Quantified Boolean Formulae: Call the Plumber!

Josef Lindsberger Alexander Maringele Georg Moser

Department of Computer Science, Universität Innsbruck, Austria
josef.lindsberger@web.de,
{alexander.maringele, georg.moser}@uibk.ac.at

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Motivation

Public relations and teaching

- https://www.uibk.ac.at/lndf/
- https://www.uibk.ac.at/jungeuni/
- Logic 101

- Is the game beatable?
- Is the game tough?
- How can we generate game instances?

Super Formula World

SFW is a simple platform game with two-dimensional game maps. It uses the game mechanics of Nintendo's Super Mario World, which introduced more complex challenges than its predecessors.

SFW encodes a reduction from the satisfiability of (quantified) Boolean formulae to graphs of gadgets.

Our hero Mario has to cope with maps drawn from such graphs to rescue princess Peach, while in fact he is being exploited as a (Q)SAT solver.

Outline

- Satisfiability of QBF
- NP framework
- PSPACE framework
- Summary
- Demo

Satisfiability of QBF

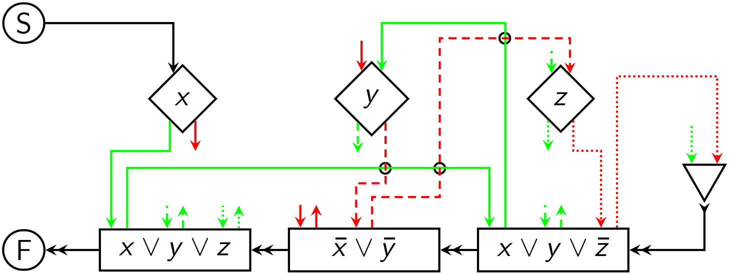
QSAT(ϕ)

$$
\begin{tikzcd}
 & \forall y\, \exists z\, (x \wedge (y \vee \bar{z}) \wedge (\bar{y} \vee z)) \arrow[d, "x"'] & \\
 & \forall y\, \exists z\, ((y \vee \bar{z}) \wedge (\bar{y} \vee z)) \arrow[dl, "y"'] \arrow[dr, "\bar{y}"] & \\
\exists z\, (z) \arrow[d, "z"'] & & \exists z\, (\bar{z}) \arrow[d, "\bar{z}"] \\
\top & & \top
\end{tikzcd}
$$

$x \;\; y \;\; z \;\; \bar{y} \;\; \bar{z}$

NP Framework

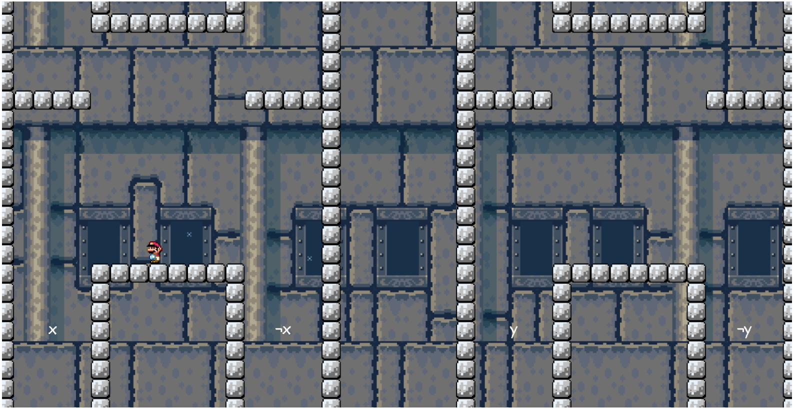
$\phi = (x \vee y \vee z) \wedge (\bar{x} \vee \bar{y}) \wedge (x \vee y \vee \bar{z})$, $\mathrm{R}_{\mathrm{NP}}(\phi)$:



$$Ⓢ\ x\ \bar{y}\ \bar{z}\ \bigtriangledown\ \underset{x,\bar{y},\bar{z}}{\checkmark}\ Ⓕ$$

