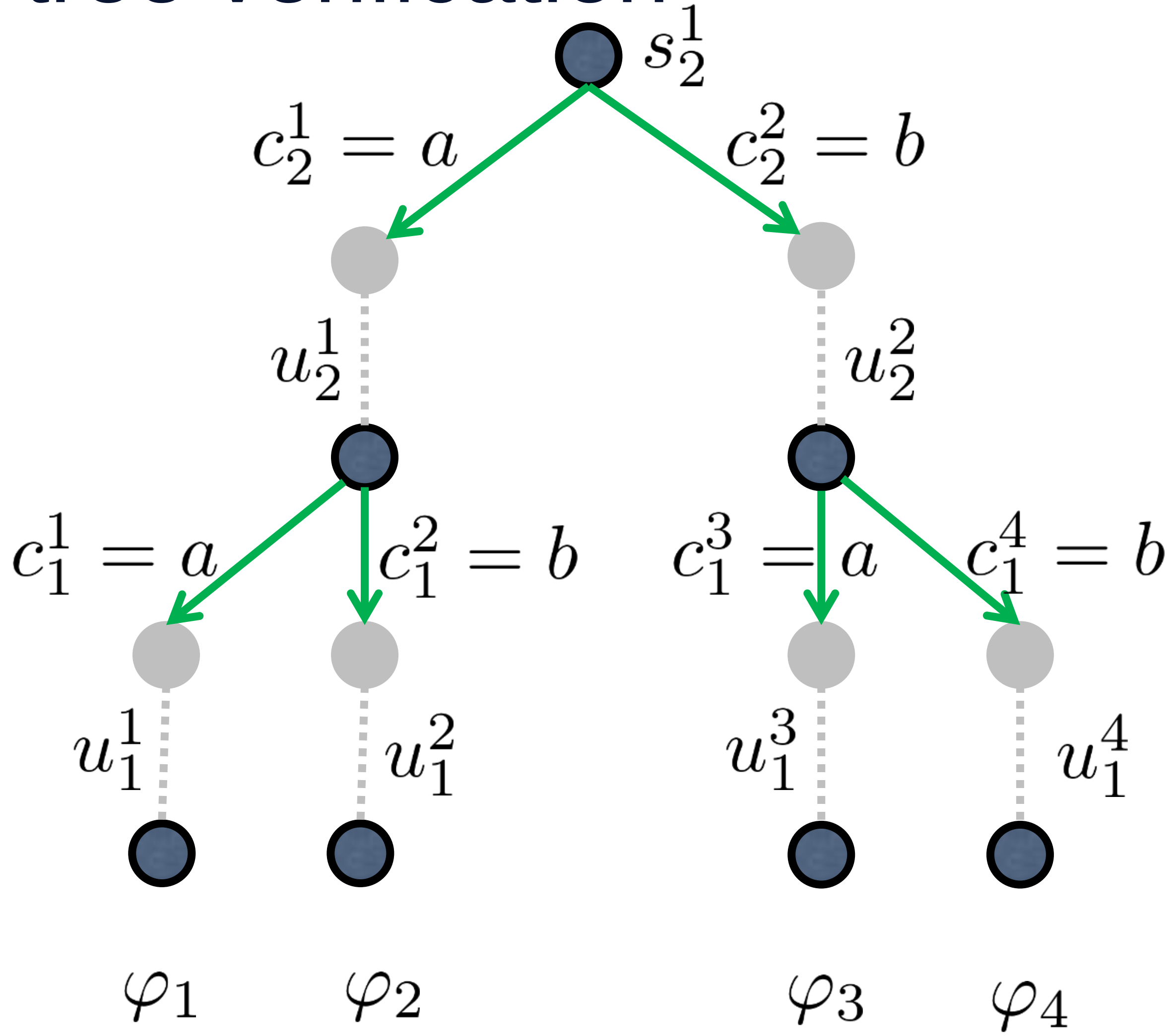
Abstract Game tree verification



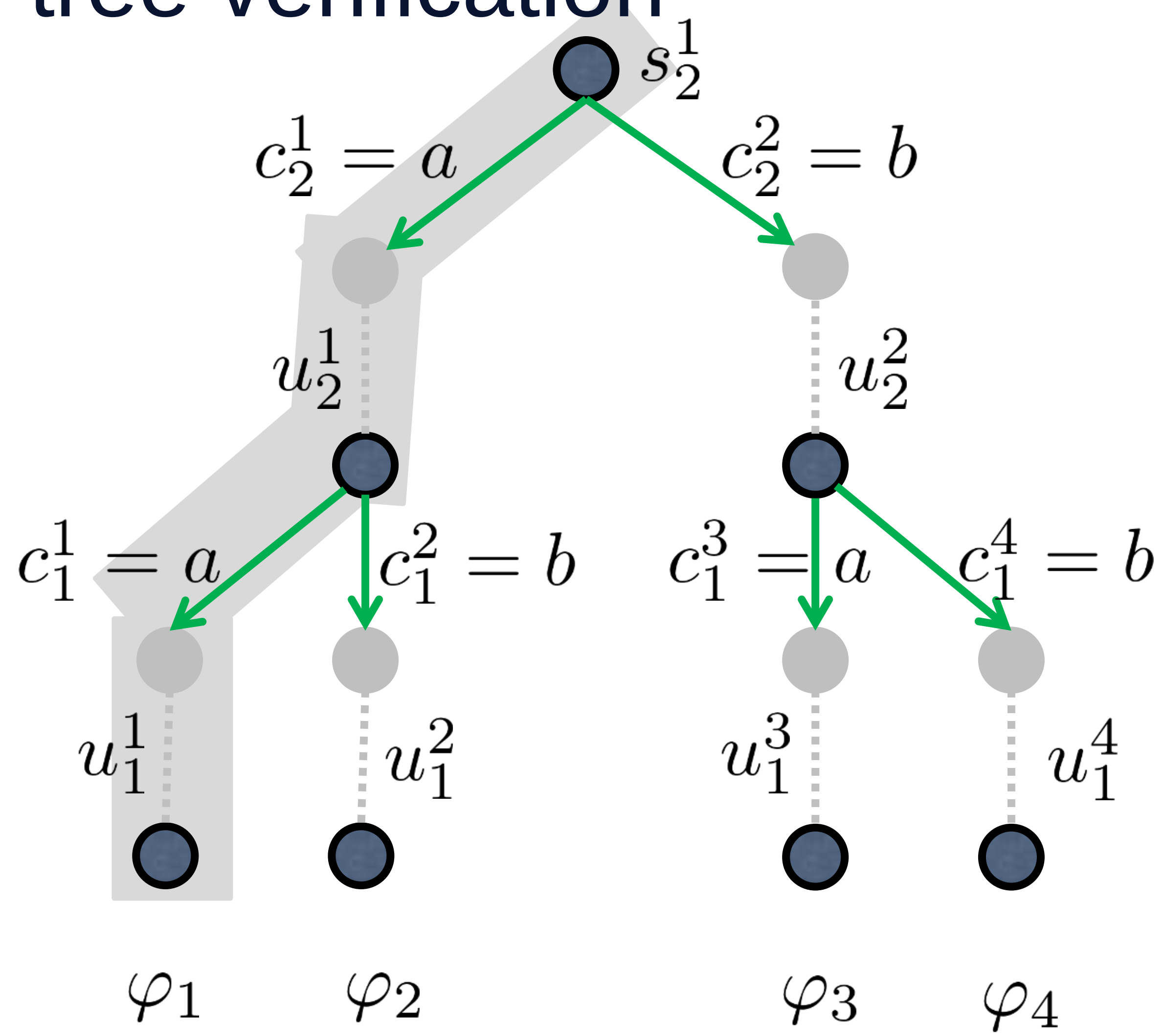
Abstract Game tree verification



Abstract Game tree verification



Abstract Game tree verification

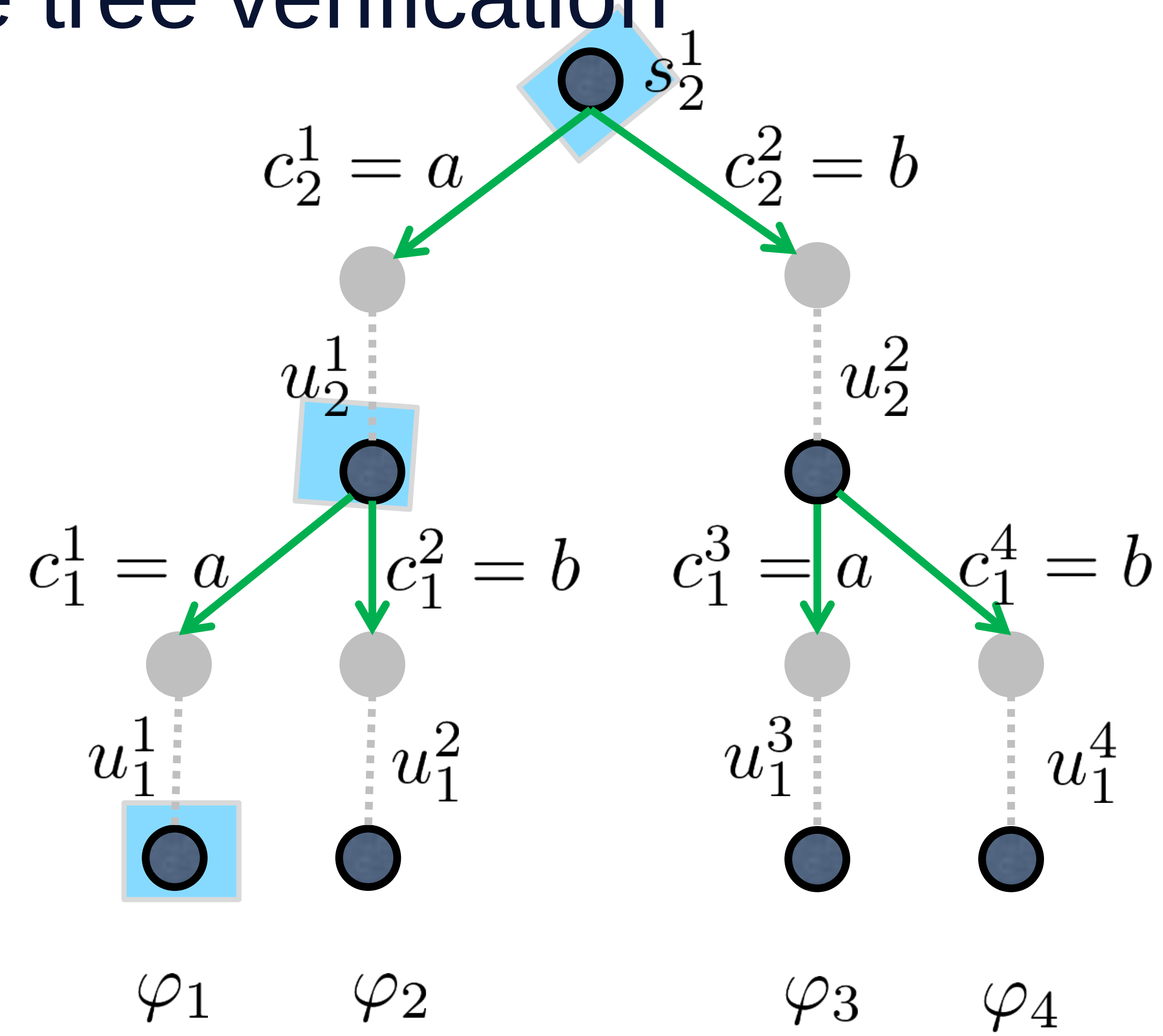


$$\varphi_1 =$$

$$\neg G(s_2^1) \wedge \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge \neg G(s_1^1) \wedge \delta(s_1^1, c_1^1, u_1^1, s_0^1) \wedge (c_1^1 = a) \wedge \neg G(s_0^1)$$

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

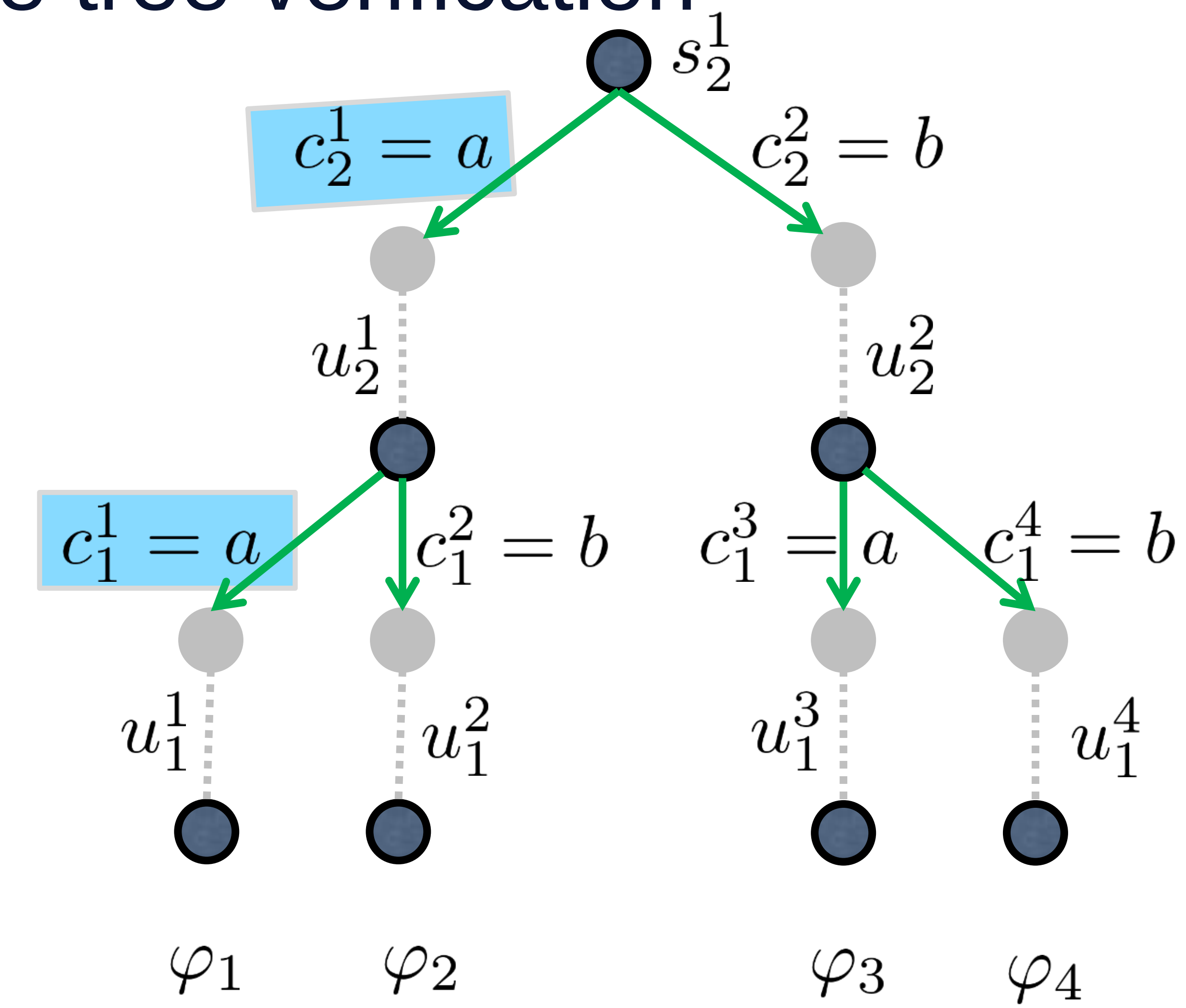
Abstract Game tree verification



$\varphi_1 =$

$$\neg G(s_2^1) \wedge \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge \neg G(s_1^1) \wedge \delta(s_1^1, c_1^1, u_1^1, s_0^1) \wedge (c_1^1 = a) \wedge \neg G(s_0^1$$

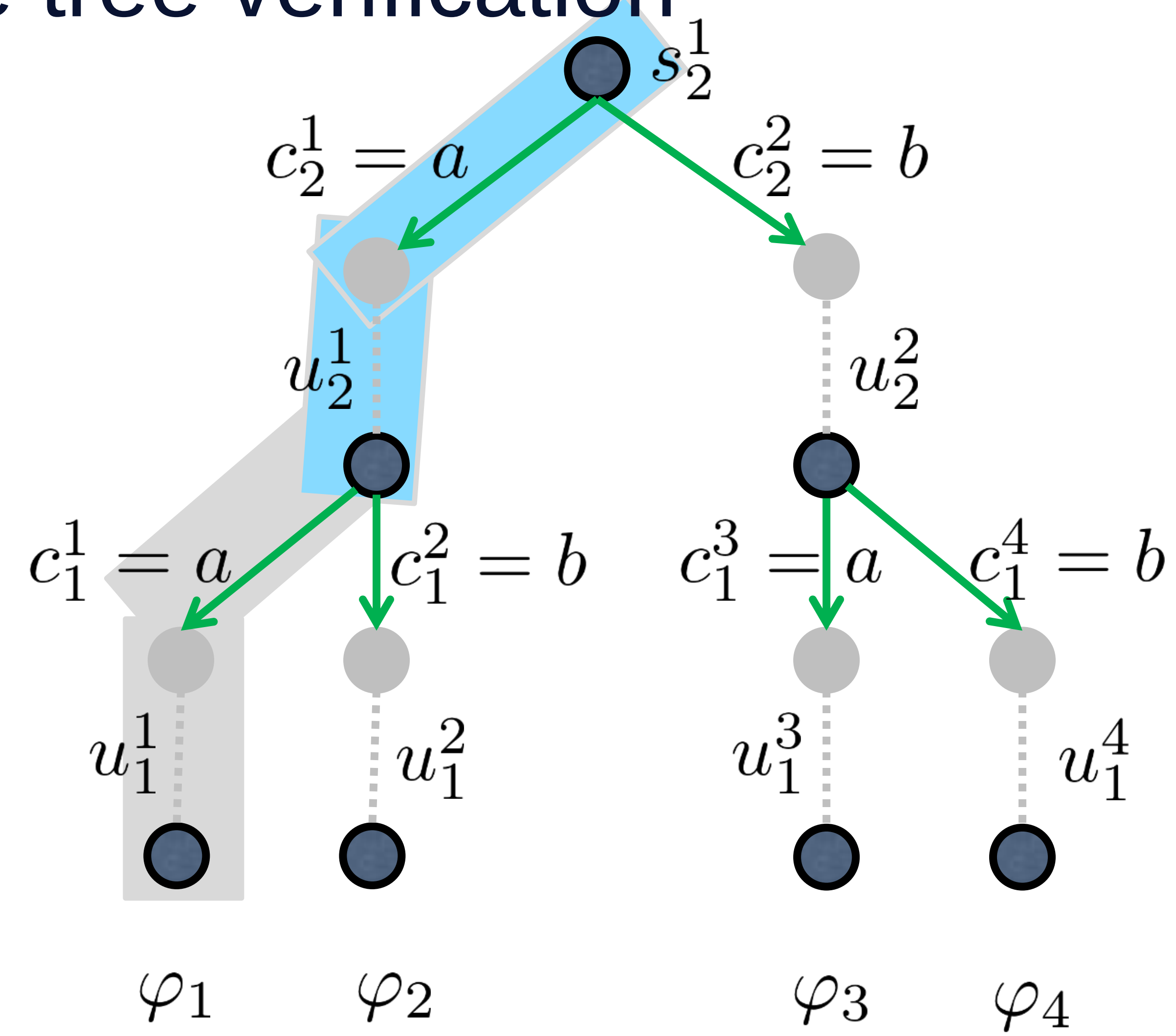
Abstract Game tree verification



$$\varphi_1 =$$

$$\neg G(s_2^1) \wedge \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge \neg G(s_1^1) \wedge \delta(s_1^1, c_1^1, u_1^1, s_0^1) \wedge (c_1^1 = a) \wedge \neg G(s_0^1)$$

