CNF From Interpolants Via BDDs

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Background

• Driver Synthesis
  – See CAV paper

• Strategy Extraction
  – Nina’s talk yesterday
Driver Synthesis

```c
// source code here
```
Driver Synthesis
Background
Background

- **Green arrow** → controllable move
- **Red arrow** → uncontrollable move
Background
State Partitioning
State Partitioning
Next State Operation
Next State Operation
Strategy Extraction

1) Our interpolants get reused
   - We need small interpolants
   - We need small CNF

2) Our interpolants are over small sets of variables
   - Interpolants are state sets (over state variables)
   - An efficient representation exists
CNF Via BDDs

• BDDs provide efficient representation
  – Interpolants are redundant and potentially large
  – BDDs are canonical

• CNF from BDD is simple and efficient
  – Get the shortest path to False
  – Block that path and repeat

• BDDs can explode
  – Small number of variables
Experimental Set Up

Interpolant → BDD → Cubes/CNF

(a \land b \land c)
(\neg a \land b \land d)
(b \land \neg c \land d)
## Results

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpolant Size</td>
<td>67.66 nodes</td>
<td>1410 nodes</td>
</tr>
<tr>
<td>Interpolant Time</td>
<td>0.24 sec</td>
<td>1.50 sec</td>
</tr>
<tr>
<td>BDD Size</td>
<td>14.8 nodes</td>
<td>58 nodes</td>
</tr>
<tr>
<td>BDD Time</td>
<td>&lt; 0.01 sec</td>
<td>&lt; 0.01 sec</td>
</tr>
<tr>
<td>Cube Size</td>
<td>2.05 clauses</td>
<td>12 clauses</td>
</tr>
<tr>
<td>Cube Time</td>
<td>&lt; 0.01 sec</td>
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**Runs of EvaSolver:** 36  
**Total Interpolants:** 872
Scalability

- Time spent solving: 1297.37s
- Time spent on interpolants: 232.33s (17%)

- BDDs do not contribute significantly to time
- Interpolant size increases with state space
Related Work

- **ALLSAT**
  - Alternative to interpolation

- **Sweeping**
  - Reduces circuit
  - Doesn’t give CNF

- **Interpolants in CNF (CAV ‘13)**
  - Needs domain specific solution
Next State Operation
ALLSAT

• Existential quantification
  – Find solution (via SAT)
  – Block solution
  – Repeat
ALLSAT

• Existential quantification
  – Find solution (via SAT)
  – Block solution
  – Repeat

• Problem:

\[(x \land x') (\neg x \land \neg x')\]

After Projection: \((x') (\neg x')\)
Related Work

- **ALLSAT**
  - Alternative to interpolation

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- **Interpolants in CNF (CAV ‘13)**
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Experimental Set Up

- Interpolant
- BDD
- Cubes/CNF
- AIG

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(a \land b \land c)
(\neg a \land b \land d)
(b \land \neg c \land d)

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a = b \land c 

b = \neg d \land e 

c = \neg (d \land \neg e) 

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<td>&lt; 0.01 sec</td>
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</tr>
<tr>
<td>Post-sweeping Size</td>
<td>14.89 nodes</td>
<td>80 nodes</td>
</tr>
<tr>
<td>Sweeping Time</td>
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Third Party Libraries

• PeRIplo (University of Lagano)
  – Interpolant library
  – Backed by MiniSAT
  – Performs some redundancy detection

• CUDD (CU Boulder)
  – BDD library

• IImc (CU Boulder)
  – Model checking library
  – Sweeping algorithms (BDD, SAT, Cut)
Other Talks

12:35 pm, July 21st, CAV
N. Narodytska, A. Legg, F. Bacchus, L. Ryzhyk and A. Walker
Solving Games without Controllable Predecessor

14:50 pm, July 21st, CAV
P. Cerny, T. Henzinger, A. Radhakrishna, L. Ryzhyk and T. Tarrach
Regression-free Synthesis for Concurrency

09:00 am, July 24th, SYNT
Leonid Ryzhyk
Automatic Device Driver Synthesis Project (Invited Talk, OSDI’14)
Questions