



Quantified Heap Invariants for Object-Oriented Programs

Temesghen Kahsai^{1,2}, *Rody Kersten*¹,
Philipp Rümmer³, Martin Schäfer²

¹Carnegie-Mellon University Silicon Valley

²Amazon Web Services

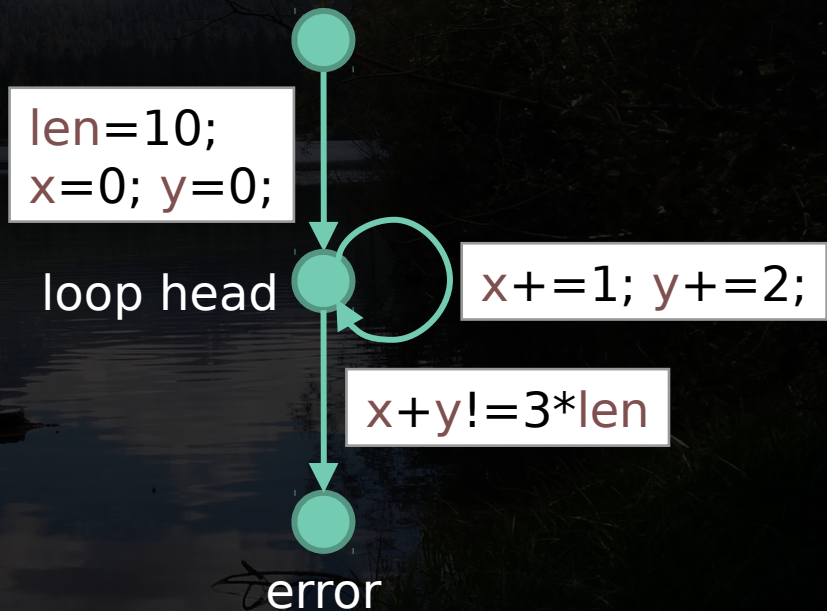
³Uppsala University

Verifying Java Programs

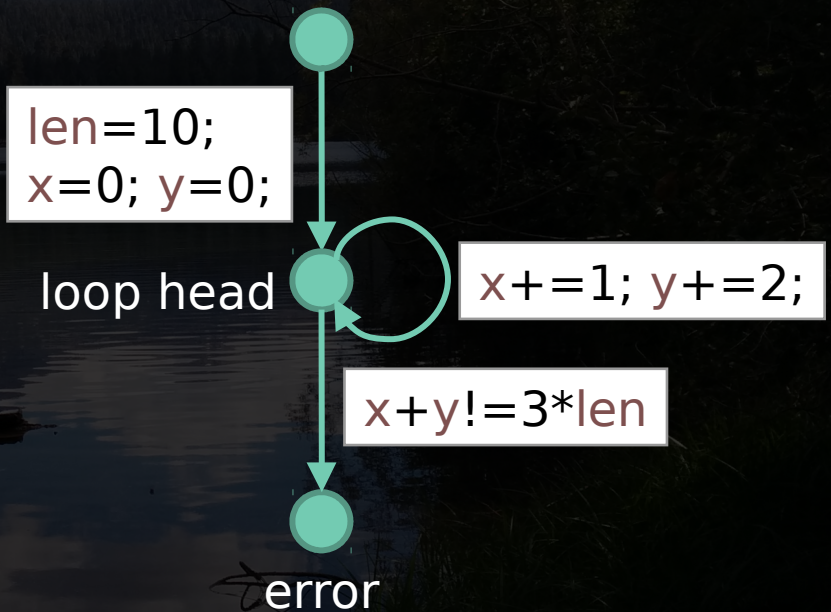
```
public class MyExample {  
    public static void main(String[] args) {  
        final int len=10;  
        int x=0, y=0;  
        while (x < len) {  
            x+=1; y+=2;  
        }  
        assert x+y==3*len;  
    }  
}
```


Verifying Java Programs

```
public class MyExample {  
  public static void main(String[] args) {  
    final int len=10;  
    int x=0, y=0;  
    while (x < len) {  
      x+=1; y+=2;  
    }  
    assert x+y==3*len;  
  }  
}
```