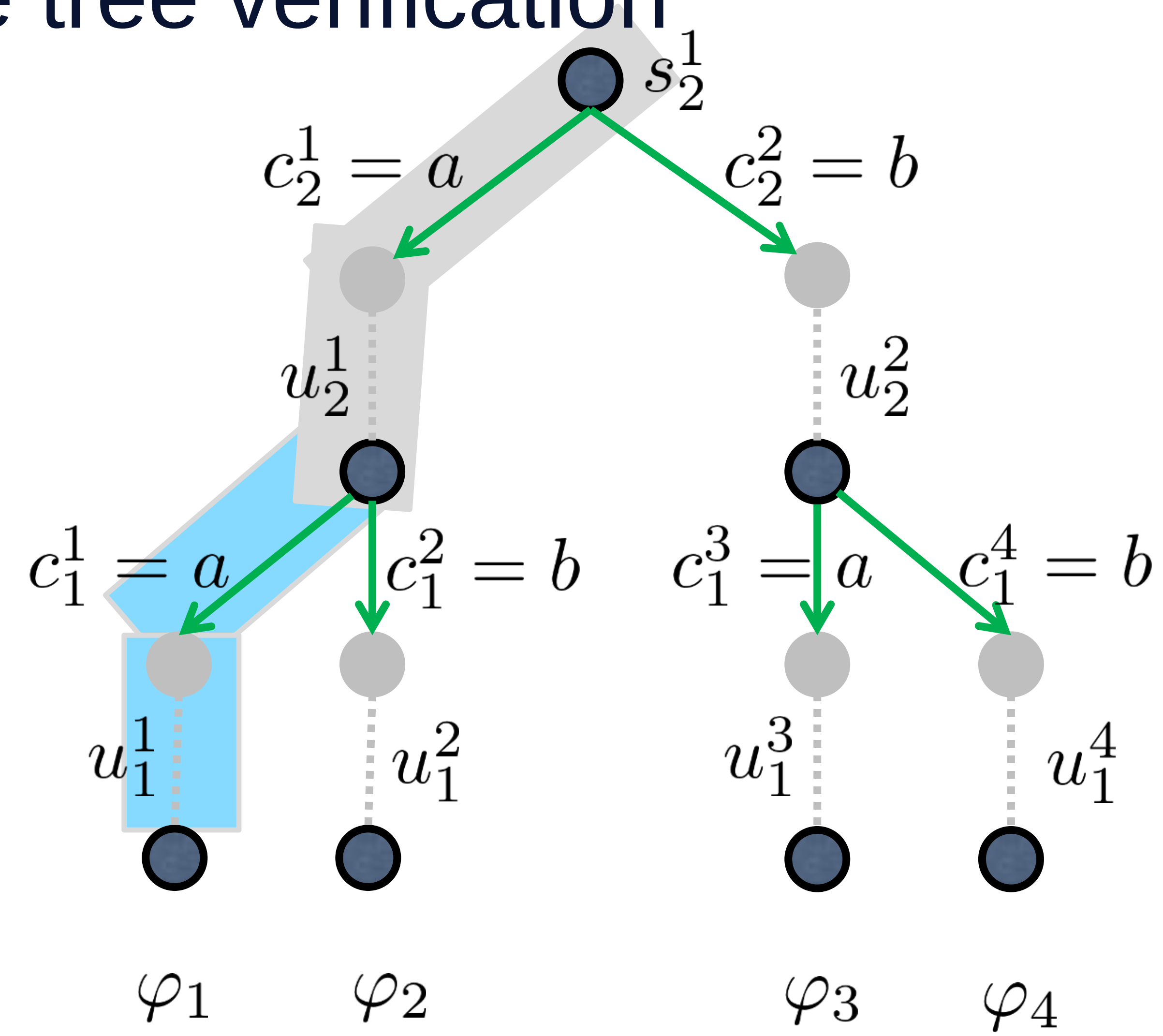
Abstract Game tree verification



$\varphi_1 =$

$$\neg G(s_2^1) \wedge \boxed{\delta(s_2^1, c_2^1, u_2^1, s_1^1)} \wedge (c_2^1 = a) \wedge \neg G(s_1^1) \wedge \delta(s_1^1, c_1^1, u_1^1, s_0^1) \wedge (c_1^1 = a) \wedge \neg G(s_0^1)$$

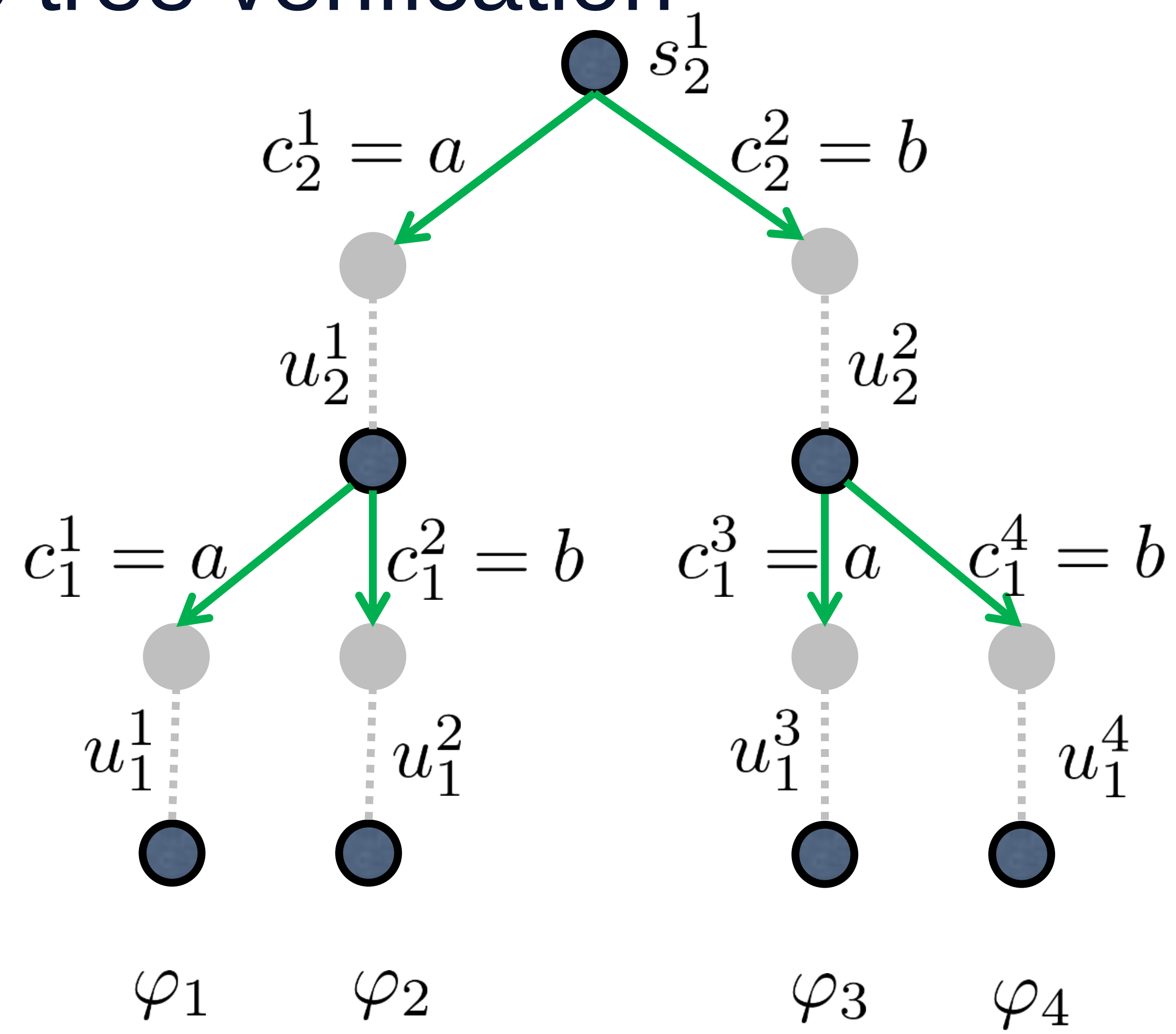
Abstract Game tree verification



$$\varphi_1 =$$

$$\neg G(s_2^1) \wedge \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge \neg G(s_1^1) \wedge \boxed{\delta(s_1^1, c_1^1, u_1^1, s_0^1)} \wedge (c_1^1 = a) \wedge \neg G(s_0^1$$

Abstract Game tree verification



$\varphi_1 =$

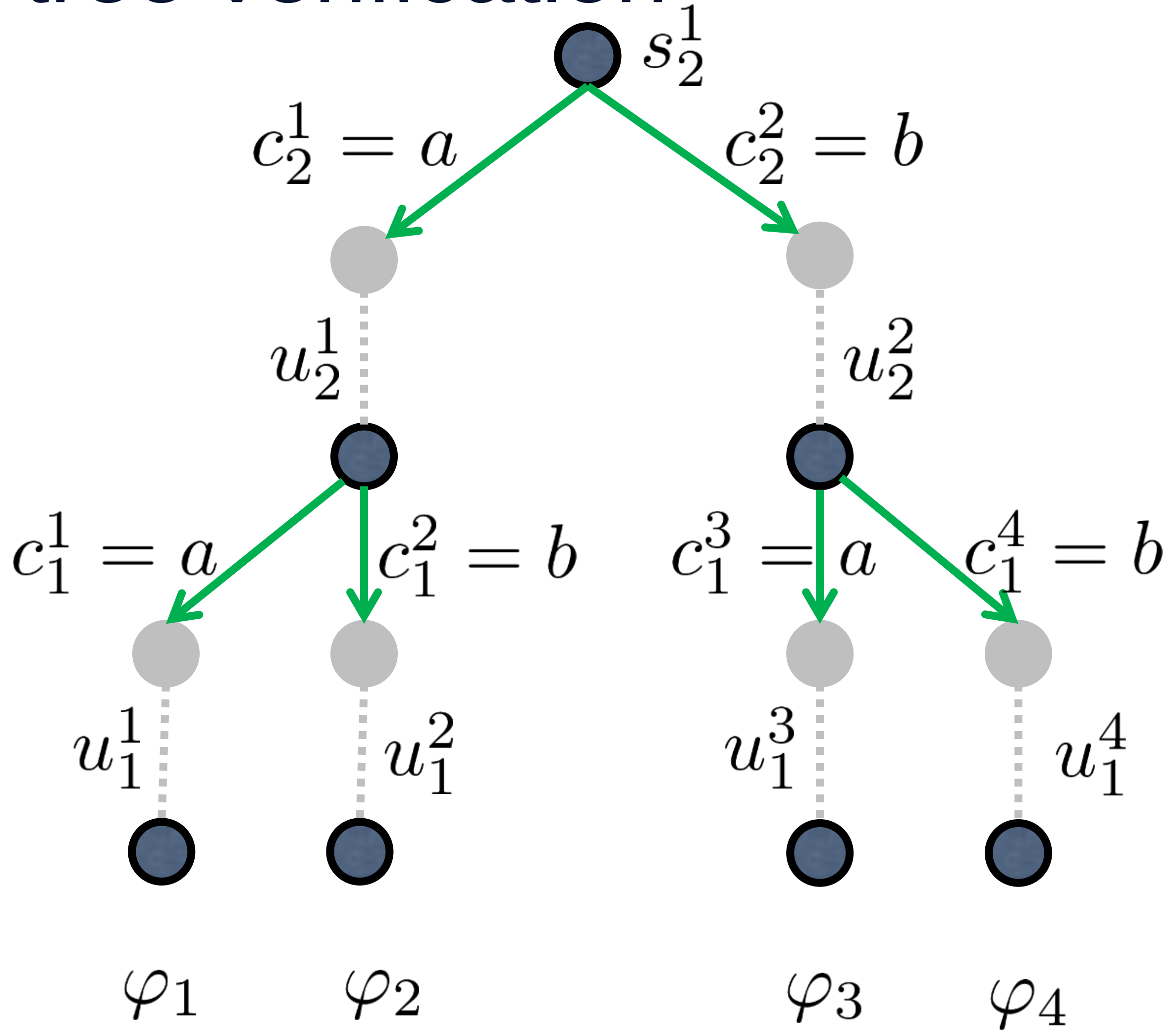
$$\neg G(s_2^1) \wedge \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge \neg G(s_1^1) \wedge \delta(s_1^1, c_1^1, u_1^1, s_0^1) \wedge (c_1^1 = a) \wedge \neg G(s_0^1)$$

$\varphi_2 = \ldots$

$\varphi_3 = \ldots$

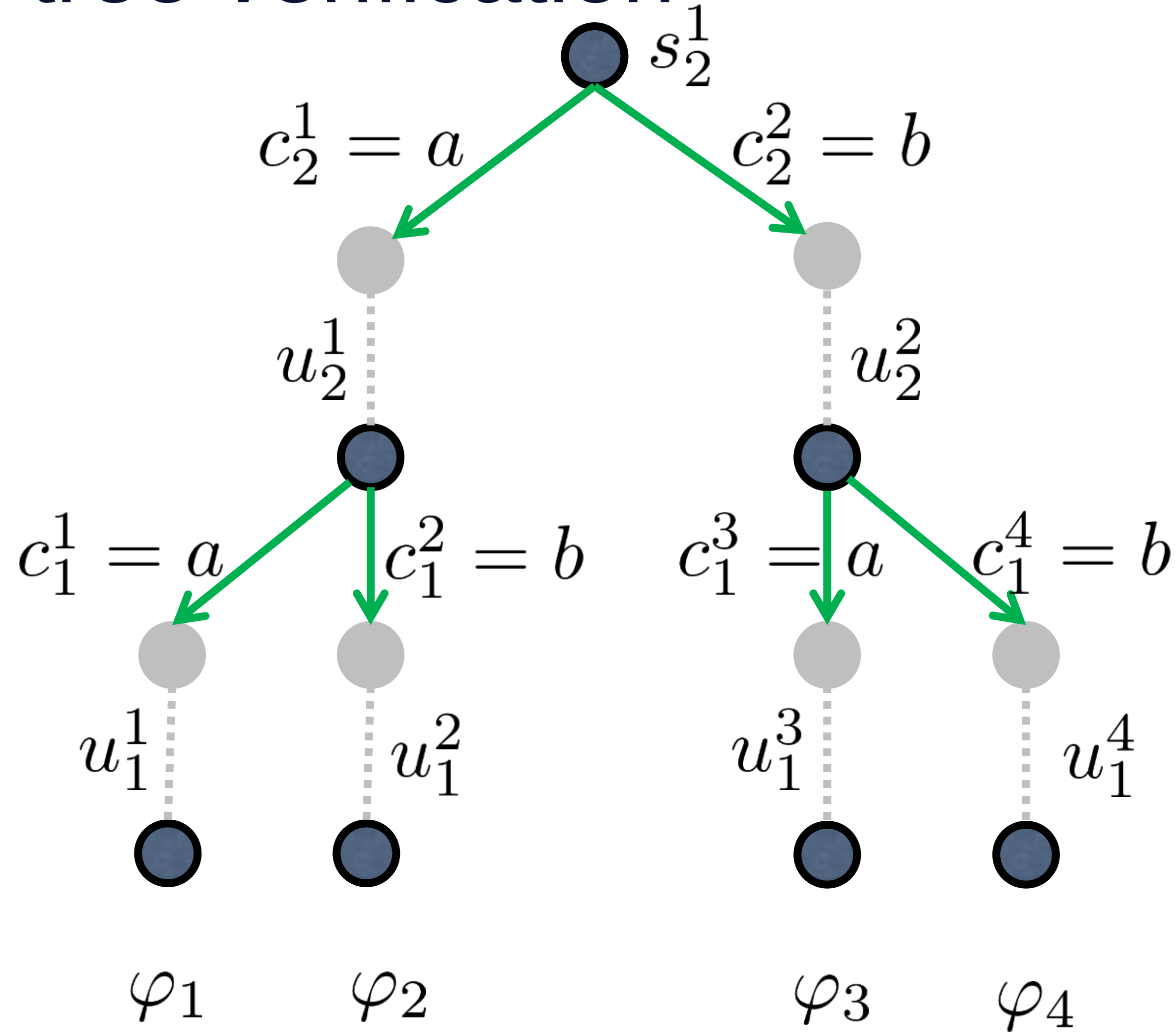
$\varphi_4 = \ldots$

Abstract Game tree verification



$$(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4$$

Abstract Game tree verification



$$(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4$$

is

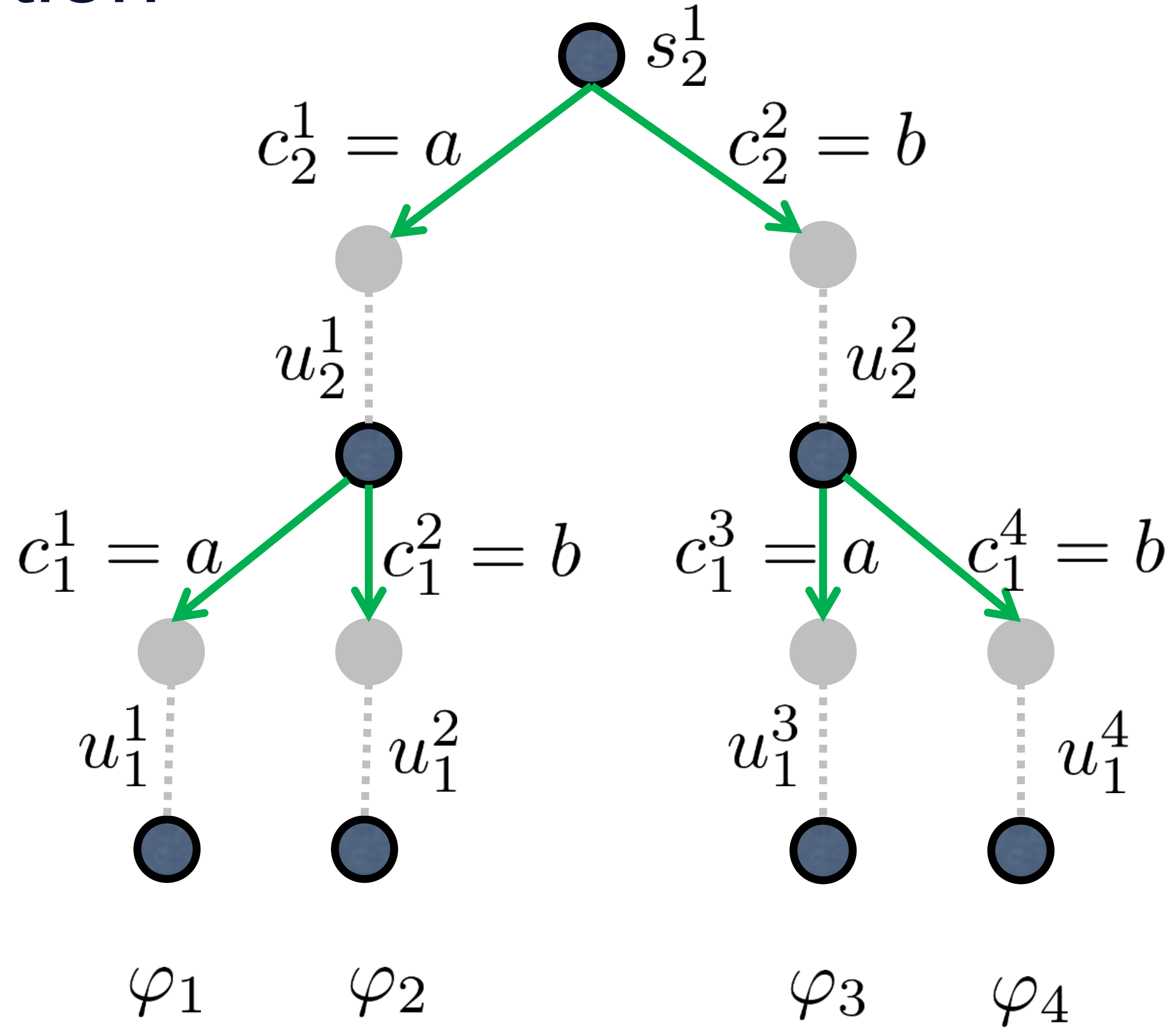
UNSAT

(by correctness of EvaSolver)