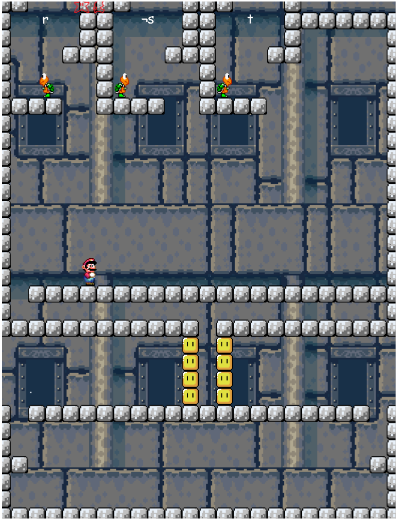
$$\mathrm{SAT}(\phi) \iff \mathrm{R}_{\mathrm{NP}}(\phi) \text{ beatable}$$

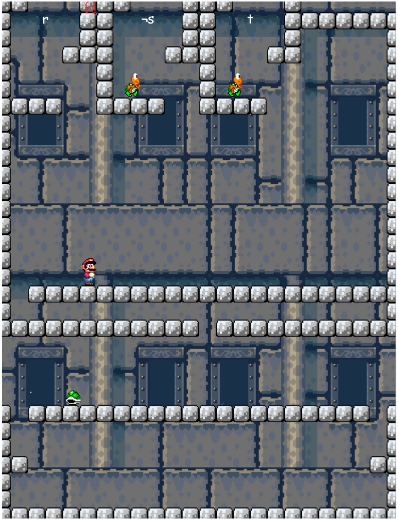
Variables and Literal Paths



Bricked and Traversable Clause

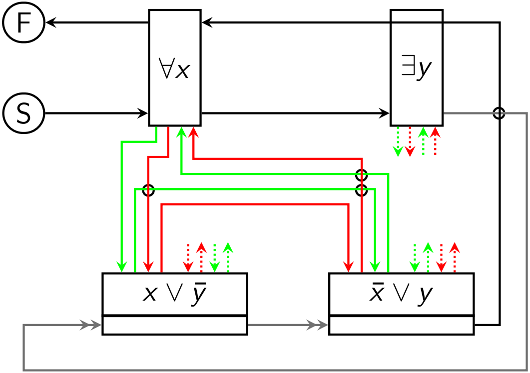
$(r \vee \bar{s} \vee t)$





PSPACE Framework

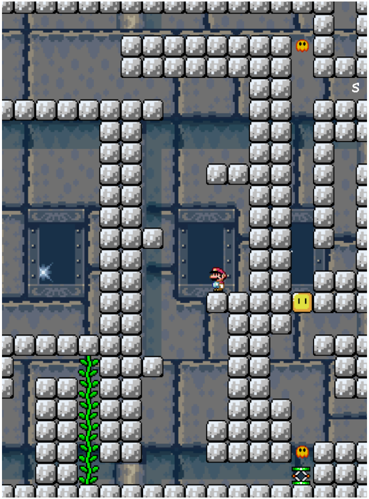
$\phi = \forall x \exists y\, (x \vee \bar{y}) \wedge (\bar{x} \vee y)$, $\mathrm{R}_{\mathrm{P}_\sqcup}(\phi)$:

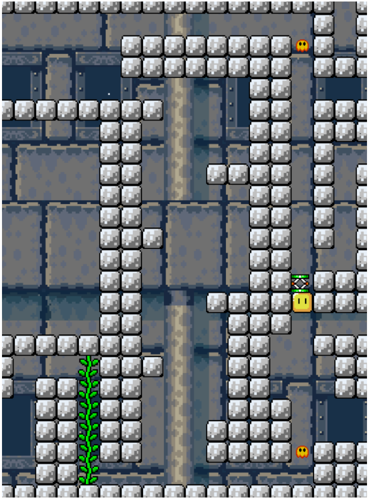


$$\text{Ⓢ}\ \forall x\ \frac{\text{open } x}{\text{close } \bar{x}}\ \exists_x y\ \frac{\text{open } y}{\text{close } \bar{y}}\ \checkmark_{x,y}\ \forall x\ \frac{\text{close } x}{\text{open } \bar{x}}\ \exists_{\bar{x}} y\ \frac{\text{close } y}{\text{open } \bar{y}}\ \checkmark_{\bar{x},\bar{y}}\ \forall x\ \text{Ⓕ}$$