Verifying Java with Horn clauses



Verifying Java with Horn clauses

$\text{entry}(x,y,\text{len}) \leftarrow \text{true}$

$\text{head}(0,0,10) \leftarrow \text{entry}(x,y,\text{len})$

$\text{len}=10;$
 $x=0; y=0;$

loop head

$x+=1; y+=2;$

$x+y \neq 3*\text{len}$

error



```
graph TD; A(( )) --> B(( )); B -- loop --> B; B --> C(( )); C --> D((error));
```

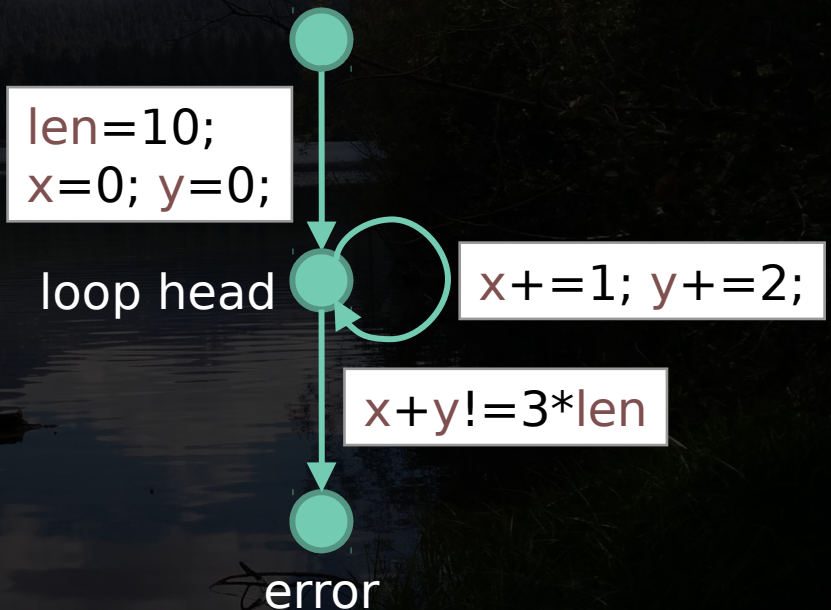

Verifying Java with Horn clauses

$\text{entry}(x,y,\text{len}) \leftarrow \text{true}$

$\text{head}(0,0,10) \leftarrow \text{entry}(x,y,\text{len})$

$\text{head}(x+1,y+2,\text{len}) \leftarrow x < \text{len} \wedge \text{head}(x,y,\text{len})$

$3 * \text{len} = x + y \leftarrow x \geq \text{len} \wedge \text{head}(x,y,\text{len})$



Solving Horn clauses

$\text{entry}(x,y,\text{len}) \leftarrow \text{true}$

$\text{head}(0,0,10) \leftarrow \text{entry}(x,y,\text{len})$

$\text{head}(x+1,y+2,\text{len}) \leftarrow x < \text{len} \wedge \text{head}(x,y,\text{len})$

$3 * \text{len} = x + y \leftarrow x \geq \text{len} \wedge \text{head}(x,y,\text{len})$

Use off-the-shelf solver

- Spacer (Z3)
- Eldarica
- ...

Solving Horn clauses

$\text{entry}(x,y,\text{len}) \leftarrow \text{true}$

$\text{head}(0,0,10) \leftarrow \text{entry}(x,y,\text{len})$

$\text{head}(x+1,y+2,\text{len}) \leftarrow x < \text{len} \wedge \text{head}(x,y,\text{len})$

$3 * \text{len} = x + y \leftarrow x \geq \text{len} \wedge \text{head}(x,y,\text{len})$

Predicate assignments:

$\text{entry}(x,y,\text{len})$?
$\text{head}(x,y,\text{len})$?

Solving Horn clauses

$\text{entry}(x,y,\text{len}) \leftarrow \text{true}$

$\text{head}(0,0,10) \leftarrow \text{entry}(x,y,\text{len})$

$\text{head}(x+1,y+2,\text{len}) \leftarrow x < \text{len} \wedge \text{head}(x,y,\text{len})$

$3 * \text{len} = x + y \leftarrow x \geq \text{len} \wedge \text{head}(x,y,\text{len})$

Predicate assignments:

$\text{entry}(x,y,\text{len})$	true
$\text{head}(x,y,\text{len})$	$3 * \text{len} = x + y$