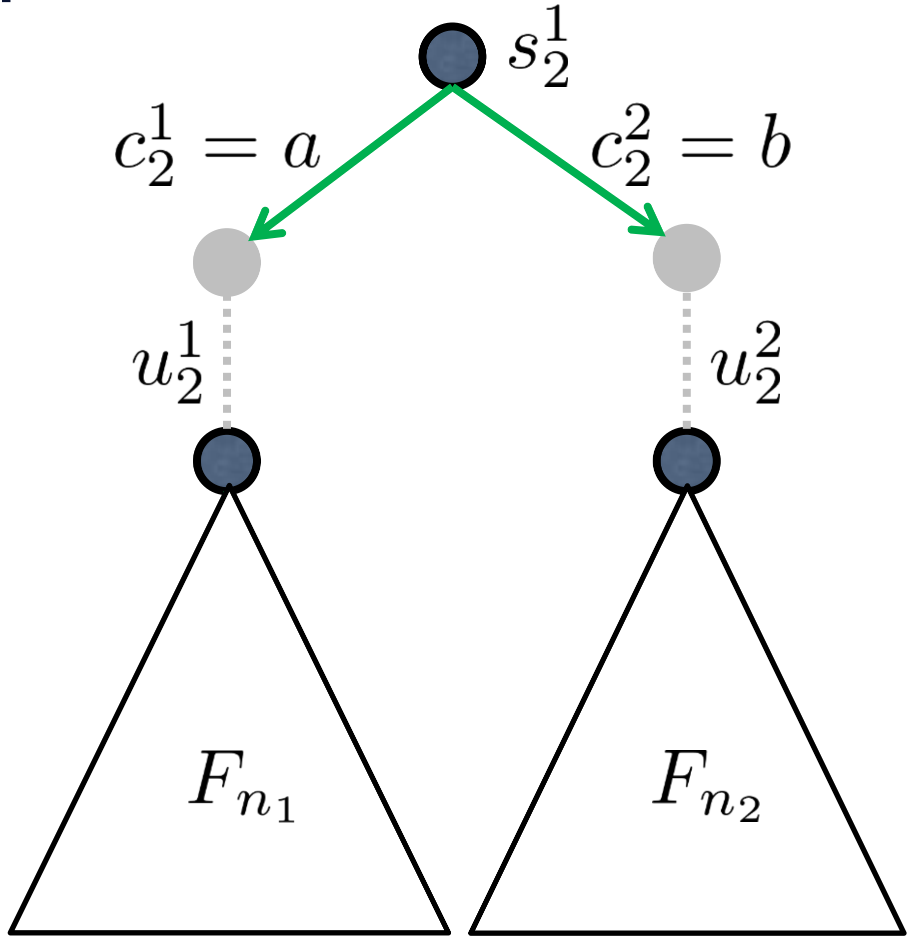
How to extract a winning strategy

Partitioning operation

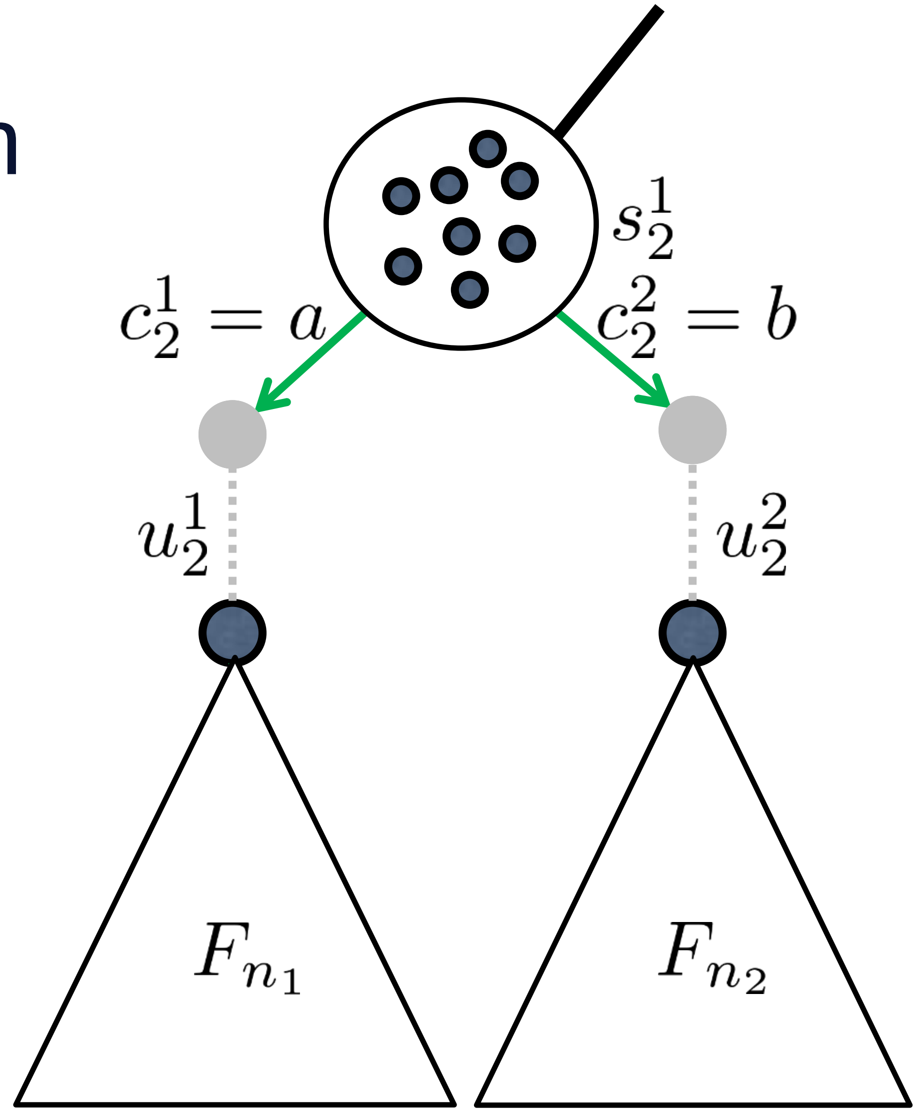
Strategy extraction



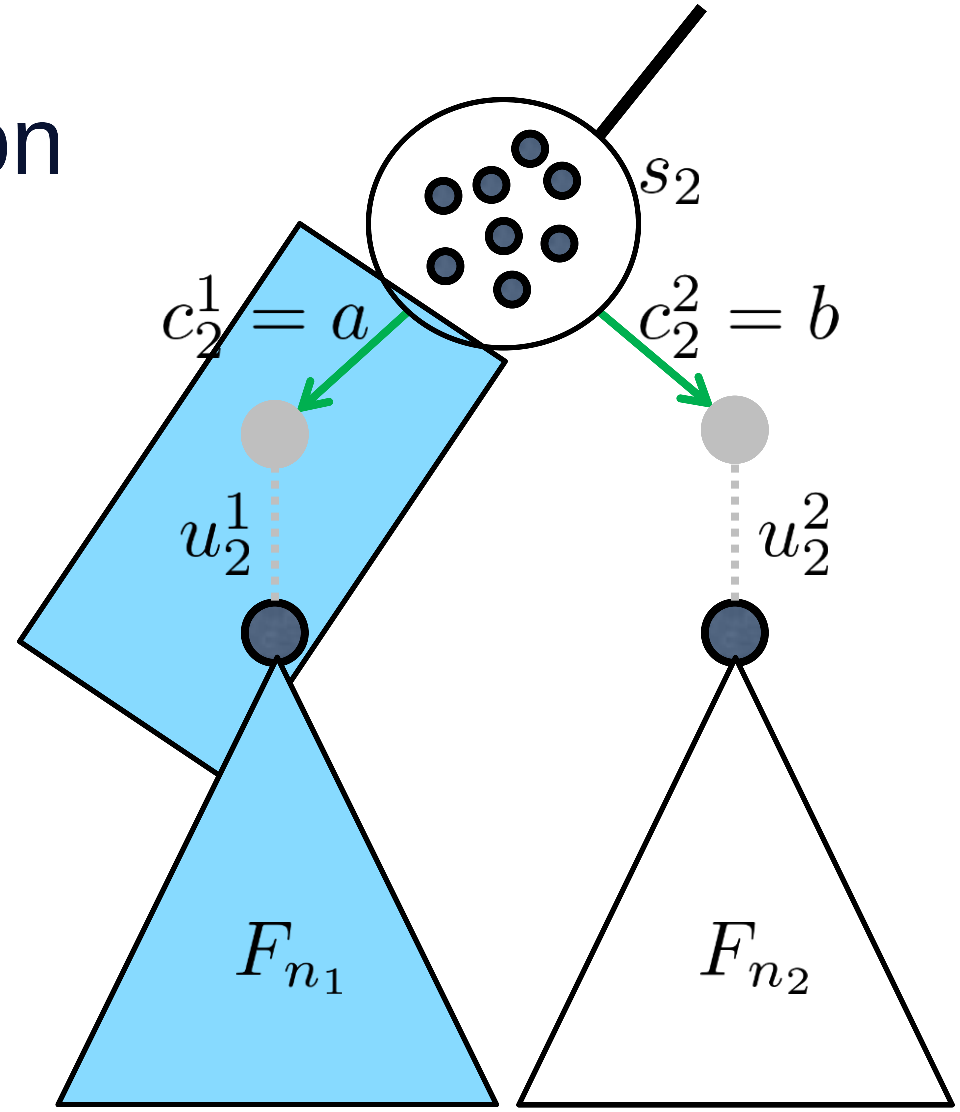
Strategy extraction



Strategy extraction

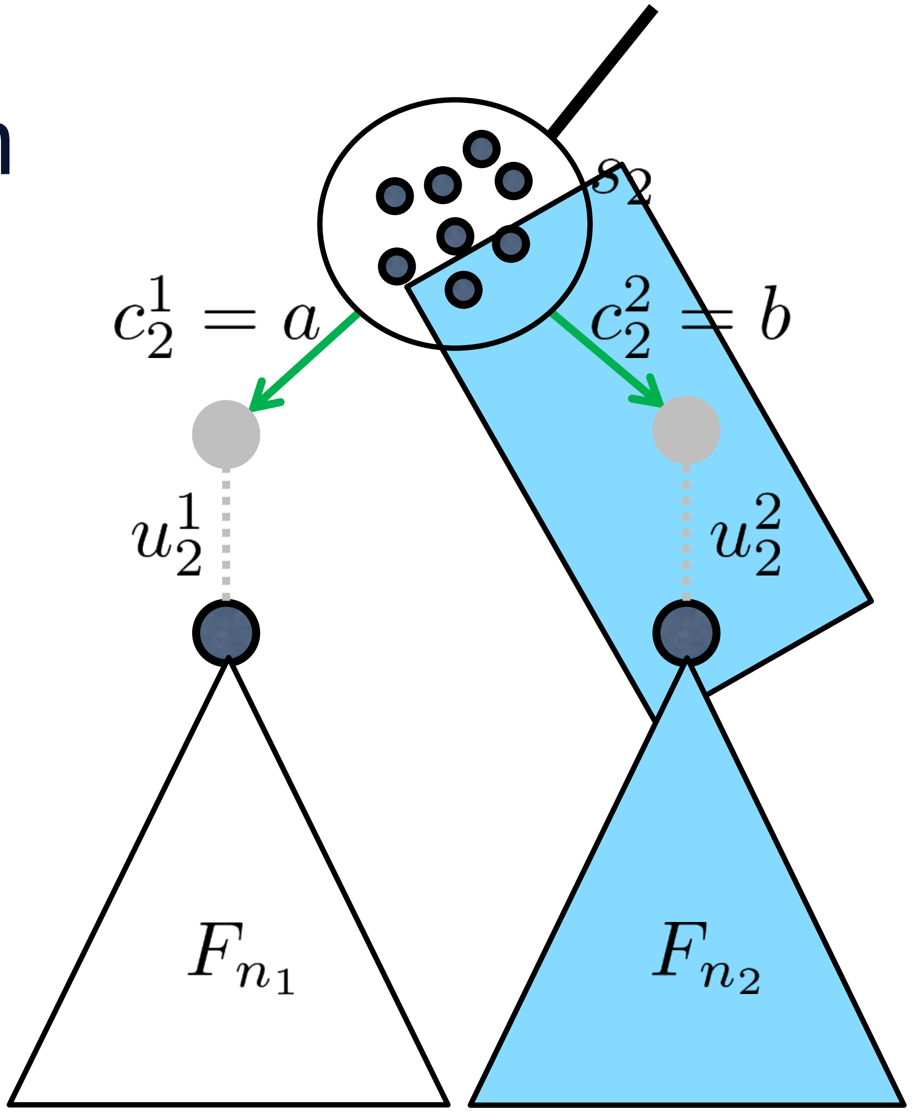


Strategy extraction



$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

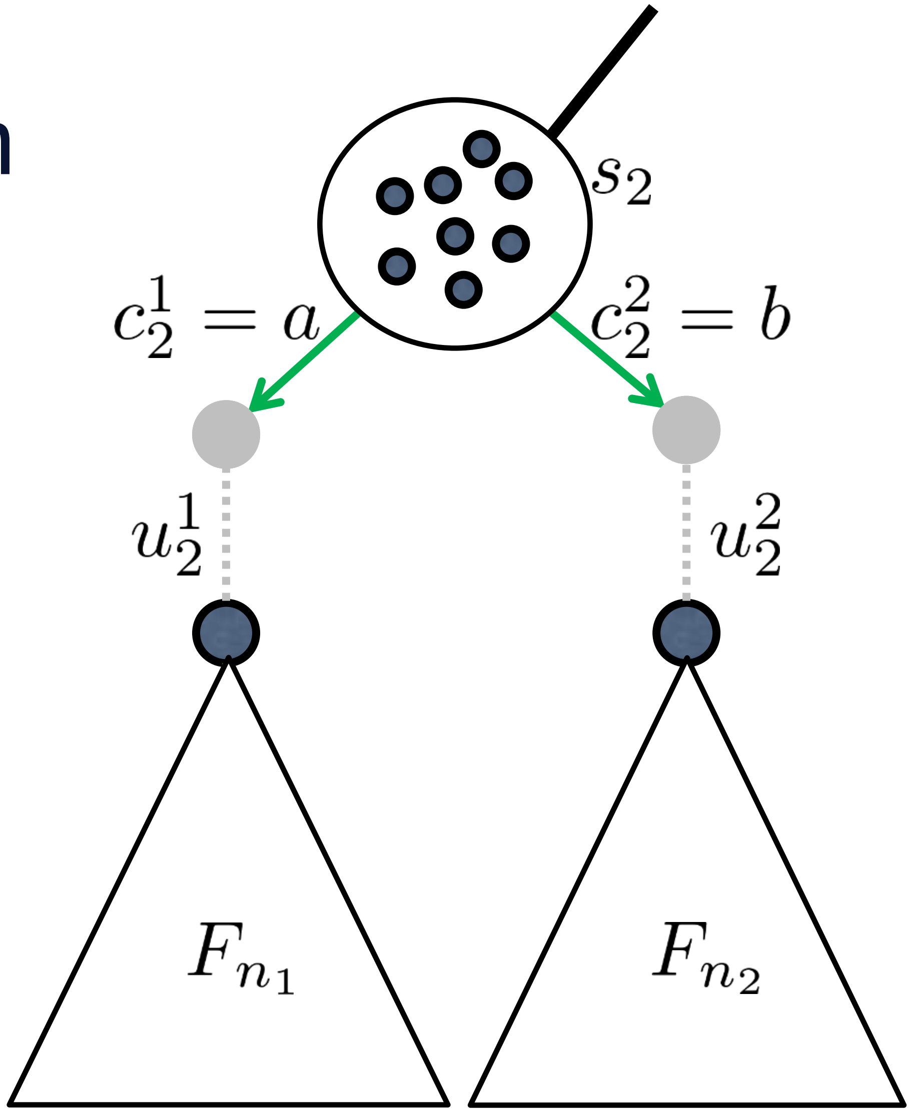
Strategy extraction



$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

Strategy extraction



$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b]$$

Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b] = \perp$$

Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b] = \perp$$

$$vars(F_a) \cap vars(F_b) = s_2^1$$

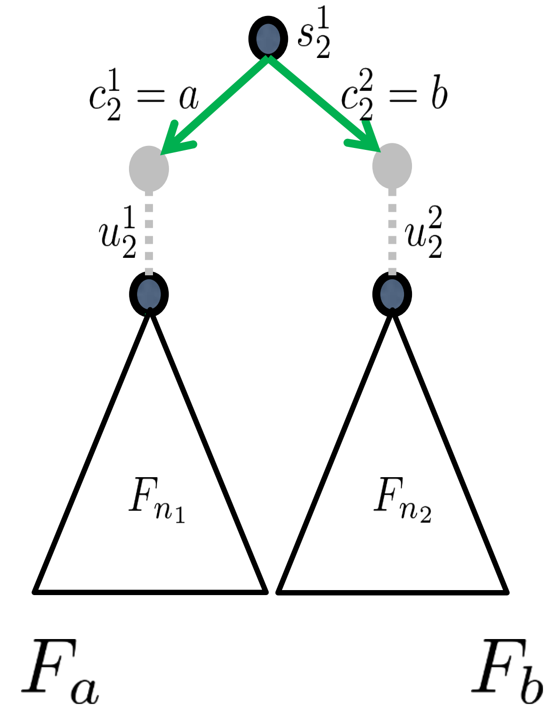
Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b] = \bot$$

$$vars(F_a) \cap vars(F_b) = s_2^1$$



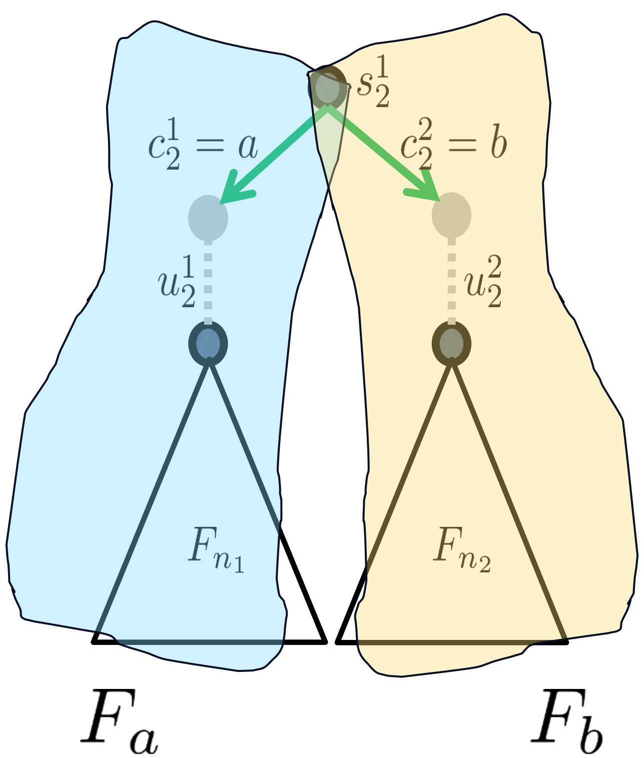
Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b] = \perp$$

$$vars(F_a) \cap vars(F_b) = s_2^1$$



Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

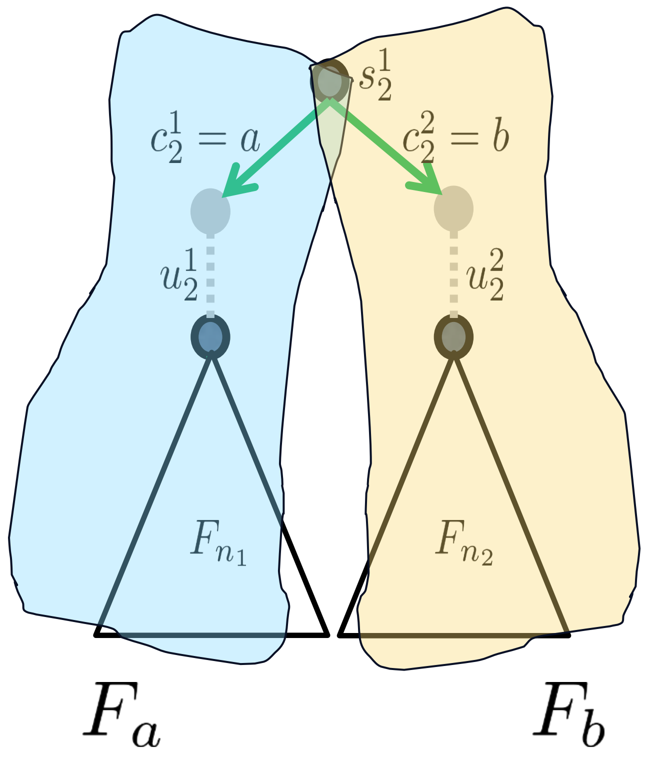
$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b] = \bot$$

$$vars(F_a) \cap vars(F_b) = s_2^1$$

$\mathcal{I}(s_2^1) = Interpolant(F_a, F_b)$:

- $F_a \implies \mathcal{I}$
- $\mathcal{I} \wedge F_b = \bot$



Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

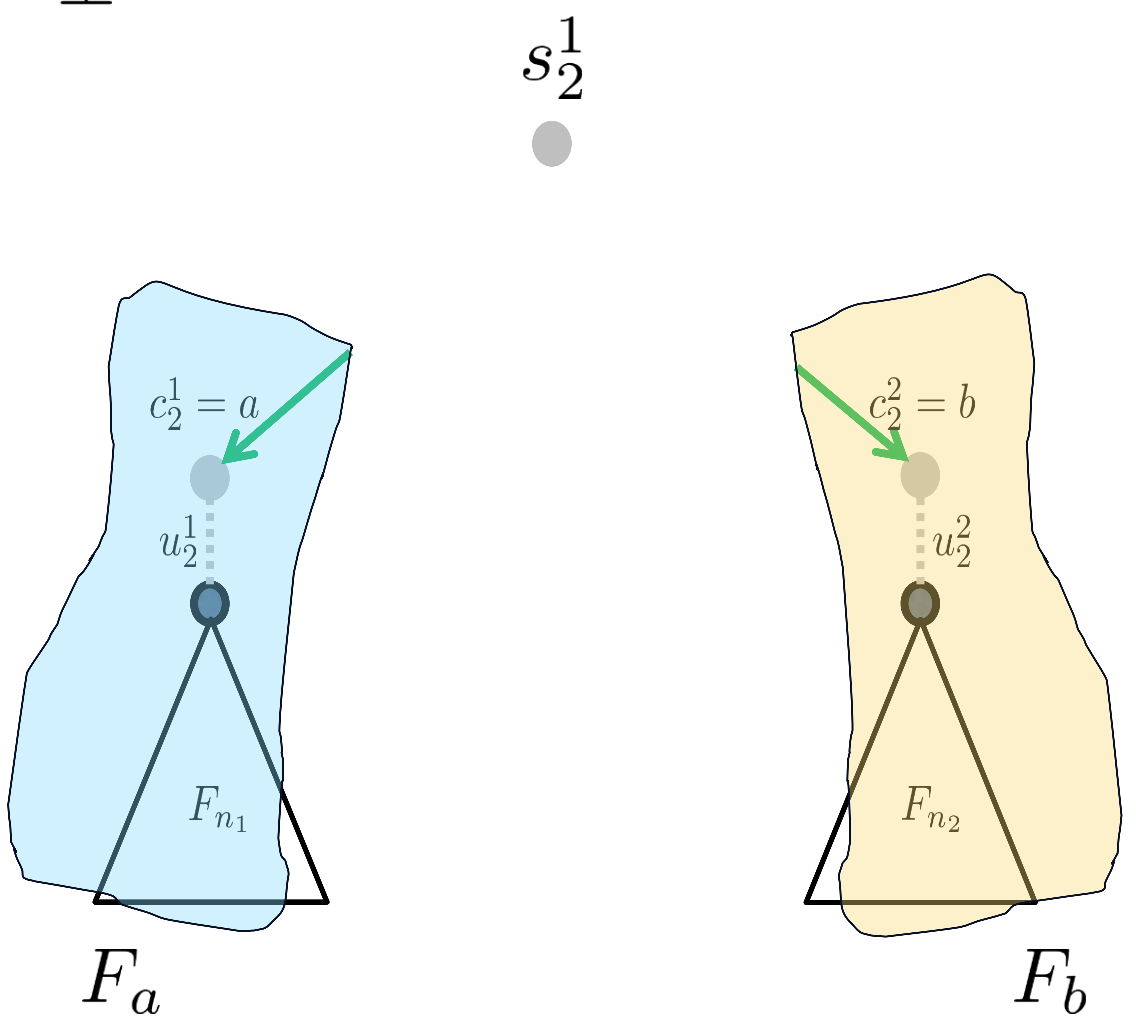
$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b] = \bot$$

$$vars(F_a) \cap vars(F_b) = s_2^1$$

$\mathcal{I}(s_2^1) = Interpolant(F_a, F_b)$:

- $F_a \implies \mathcal{I}$
- $\mathcal{I} \wedge F_b = \bot$



Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a) \wedge F_{n_1}.$$

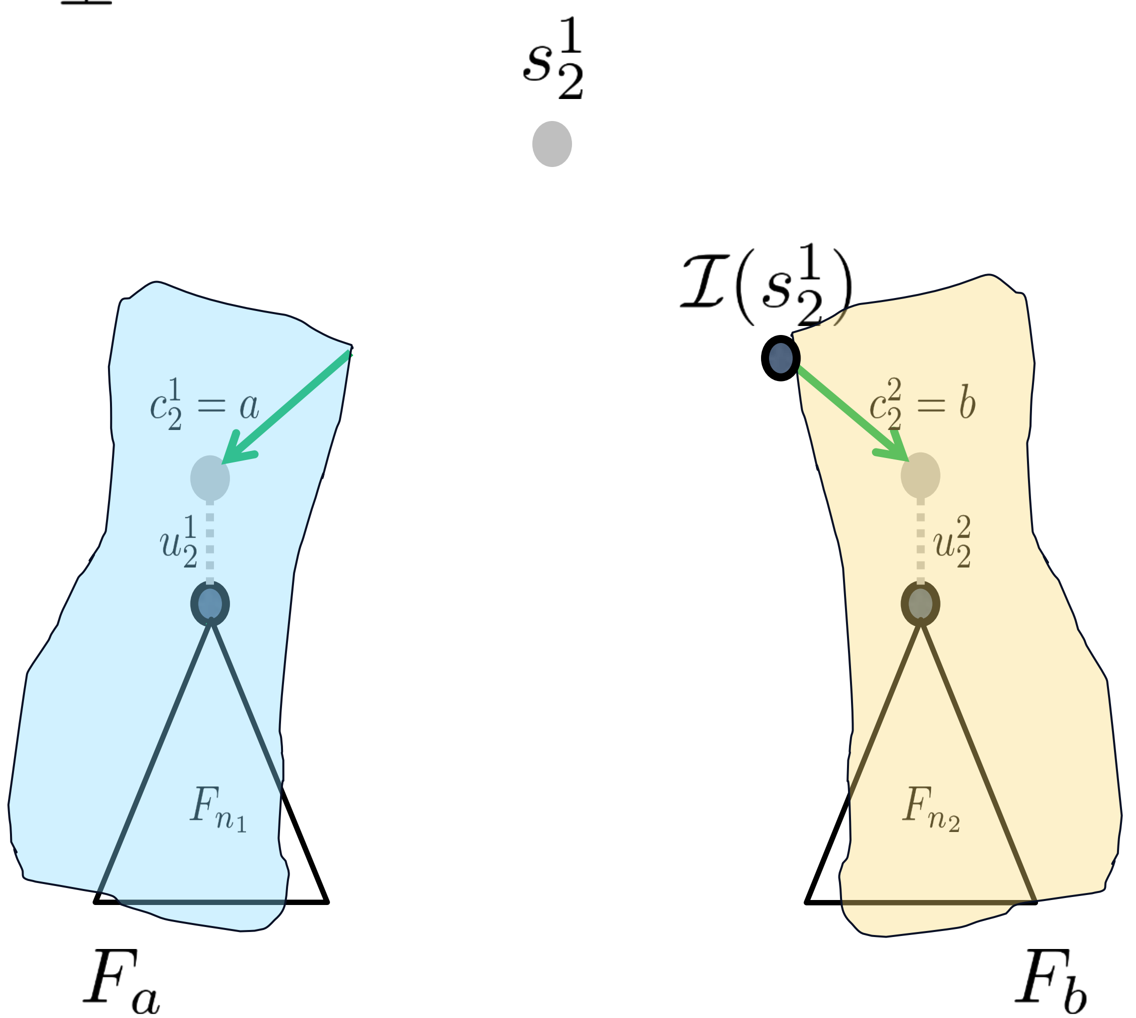
$$F_b = (s_2^1 = I) \wedge \neg G(s_2^1) \wedge \delta(s_2^1, c_2^2, u_2^2, s_1^2) \wedge (c_2^2 = b) \wedge F_{n_2}.$$

$$[(s_2^1 = I) \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3 \wedge \varphi_4] \equiv [F_a \wedge F_b] = \bot$$

$$vars(F_a) \cap vars(F_b) = s_2^1$$

$\mathcal{I}(s_2^1) = Interpolant(F_a, F_b)$:

- $F_a \implies \mathcal{I}$
- $\mathcal{I} \wedge F_b = \bot$



Strategy extraction

$$F_a = \delta(s_2^1, c_2^1, u_2^1, s_1^1) \land (c_2^1 = a) \land F_{n_1}.$$

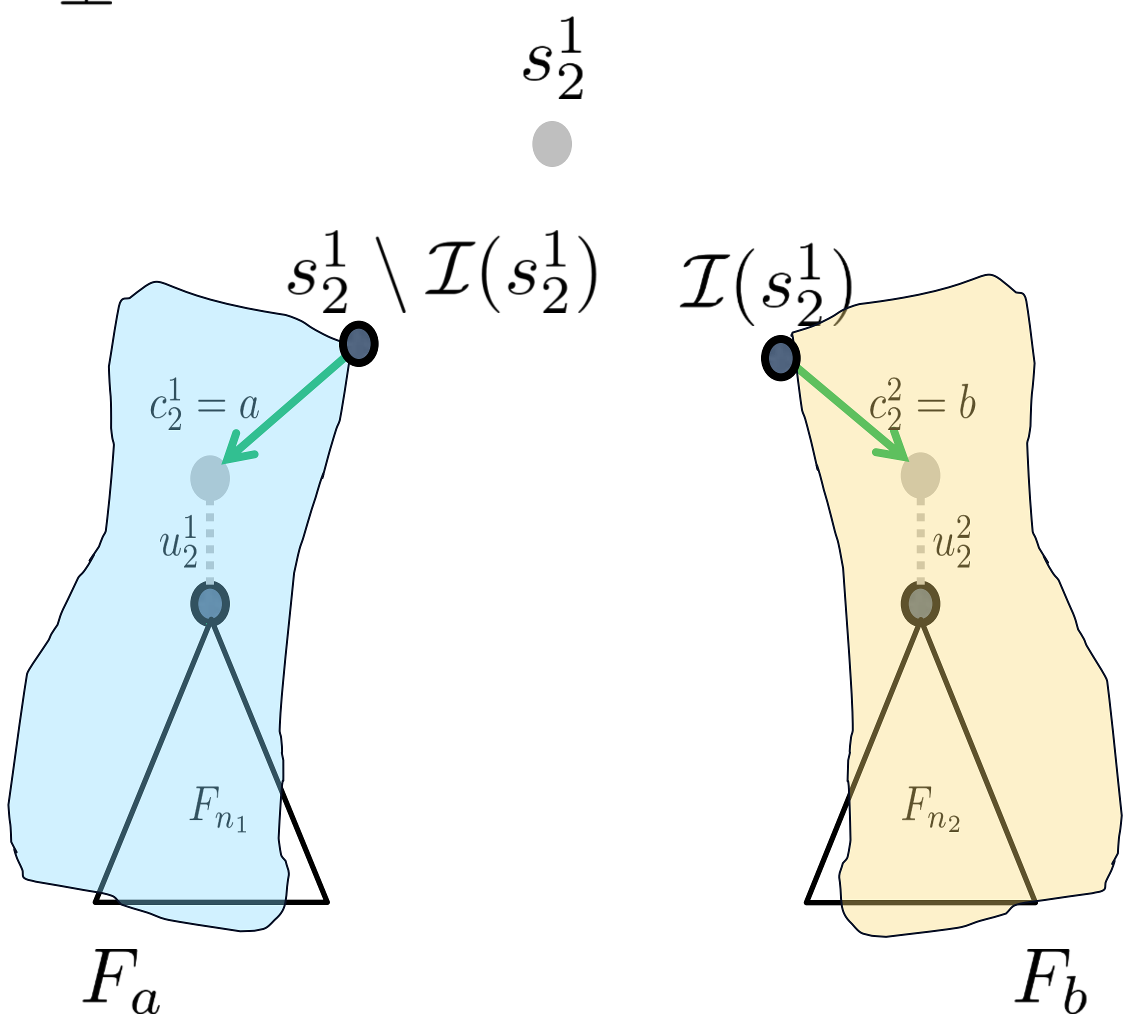
$$F_b = (s_2^1 = I) \land \neg G(s_2^1) \land \delta(s_2^1, c_2^2, u_2^2, s_1^2) \land (c_2^2 = b) \land F_{n_2}.$$

$$[(s_2^1 = I) \land \varphi_1 \land \varphi_2 \land \varphi_3 \land \varphi_4] \equiv [F_a \land F_b] = \bot$$

$$vars(F_a) \cap vars(F_b) = s_2^1$$

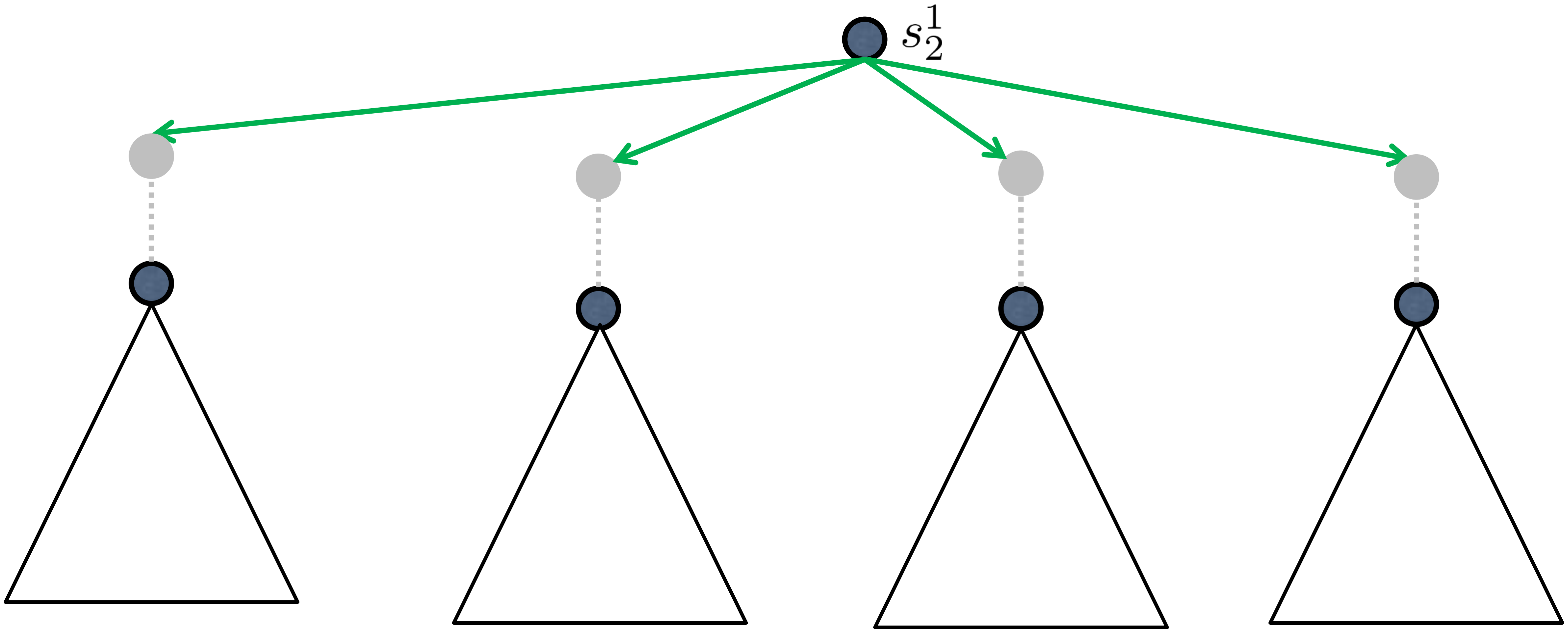
$\mathcal{I}(s_2^1) = Interpolant(F_a, F_b)$:

- $F_a \implies \mathcal{I}$
- $\mathcal{I} \land F_b = \bot$

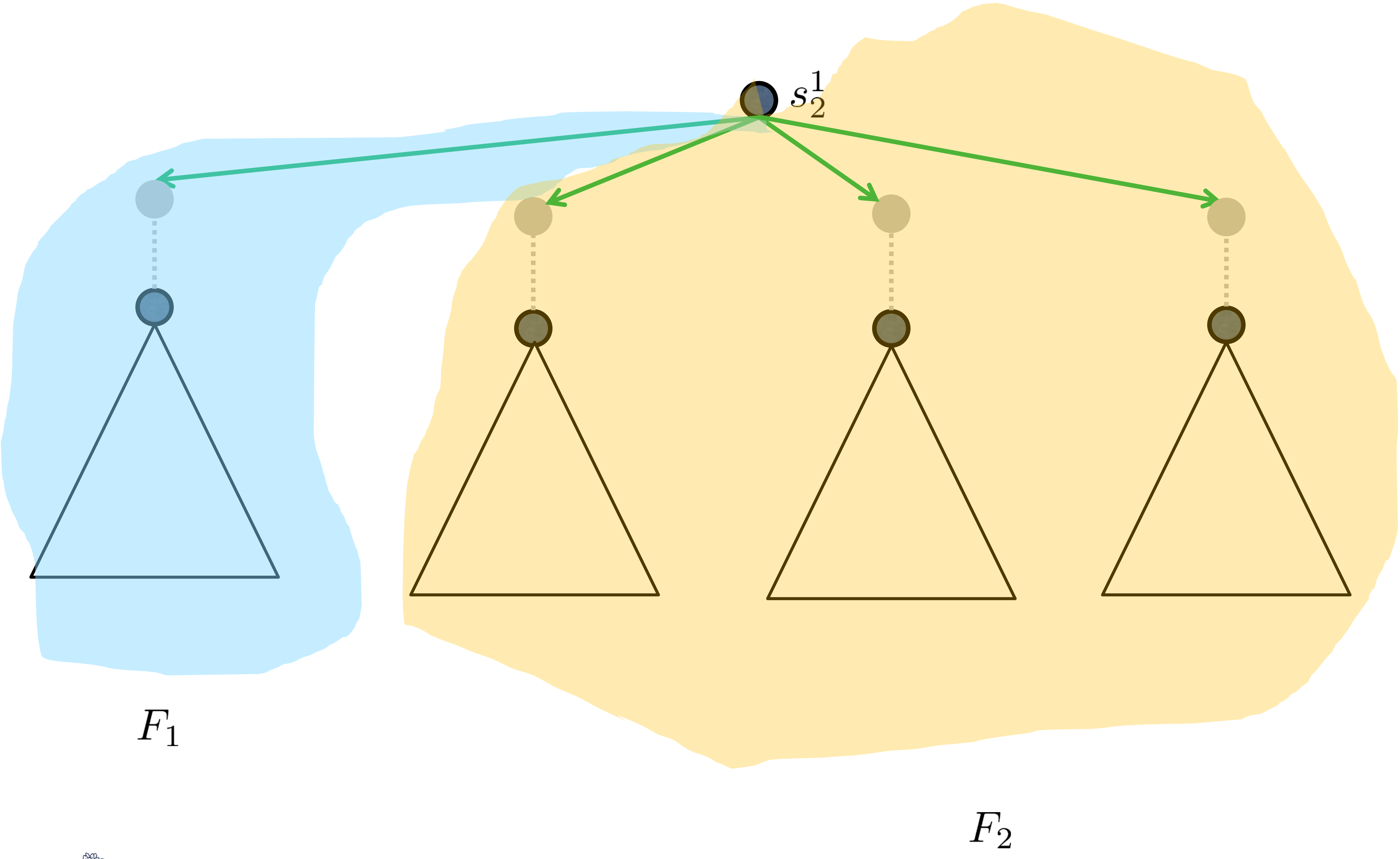


UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

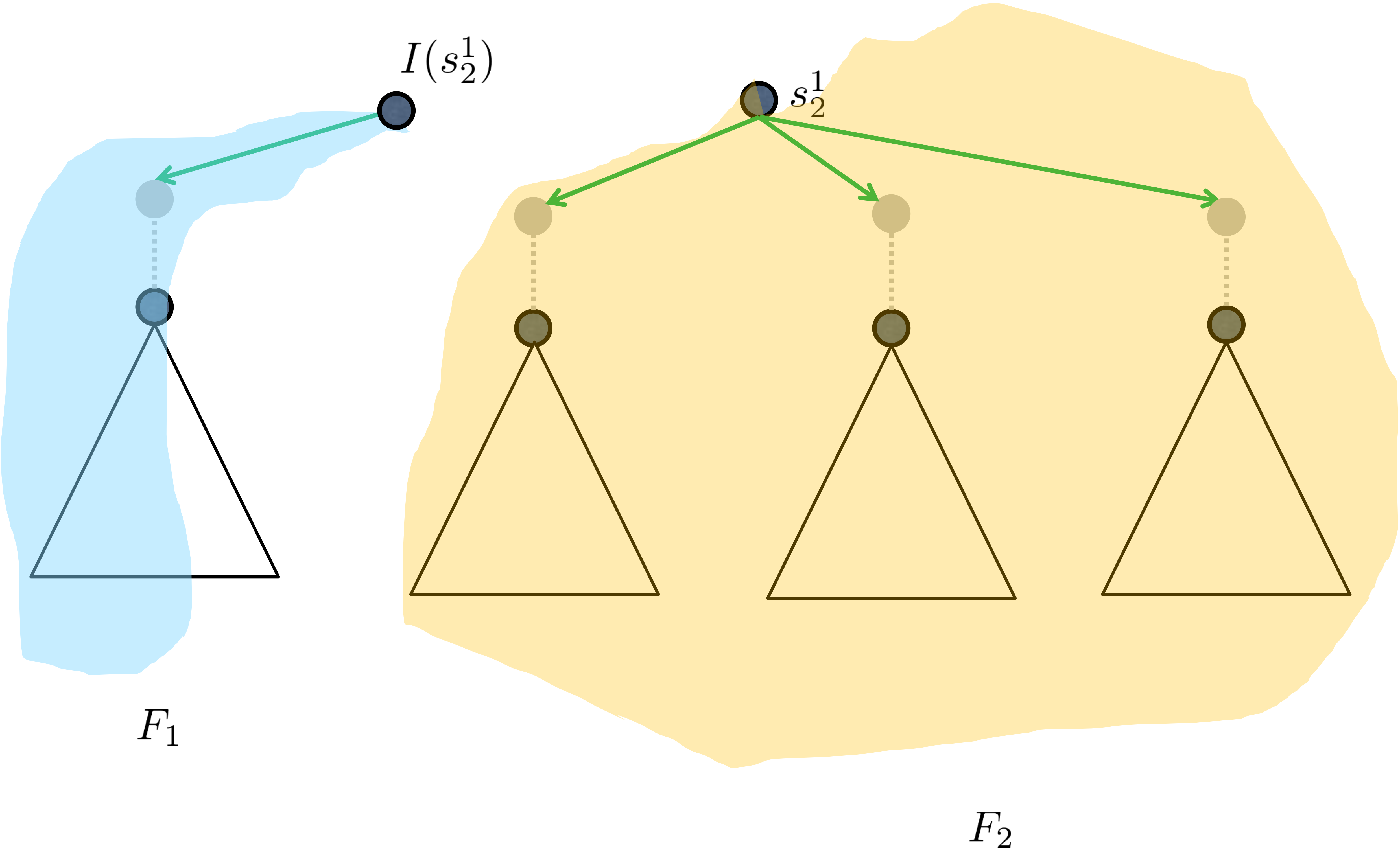
Strategy extraction



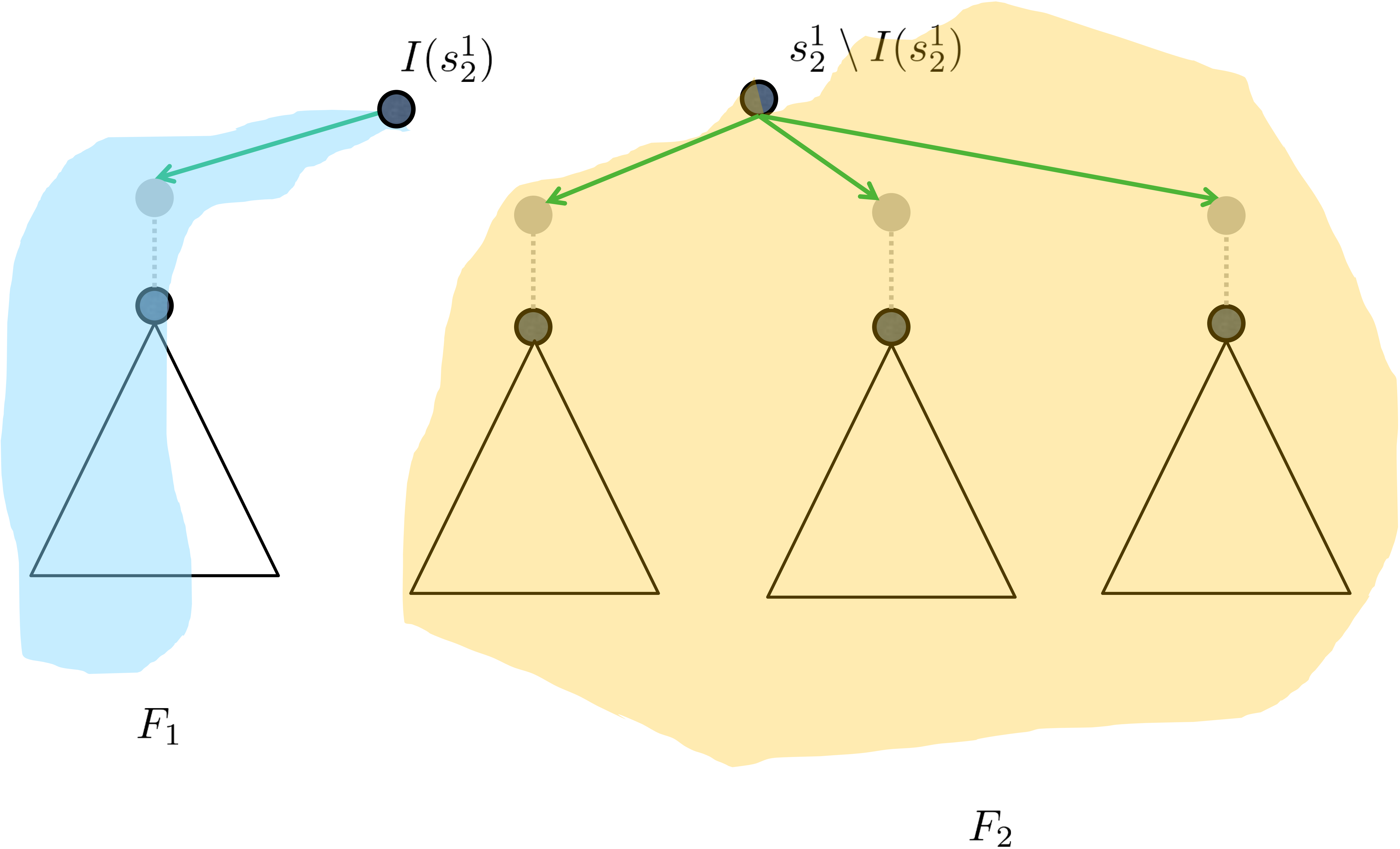
Strategy extraction



Strategy extraction



Strategy extraction

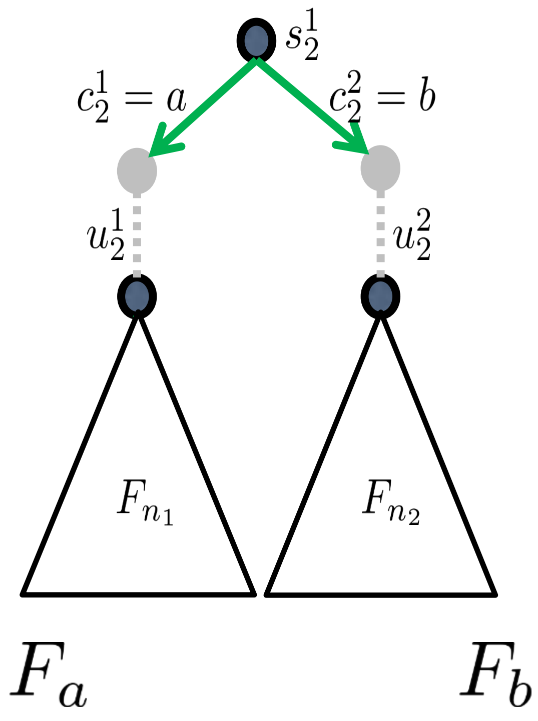


Size of interpolants

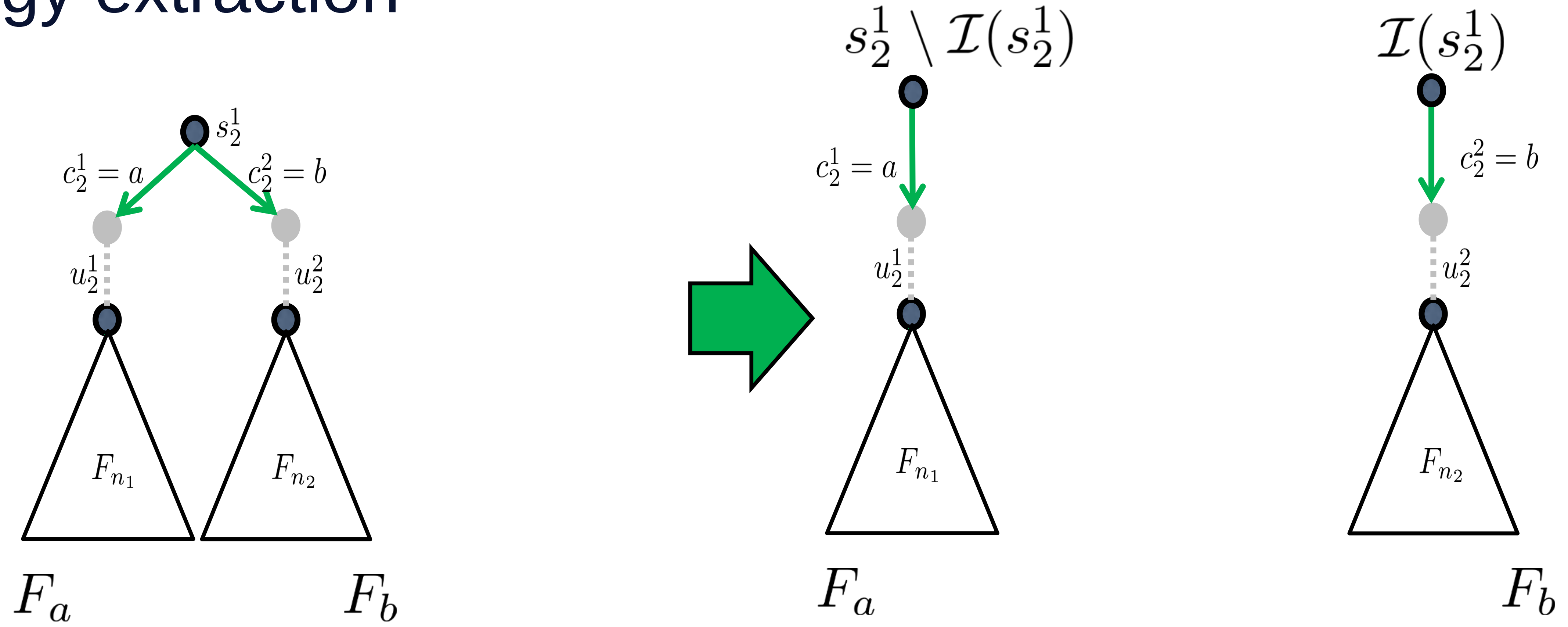
Size of interpolants is reasonable (tomorrow's talk).