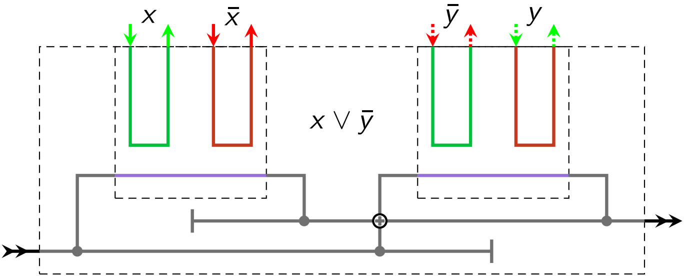
$$\mathrm{QSAT}(\phi) \iff \mathrm{R}_{\mathrm{P}_\sqcup}(\phi) \text{ beatable}$$

Closed and Traversable Door





Clause with Doors



- open the door
- close the door
- traverse the clause
- traverse the door

Existential quantifier

enter ℓ^{+}

open:$\rightharpoonup$ traverse:$\rightharpoonup$ close:$\rightharpoondown$

next ... return

previous

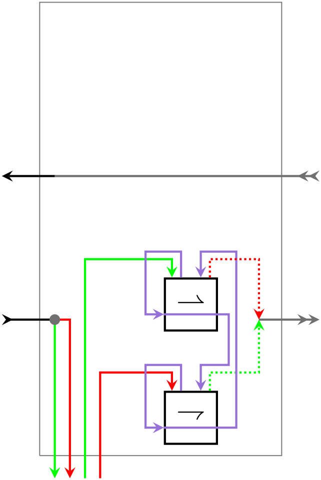
...

enter ℓ^{-}

open:$\rightharpoondown$ traverse:$\rightharpoondown$ close:$\rightharpoonup$

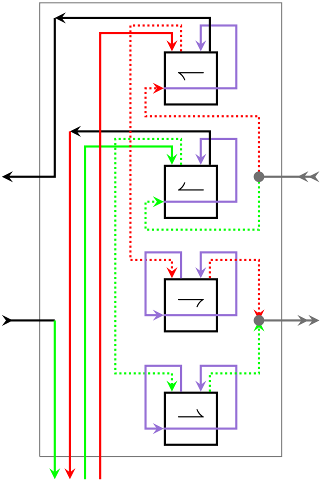
next ... return

previous



Universal quantifier

enter
ℓ^+ open:$\leftharpoonup$
open:$\rightharpoonup$ traverse:$\rightharpoonup$ close:$\rightharpoonup$
next . . . return
traverse:$\leftharpoonup$ close:$\leftharpoonup$
ℓ^- open:$\leftharpoondown$
open:$\rightharpoondown$ traverse:$\rightharpoondown$ close:$\rightharpoondown$
next . . . return
traverse:$\leftharpoondown$ close:$\leftharpoondown$
previous



Summary

Super Formula World

We have

- Refinements of the NP and PSPACE frameworks etablished by Aloupis et al.
- Instantiations of the frameworks for Super Mario World with precise definitions and fixed topologies
- Running implementations of the frameworks
- Exposition of computational complexity of video games
- Improvable game experience
  - Game play is too repetitive
  - Maps are limited to prenex CNF
  - User cannot be smart

Super Formula World

Life Demo

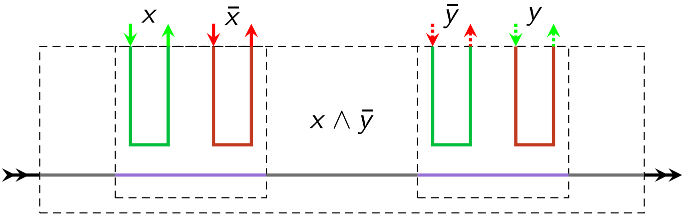
- `NP` framework $\bar{x} \vee y$

or

- `PSPACE` framework $\forall x\,(\bar{x} \vee y)$

Future Work

- Less repetition
- Smaller maps
- Construct from NNF



References

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Available online at `https://github.com/DwarfVader/mario`.