Solving Horn clauses

true \leftarrow true

$3*10=0+0$ \leftarrow true

$3*len=x+1+y+2$ \leftarrow $x < len \wedge (3*len=x+y)$

$3*len=x+y$ \leftarrow $x \geq len \wedge (3*len=x+y)$

Predicate assignments:

entry(x,y,len)	true
head(x,y,len)	$3*len=x+y$

Solving Horn clauses

$\text{entry}(x,y,\text{len}) \leftarrow \text{true}$

$\text{head}(0,0,10) \leftarrow \text{entry}(x,y,\text{len})$

$\text{head}(x+1,y+2,\text{len}) \leftarrow x < \text{len} \wedge \text{head}(x,y,\text{len})$

$3 * \text{len} = x + y \leftarrow x \geq \text{len} \wedge \text{head}(x,y,\text{len})$

Predicate assignments:

$\text{entry}(x,y,\text{len})$	true
$\text{head}(x,y,\text{len})$	$y = 2 * x$

Solving Horn clauses

true \leftarrow true

$0=2*0$ \leftarrow true

$y+2=2*(x+1)$ \leftarrow $x < \text{len} \wedge (y=2*x)$

$3*\text{len}=x+y$ \leftarrow $x \geq \text{len} \wedge (3*\text{len}=x+y)$

Predicate assignments:

entry(x,y,len)	true
head(x,y,len)	$y=2*x$

A dark, moody landscape photograph of a lake and mountains. The scene is framed by a white border. The foreground is dominated by dark, silhouetted trees and foliage. In the middle ground, a calm lake reflects the sky and the distant mountains. The background features a range of mountains under a cloudy sky. The overall tone is somber and atmospheric.

So what's so difficult?

How do we encode heap?

```
public static void main(String[] args) {  
    final int size = 10;  
    final int[] table = new int[size];  
    Node l1 = null, l2 = null;  
    for (int i=0; i<args.length; i++) {  
        int d = Integer.parseInt(args[i]);  
        if (d >= 0 && d < size)  
            l1 = new Node(l1, d);  
        else l2 = new Node(l2, d);  
    }  
    while (l1 != null) {  
        table[l1.data]++;  
        l1 = l1.next;  
    }  
}
```

```
public static class Node {  
    final Node next;  
    final int data;  
    public Node(Node next, int data) {  
        this.next = next;  
        this.data = data;  
    }  
}
```

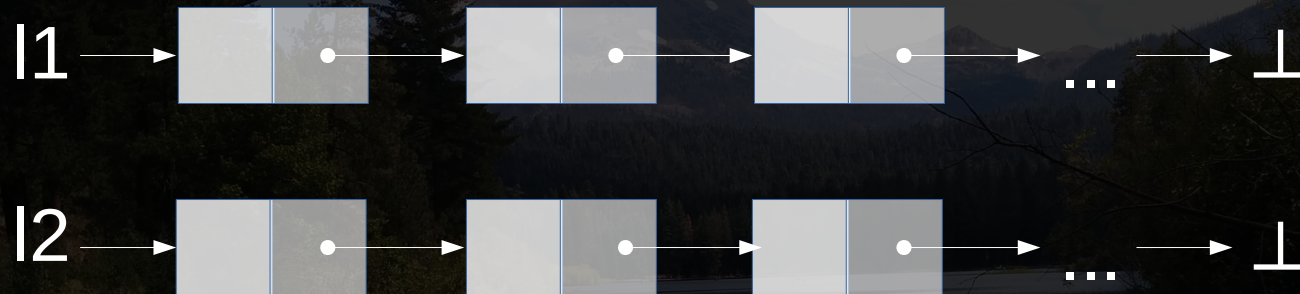

How do we encode heap?

```
public static void main(String[] args) {  
    final int size = 10;  
    final int[] table = new int[size];  
    Node l1 = null, l2 = null;  
    for (int i=0; i<args.length; i++) {  
        int d = Integer.parseInt(args[i]);  
        if (d >= 0 && d < size)  
            l1 = new Node(l1, d);  
        else l2 = new Node(l2, d);  
    }  
    while (l1 != null) {  
        table[l1.data]++;  
        l1 = l1.next;  
    }  
}
```

```
public static class Node {  
    final Node next;  
    final int data;  
    public Node(Node next, int data) {  
        this.next = next;  
        this.data = data;  
    }  
}
```

For all Node objects reachable from l1 at this location: $0 \leq l1.data < table.length$

Space invariants



- Lightweight notion of invariants
- Summarize states of all objects of a particular class
- Specific to program and properties to be proven
- Can be inferred effectively using Horn solvers