Next state operation

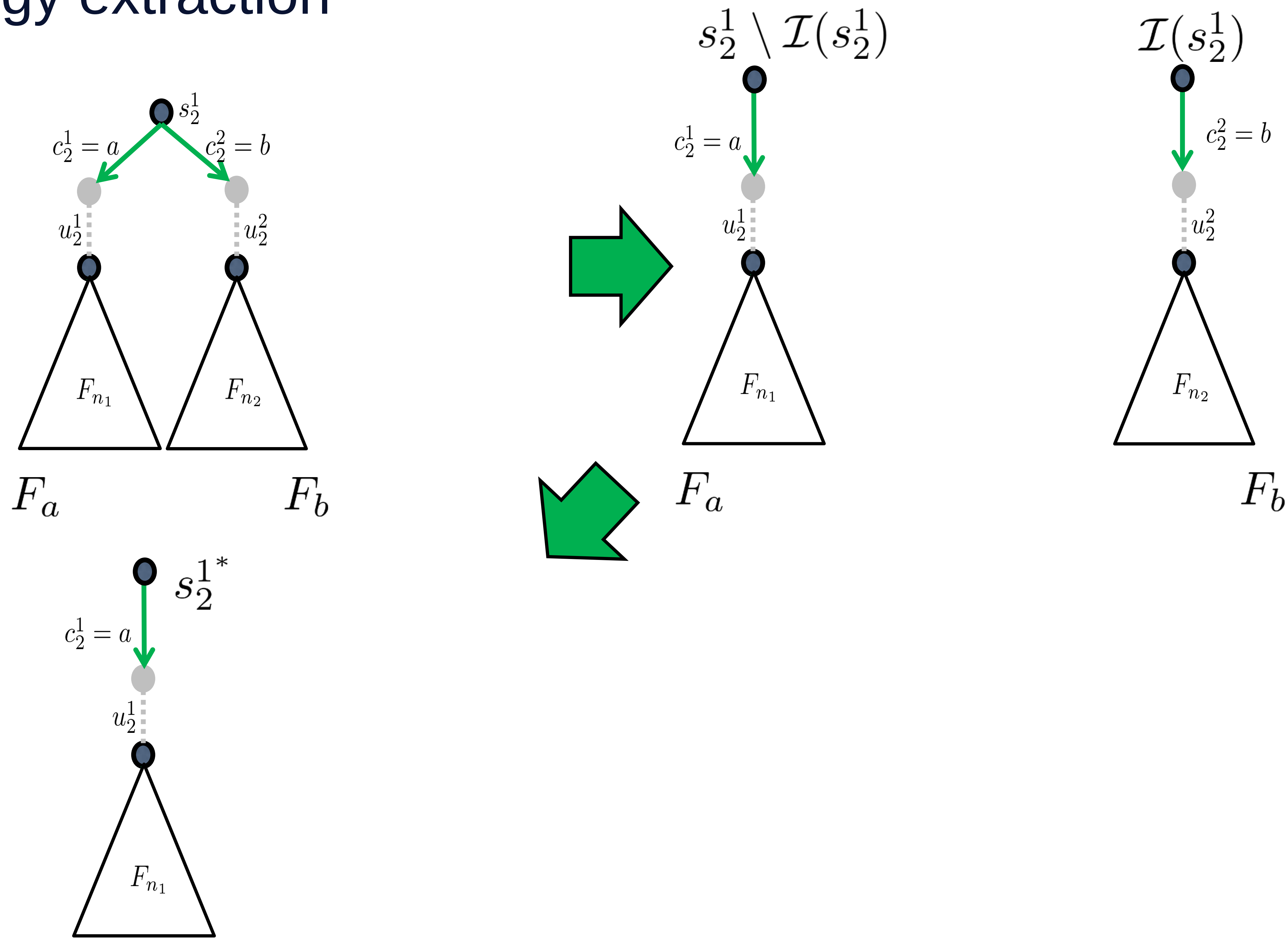
Strategy extraction



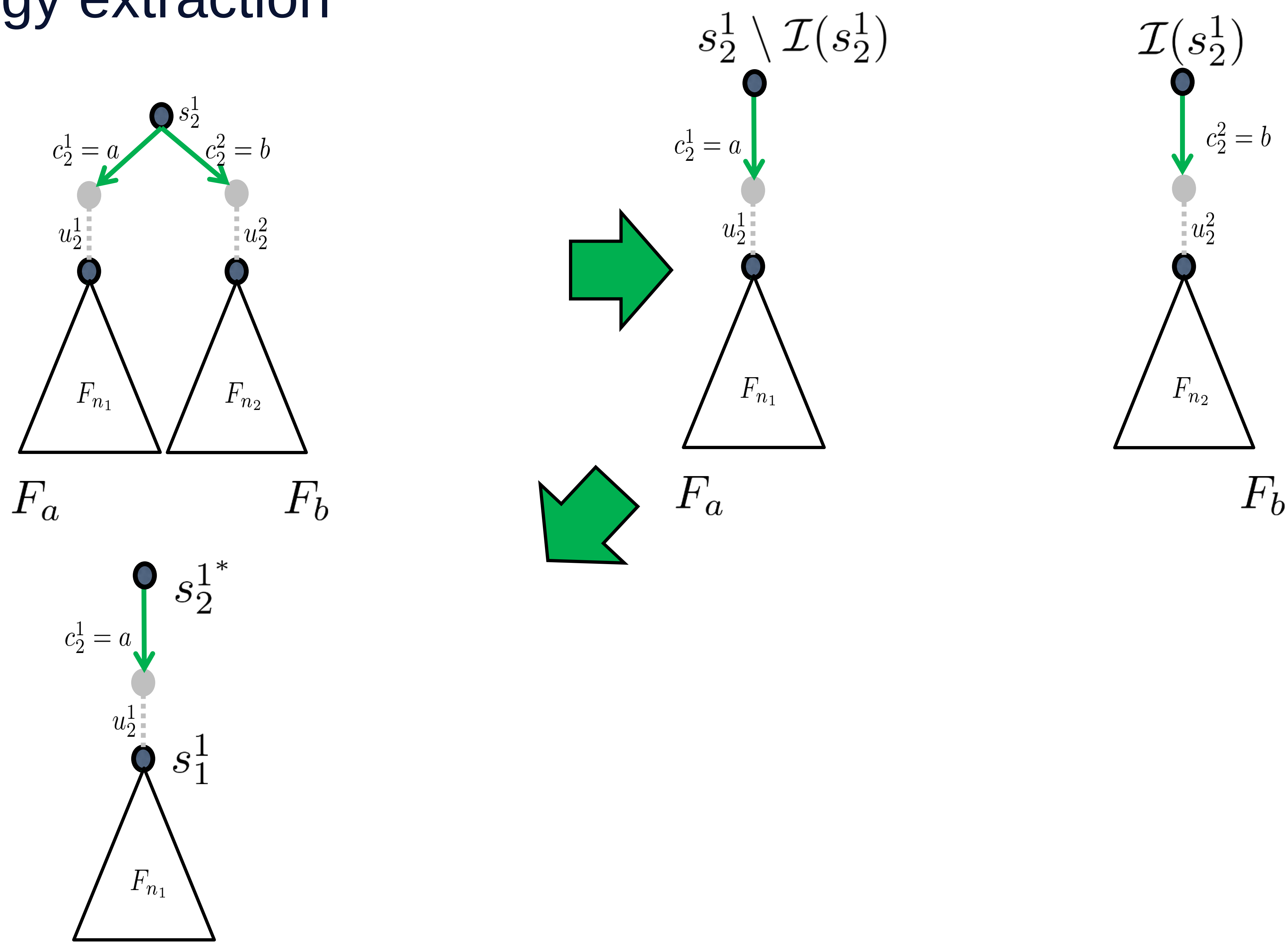
Strategy extraction



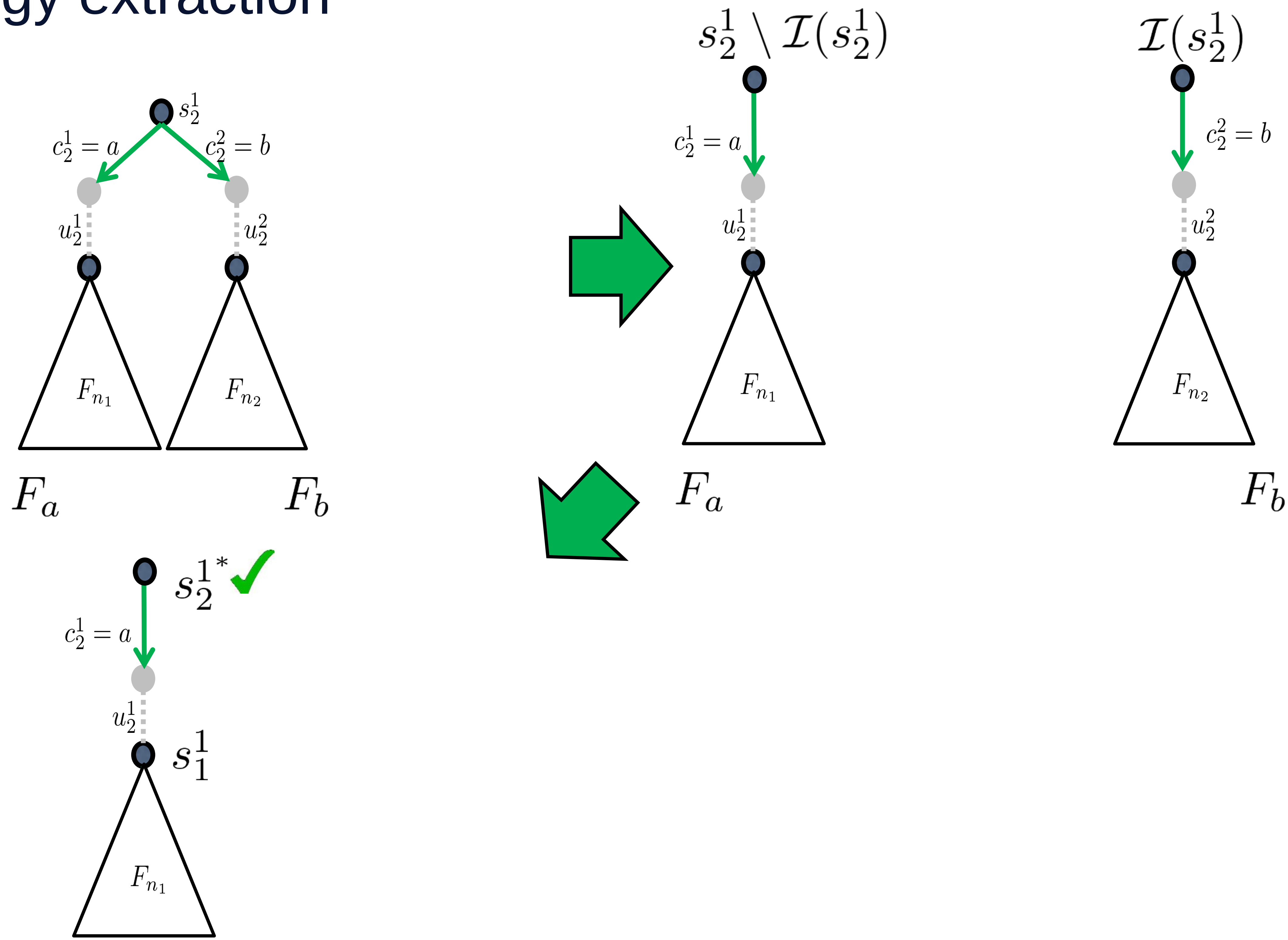
Strategy extraction



Strategy extraction

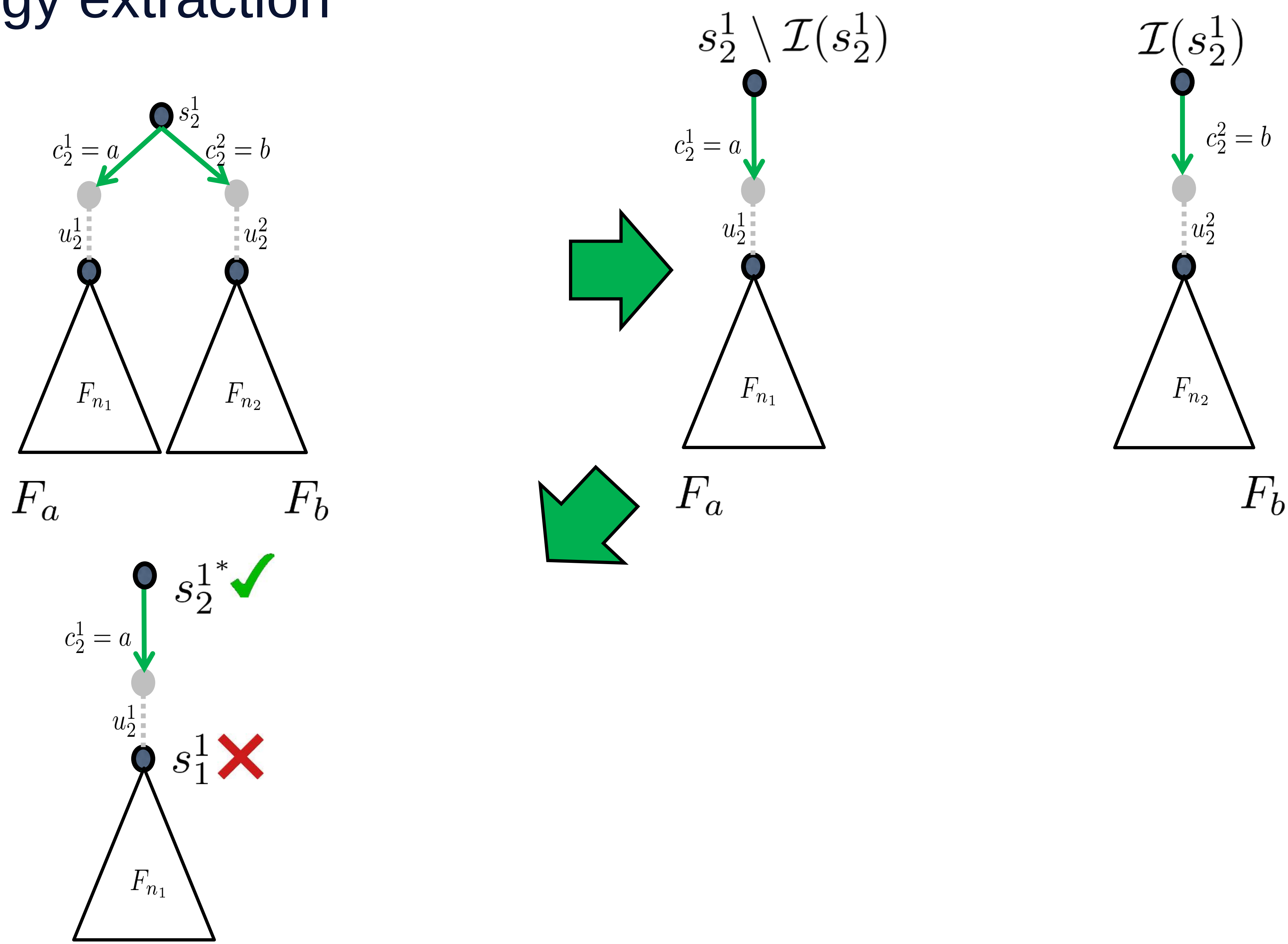


Strategy extraction

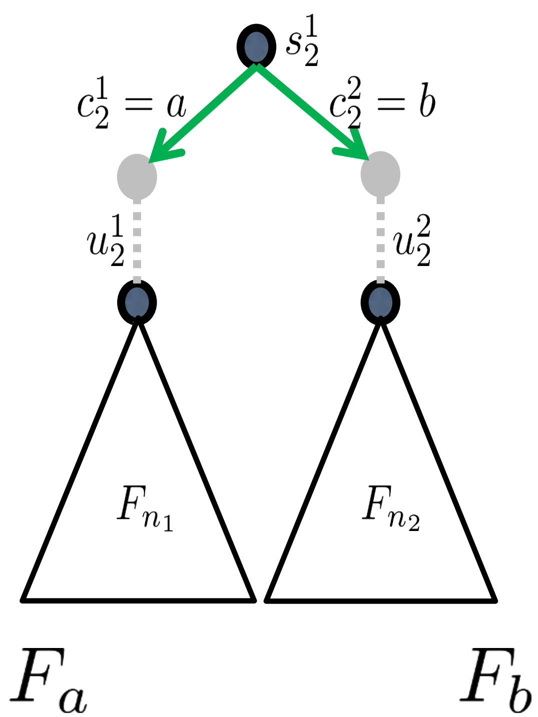


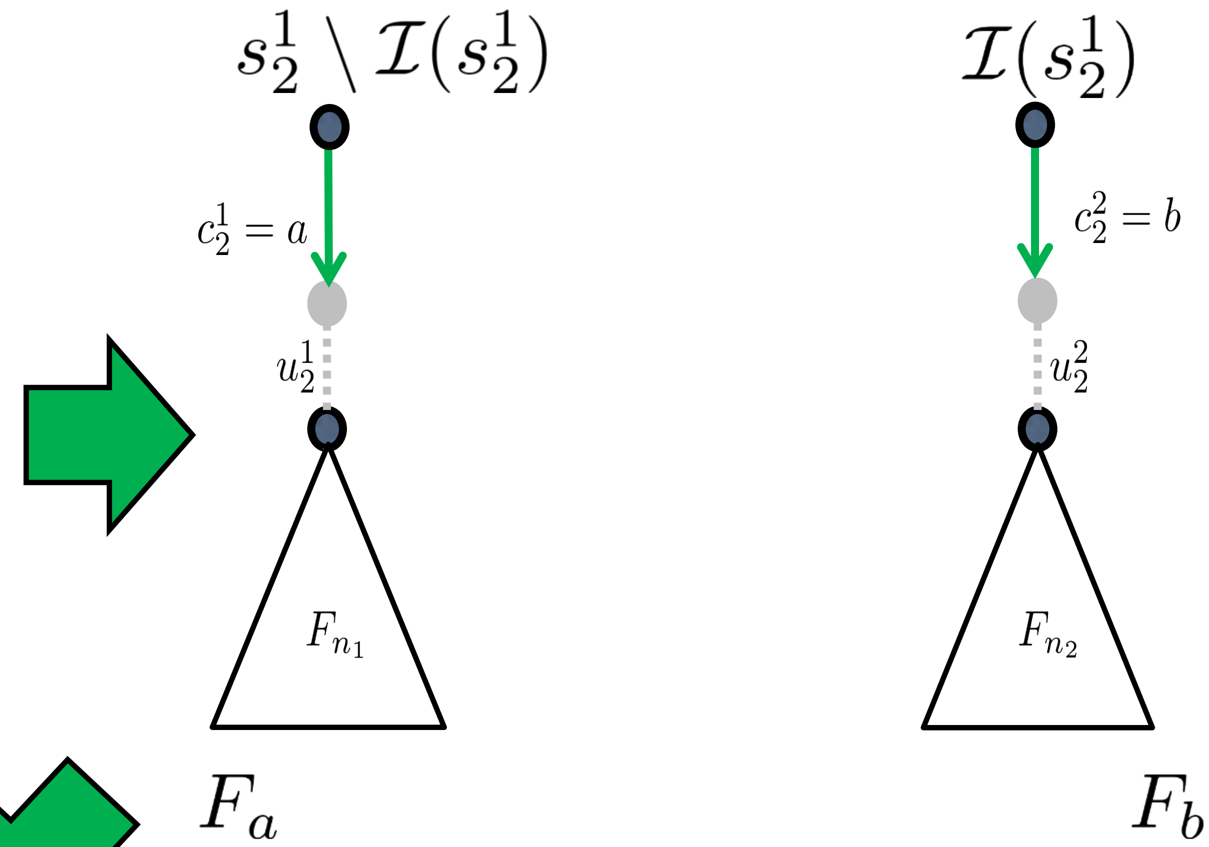
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

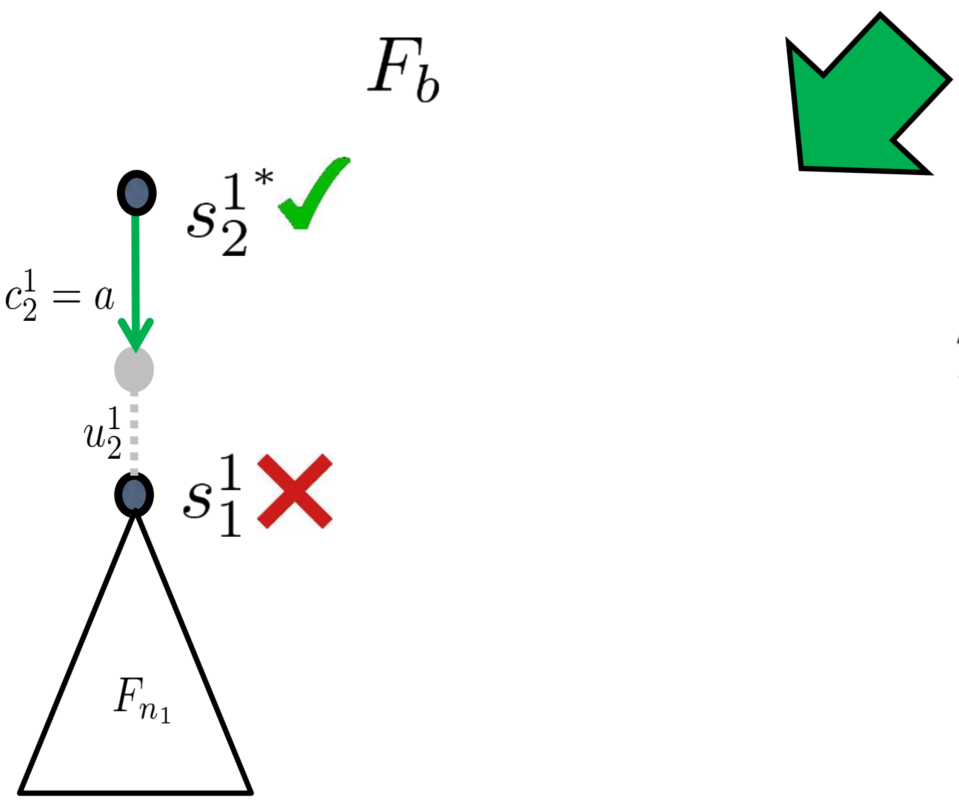
Strategy extraction



Strategy extraction

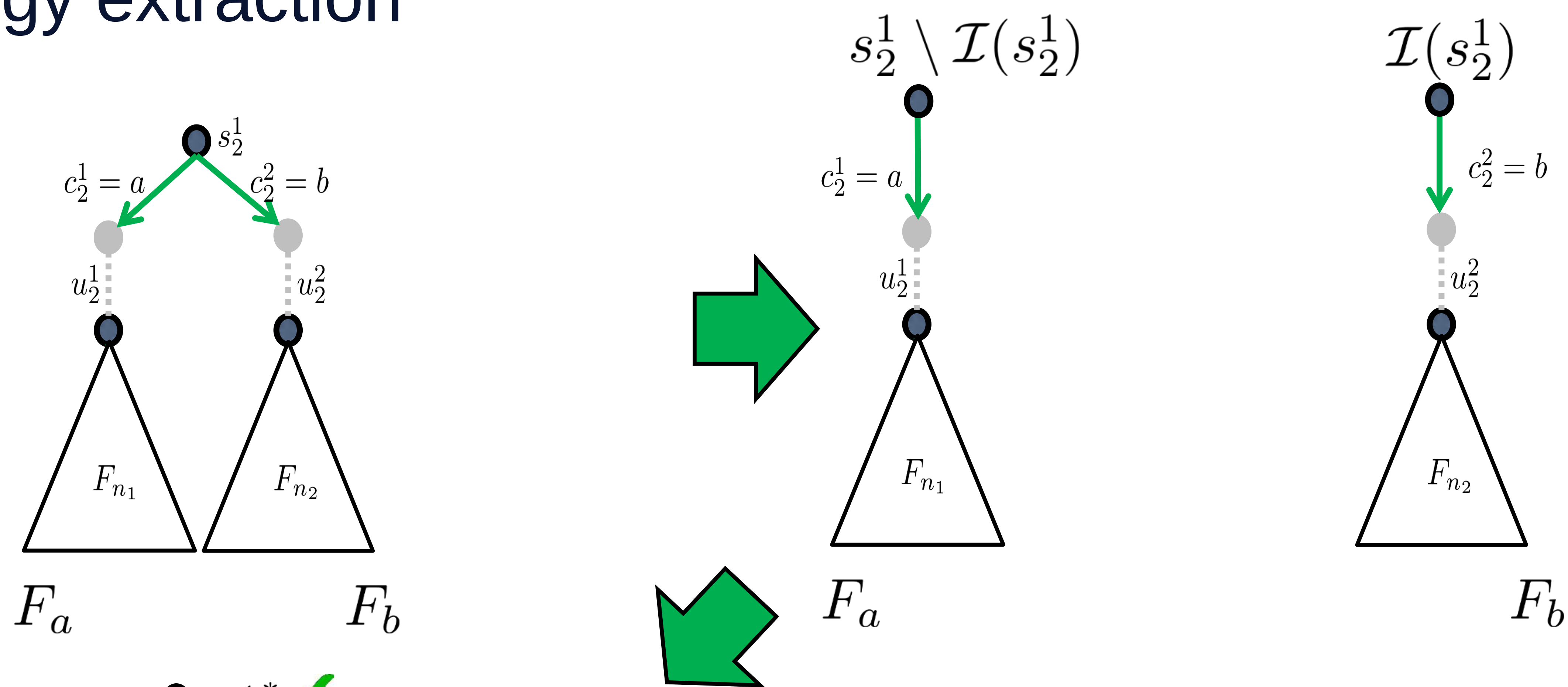


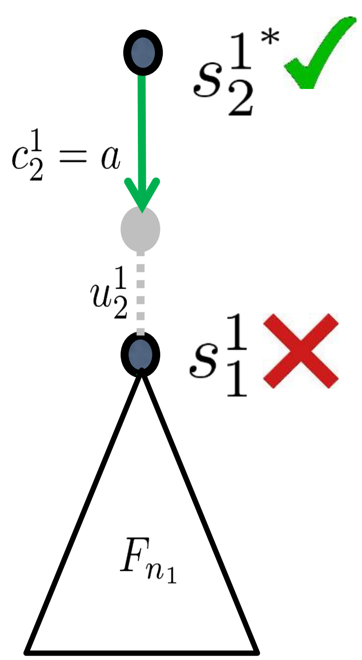




$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

Strategy extraction

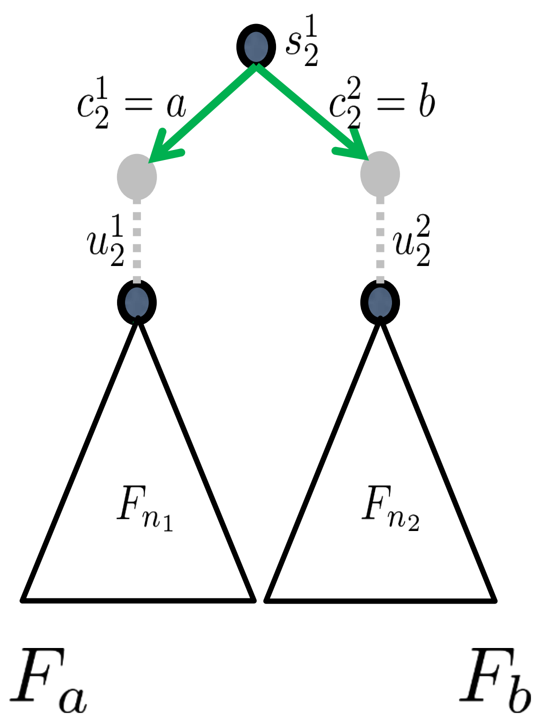


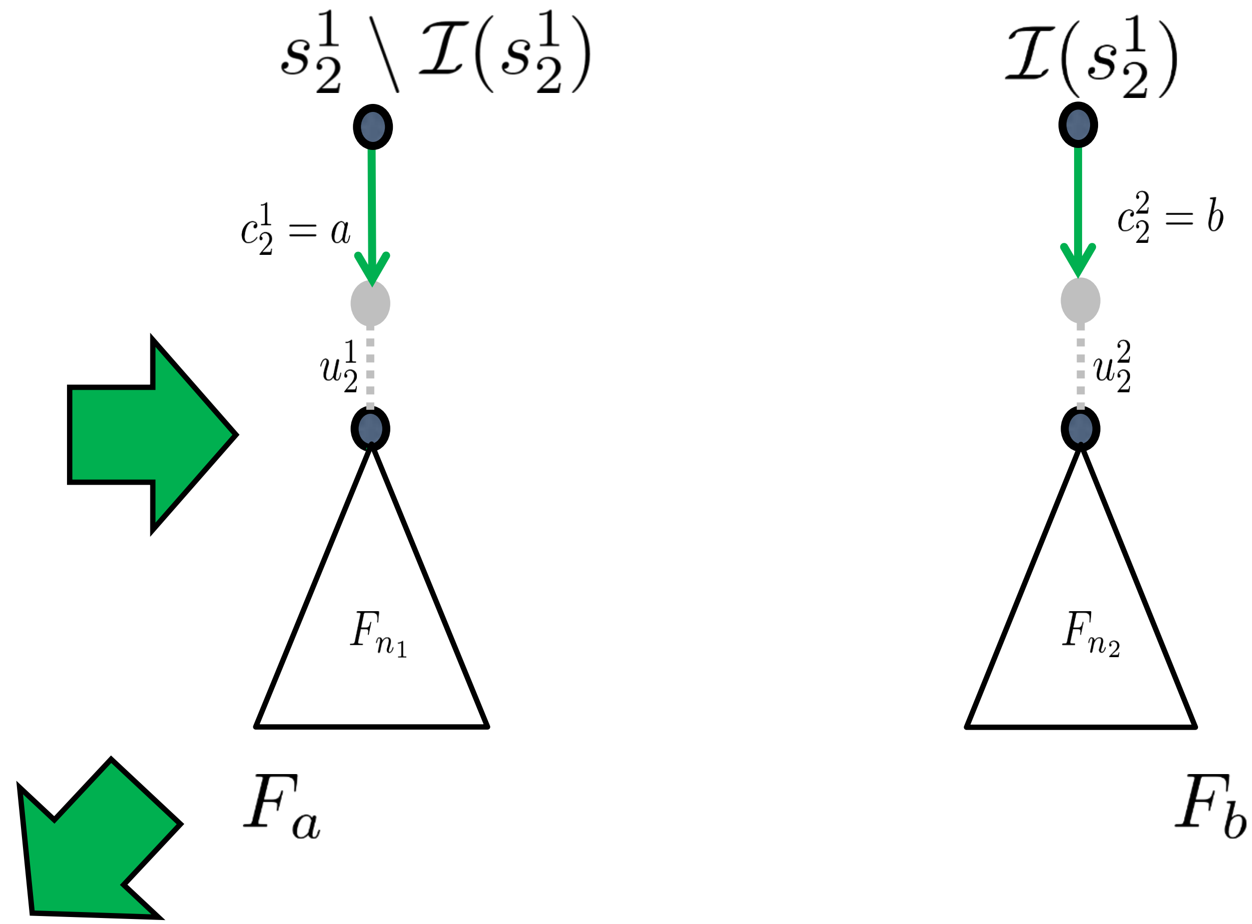


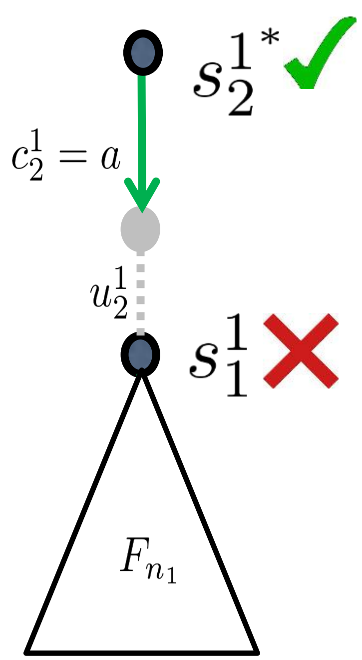
$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

AllSAT(T)

Strategy extraction





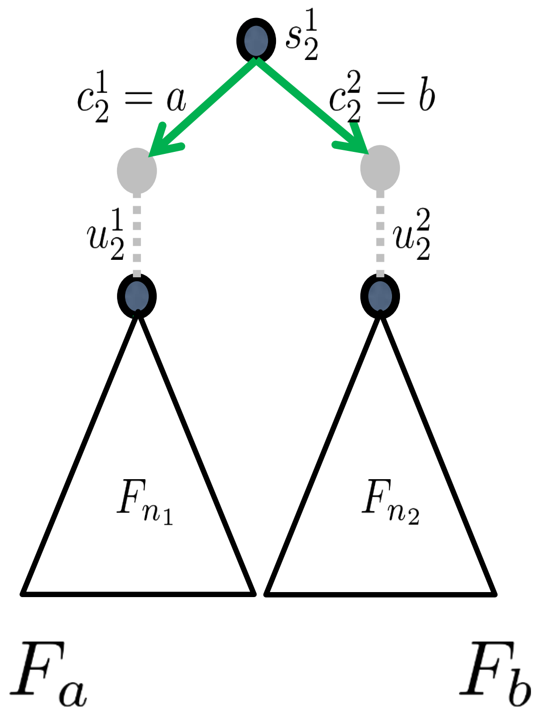


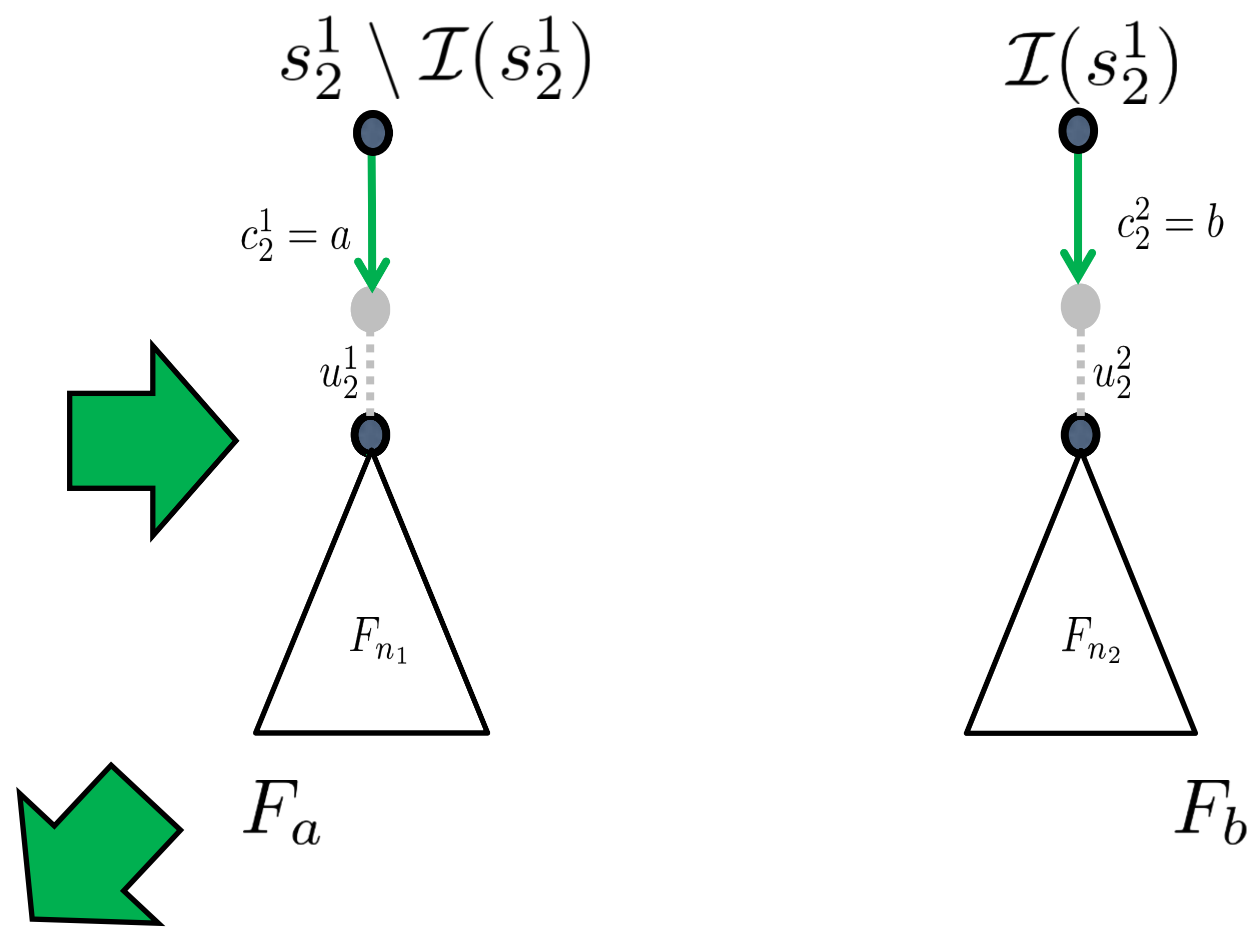
$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

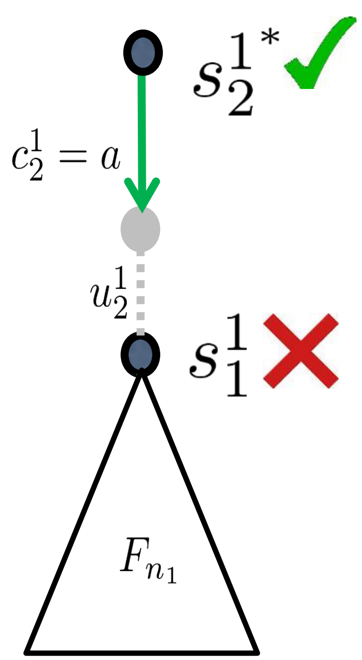
$$\mathrm{AllSAT(T)}$$

$$\mathrm{AllSAT(T)}|_{s_1^1}$$

Strategy extraction







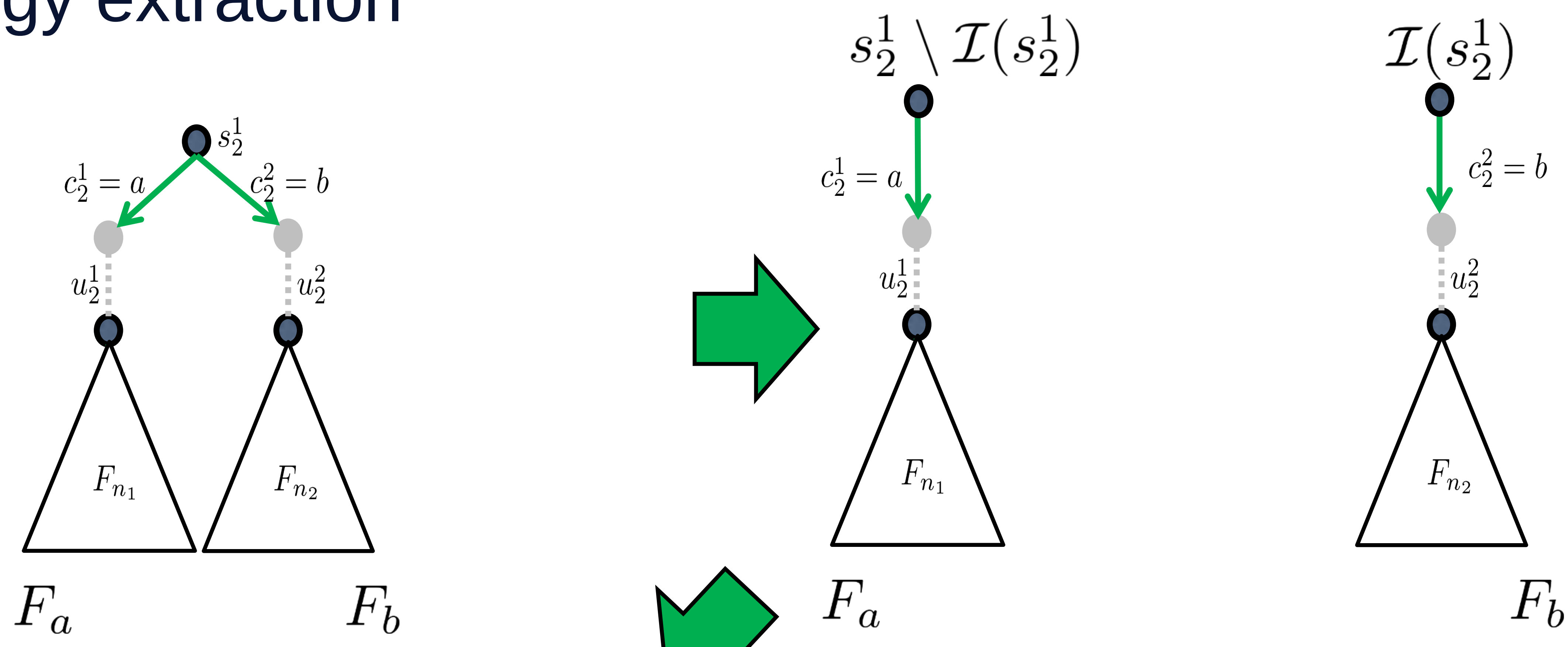
$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

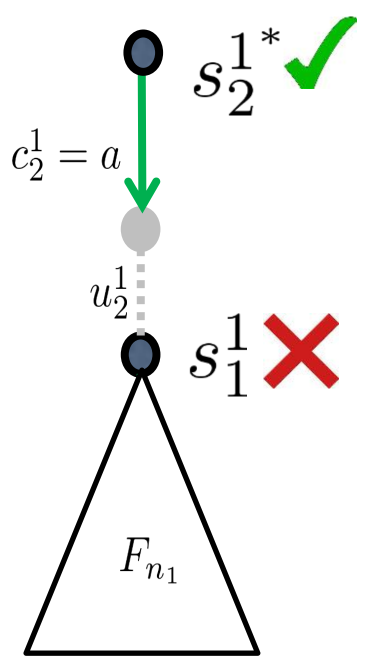
$$\mathrm{AllSAT(T)}$$

$$\mathrm{AllSAT(T)}|_{s_1^1}$$

Y. Yu, P. Subramanyan, N. Tsiskaridze, S. Malik: All-SAT Using Minimal Blocking Clauses, 2014

Strategy extraction





$$T = s_2^{1*} \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

AllSAT(T)

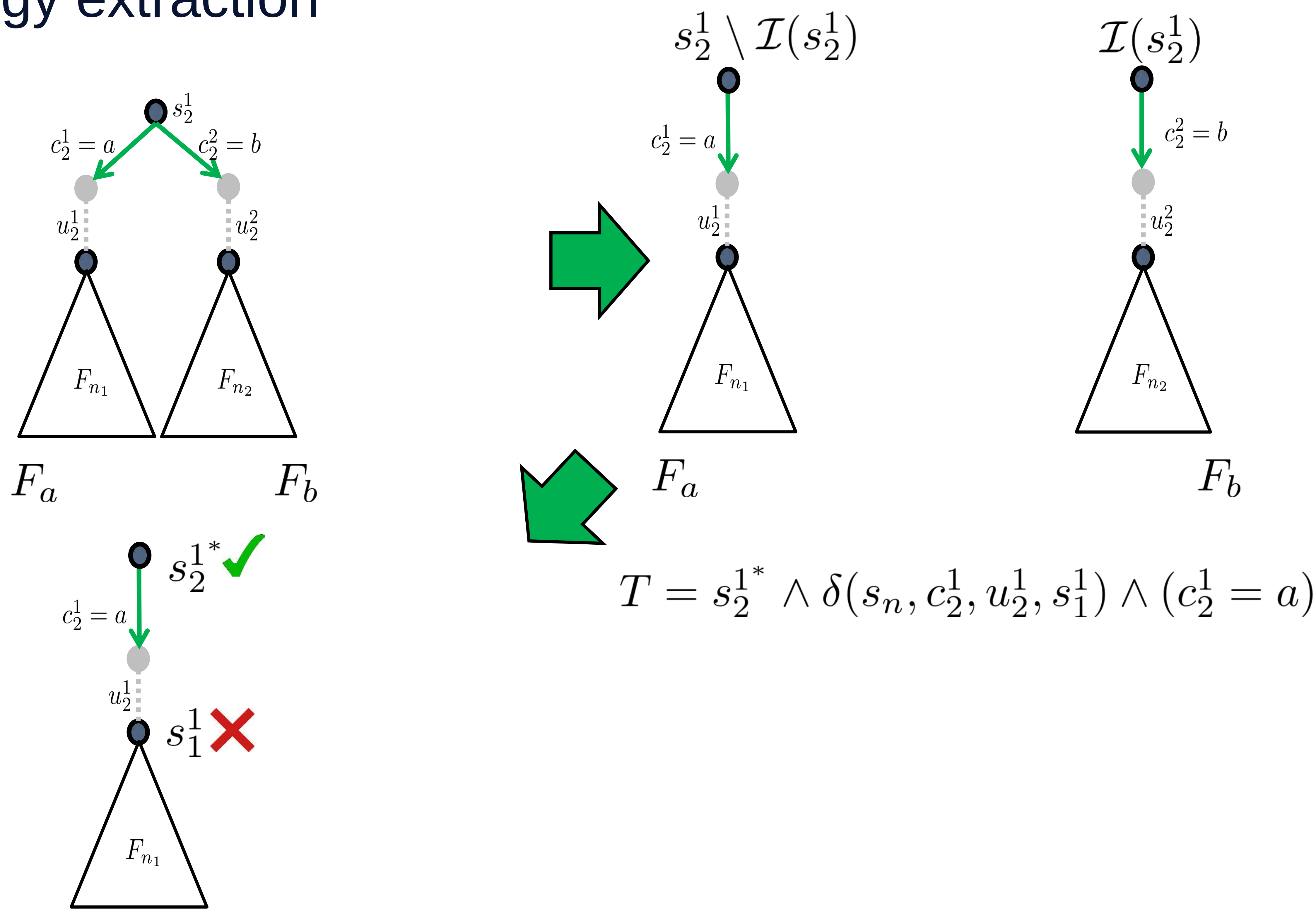
$\text{AllSAT}(T)|_{s_1^1}$

Does not scale!

Y. Yu, P. Subramanyan, N. Tsiskaridze, S. Malik: All-SAT Using Minimal Blocking Clauses, 2014

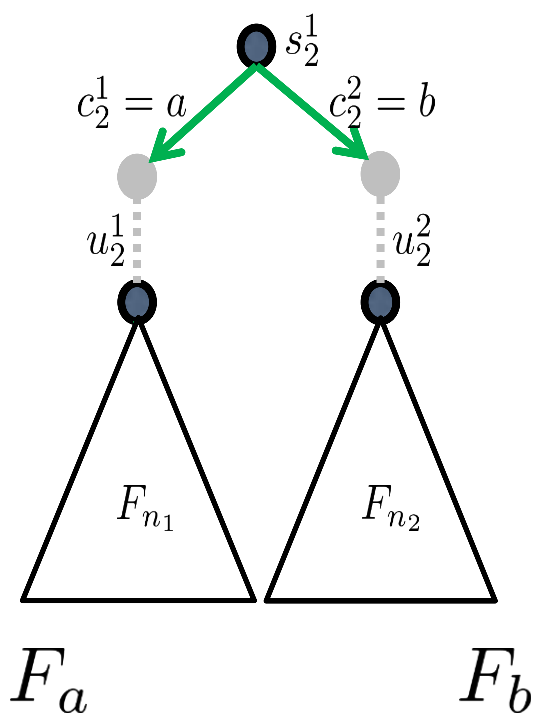
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

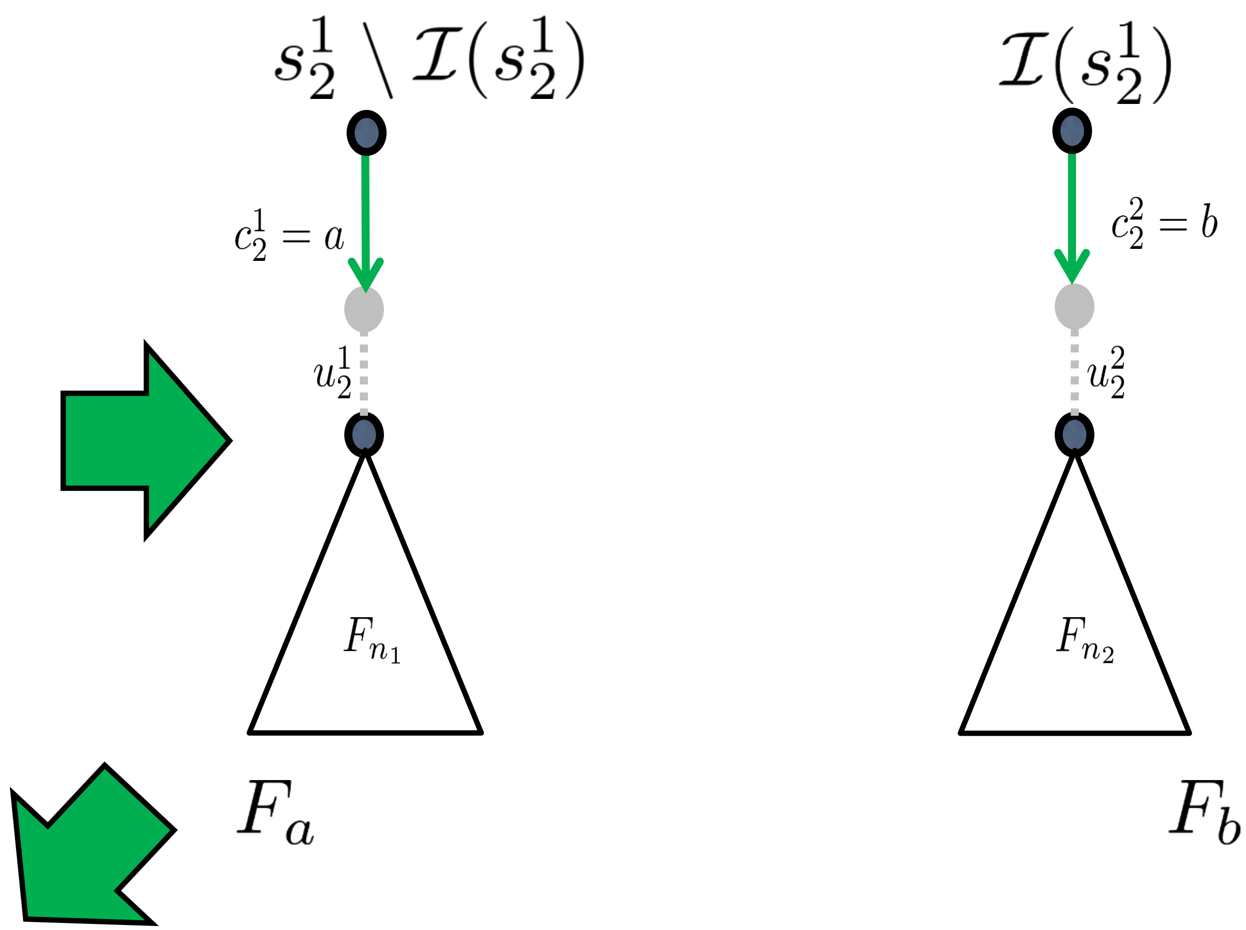
Strategy extraction



$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

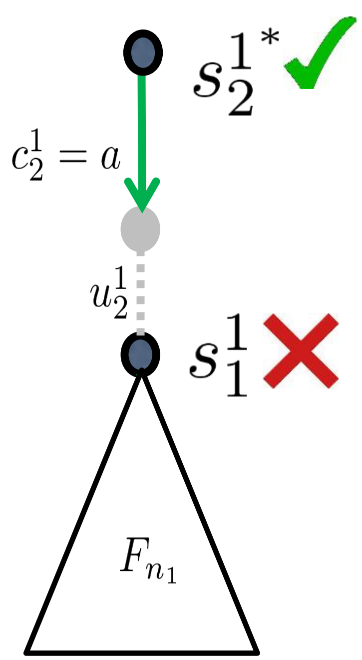
Strategy extraction



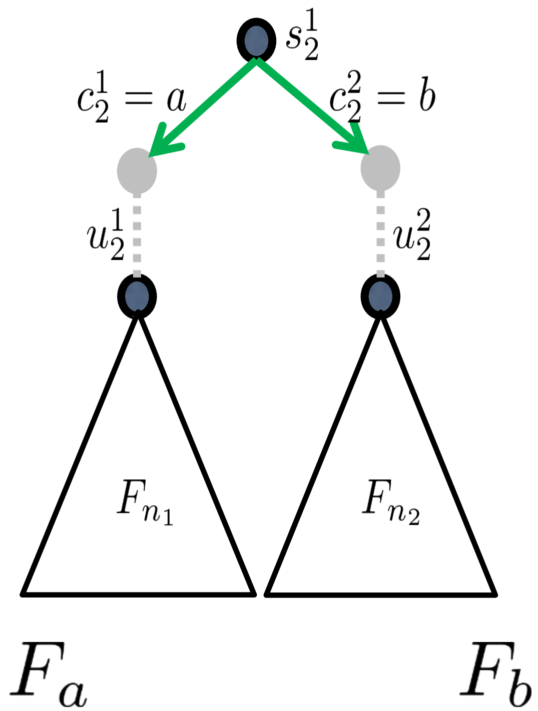


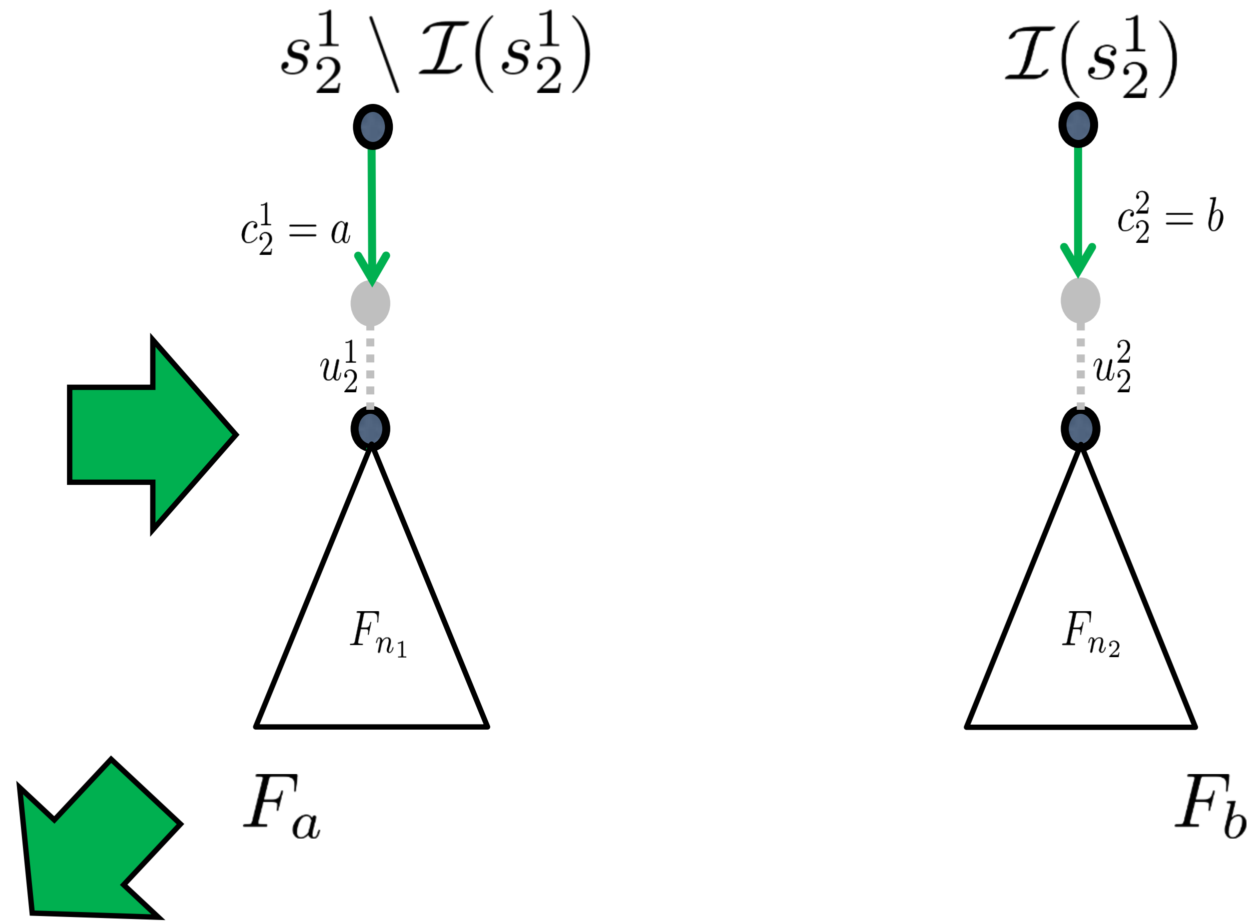
$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

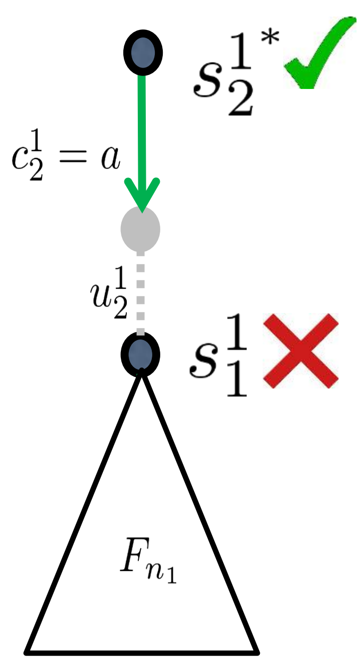
F_{n_1}



Strategy extraction



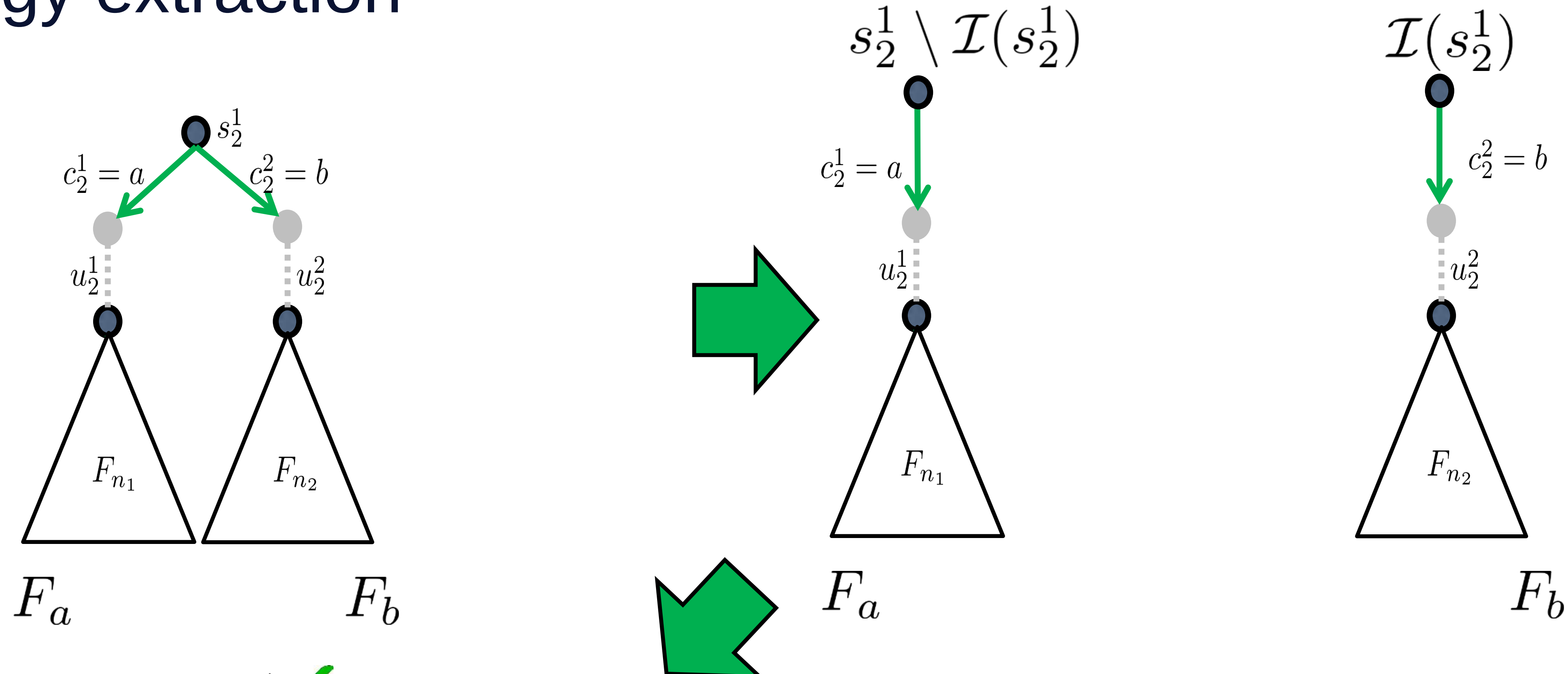


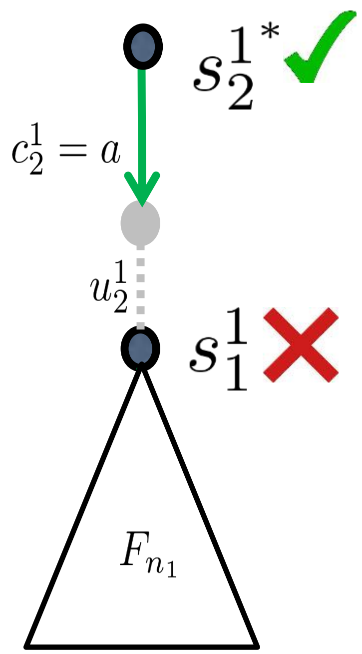


$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

$$F_{n_1} \wedge T = \bot$$

Strategy extraction

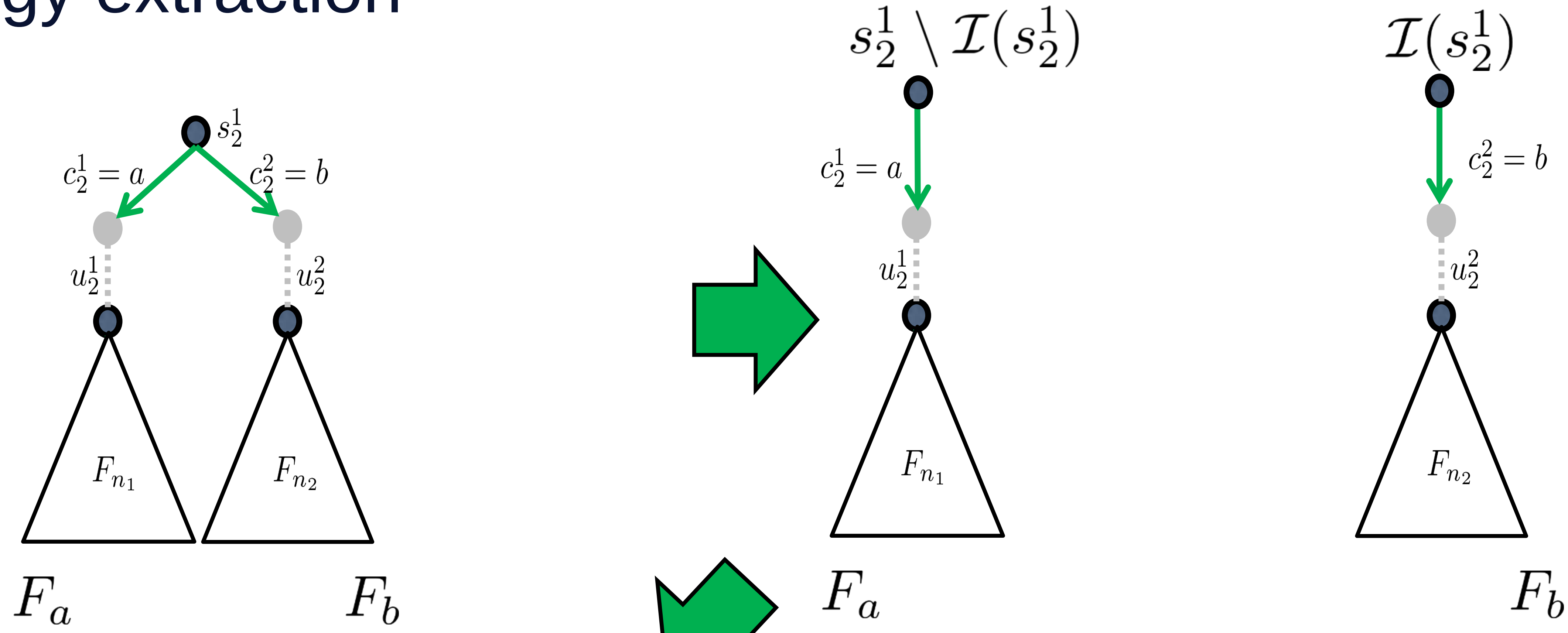


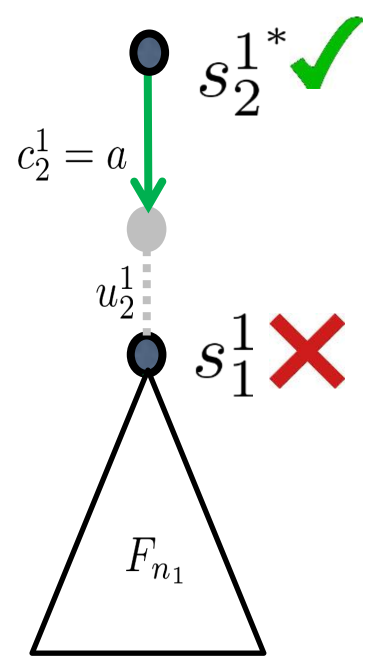


$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

$$F_{n_1} \wedge T = \bot, vars(F_{n_1}) \cap vars(T) = s_1^1$$

Strategy extraction





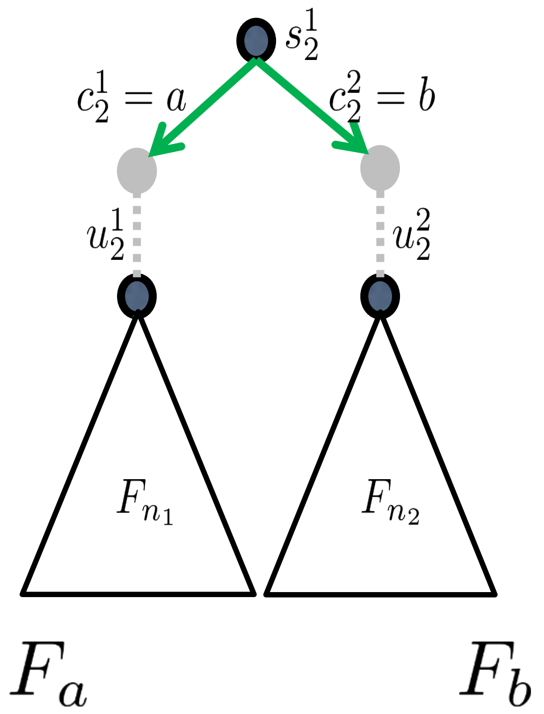
$$T = {s_2^1}^* \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

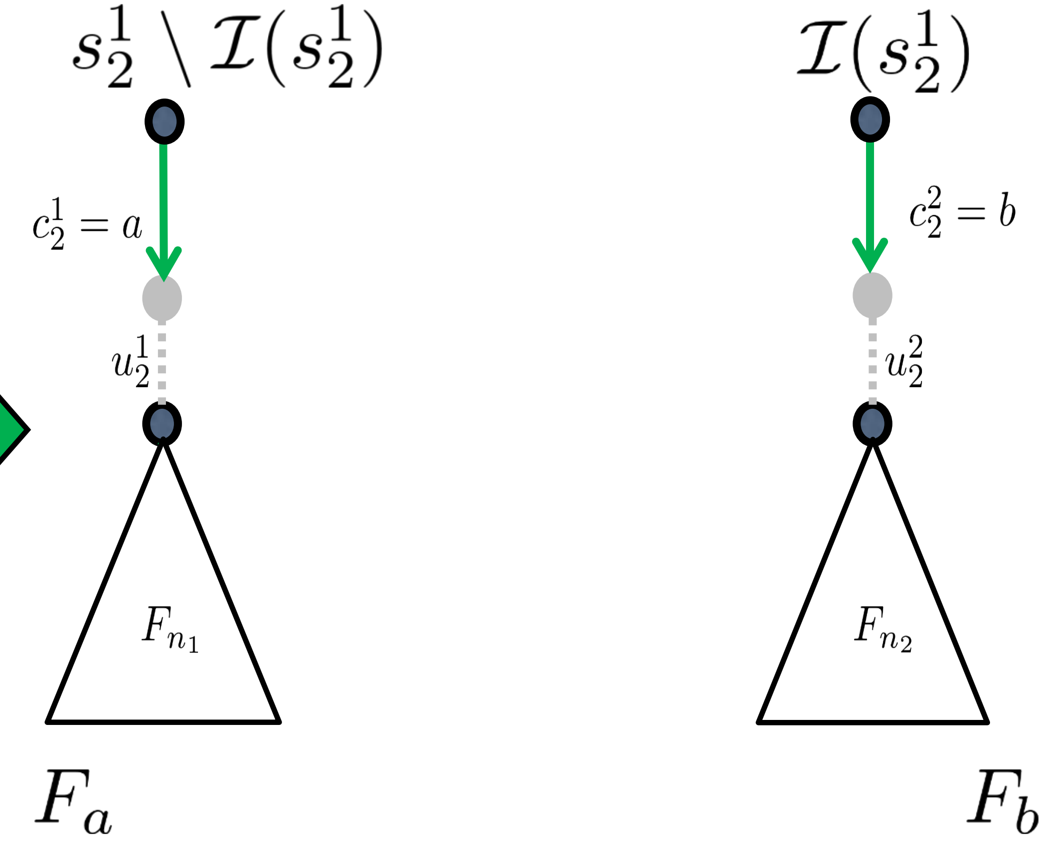
$$F_{n_1} \wedge T = \perp, vars(F_{n_1}) \cap vars(T) = s_1^1$$

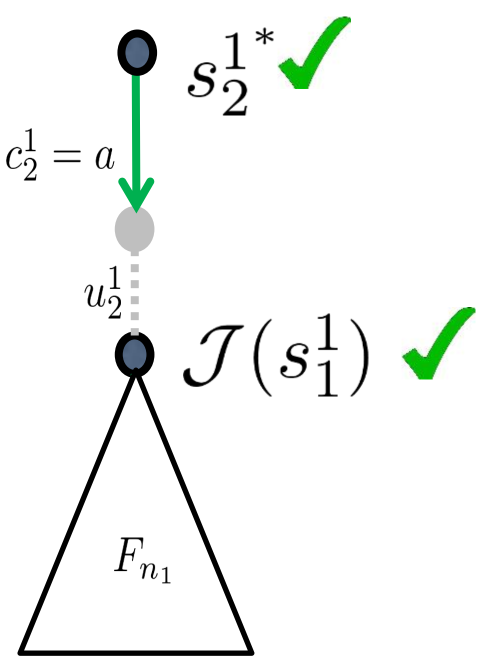
$\mathcal{J} = Interpolant(T, F_{n_1})$:

- $T \implies \mathcal{J}$
- $\mathcal{J} \wedge F_{n_1} = \perp$

Strategy extraction







$$T = s_2^{1*} \wedge \delta(s_n, c_2^1, u_2^1, s_1^1) \wedge (c_2^1 = a)$$

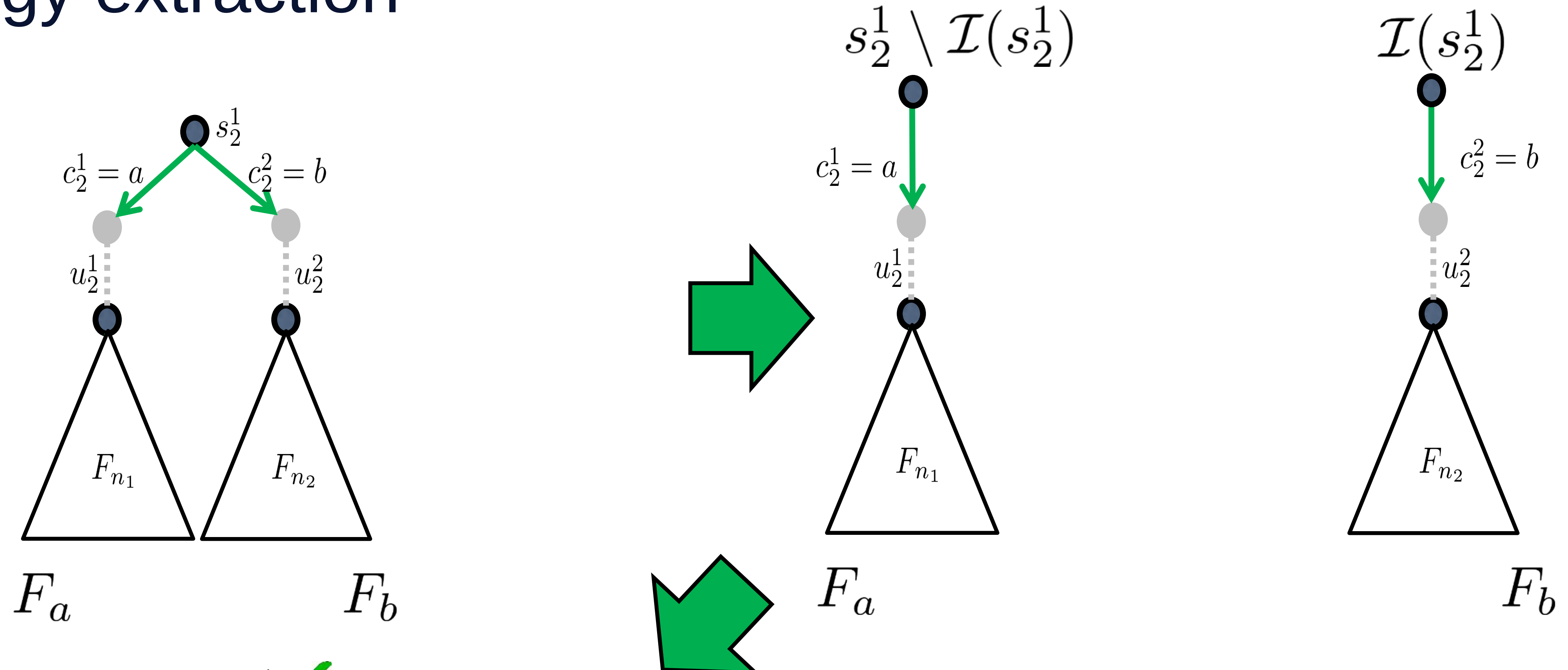
$$F_{n_1} \wedge T = \bot, vars(F_{n_1}) \cap vars(T) = s_1^1$$

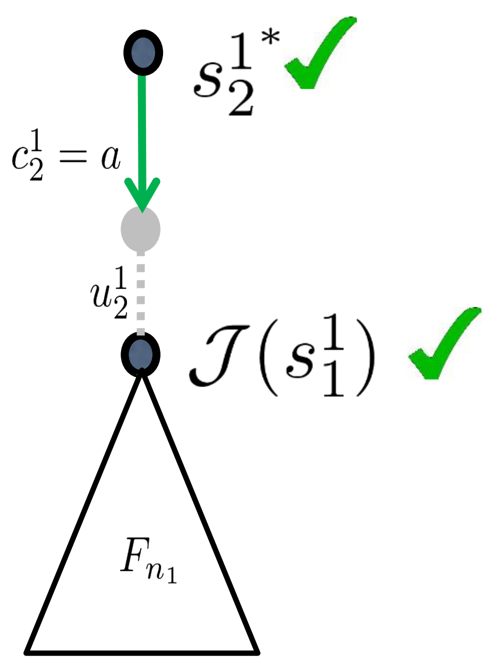
$\mathcal{J} = Interpolant(T, F_{n_1})$:

- $T \implies \mathcal{J}$
- $\mathcal{J} \wedge F_{n_1} = \bot$

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Strategy extraction





$\mathcal{J}(s_1^1)$ is an overapproximation of reachable states.

Conclusion

We proposed a strategy extraction algorithm based on interpolants

11:15 pm, July 18th, IPRA
Alexander Legg, Nina Narodytska and Leonid Ryzhyk
Practical CNF Interpolants Via BDDs

12:35 pm, July 21st, CAV
N. Narodytska, A. Legg, F. Bacchus, L. Ryzhyk and A. Walker
Solving Games without Controllable Predecessor

14:50 pm, July 21st, CAV
P. Cerny, T. Henzinger, A. Radhakrishna, L. Ryzhyk and T. Tarrach
Regression-free Synthesis for Concurrency

09:00 am, July 24th, SYNT
Leonid Ryzhyk
Automatic Device Driver Synthesis Project (Invited Talk, OSDI'14)

Thanks!

Questions?