Java to IVL

```
while (l1 != null) {  
    table[l1.data]++;  
    l1 = l1.next;  
}
```

```
while l1 != null  
    data, next = pull(l1)  
    assert 0<=data<10  
    tmp = pull(table, data)  
    push(table, data, tmp+1)  
    l1 = next
```


Removing heap interaction

```
while l1 != null
  data, next = pull(l1)
  assert 0 <= data < 10
  tmp = pull(table, data)
  push(table, data, tmp+1)
  l1 = next
```

```
while l1 != null
  havoc(data, next)
  assume  $\Phi_{\text{Node}}(l1, data, next)$ 
  assert 0 <= data < 10
  havoc(tmp)
  assume  $\Phi_{\text{int[]}}(table, data, tmp)$ 
  assert  $\Phi_{\text{int[]}}(table, data, tmp+1)$ 
  l1 = next
```


IVL to Horn clauses

```
assume  $\Phi_{\text{int}[]}$ (table, data, tmp)  
assert  $\Phi_{\text{int}[]}$ (table, data, tmp+1)
```

$$p3(l1, \text{table}, \text{next}, \text{tmp}) \leftarrow \Phi_{\text{int}[]}(\text{table}, \text{data}, \text{tmp}) \wedge p2(l1, \text{table}, \text{next})$$
$$\Phi_{\text{int}[]}(\text{table}, \text{data}, \text{tmp}+1) \leftarrow p3(l1, \text{table}, \text{next}, \text{tmp})$$

Solve Horn clauses

```
public static void main(String[] args) {  
    final int size = 10;  
    final int[] table = new int[size];  
    Node l1 = null, l2 = null;  
    for (int i=0; i<args.length; i++) {  
        int d = Integer.parseInt(args[i]);  
        if (d >= 0 && d < size)  
            l1 = new Node(l1, d);  
        else l2 = new Node(l2, d);  
    }  
    while (l1 != null) {  
        table[l1.data]++;  
        l1 = l1.next;  
    }  
}
```


References as tuples

- Capture distinguishing features about reference
 - e.g. dynamic type, allocation site, values of final fields
- Immutable
- Reading/writing through pull/push would defeat their purpose
- e.g. nodes in l1 have different allocation site than nodes in l2

Solve Horn clauses

```
public static void main(String[] args) {  
    final int size = 10;  
    final int[] table = new int[size];  
    Node l1 = null, l2 = null;  
    for (int i=0; i<args.length; i++) {  
        int d = Integer.parseInt(args[i]);  
        if (d >= 0 && d < size)  
            l1 = new Node(l1, d);  
        else l2 = new Node(l2, d);  
    }  
    while (l1 != null) {  
        table[l1.data]++;  
        l1 = l1.next;  
    }  
}
```

Can be verified with these
space invariants:

$$\phi_{\text{int}[]}(\text{o}, \text{idx}, \text{data}) := \text{true}$$
$$\phi_{\text{Node}}(\text{o}, \text{data}, \text{next}) := \text{o} \downarrow_{\text{allocSite}} = 119 \rightarrow (0 \leq \text{data} < 10 \wedge (\text{next} \downarrow_{\text{id}} = 0 \vee \text{next} \downarrow_{\text{allocSite}} = 119))$$

Optimizations and extensions

- Optimized push-pull placement
- Method in-lining
- Flow sensitive space invariants
- Array invariants: include accessed index in space invariant

Limitations

- Space invariants can only express relation between multiple objects in special cases
- Ability to distinguish different objects depends on the precision of the static analysis



Implementation

- Available on GitHub
- Front-end Soot: Java bytecode \rightarrow Jimple \rightarrow Horn.
- Implementation is sound only for subset of Java. Threads, reflection, etc are not supported; integers are treated as natural numbers, etc.
- Supports different back-ends (Spacer and Eldarica)

Experimental results

		Correct		Incorrect		TO	N/A
		Safe	Unsafe	Imprecise	Unsound		
JayHorn benchmarks (41 safe, 41 unsafe)	JayHorn (Eldarica)	33	41	3	0	5	0
	JayHorn (Spacer)	37	41	3	0	1	0
	CPAChecker	5	5	0	0	0	72
MinePump (43 safe, 21 unsafe)	JayHorn (Eldarica)	32	21	11	0	0	0
	JayHorn (Spacer)	32	19	8	0	5	0
	CPAChecker	43	21	0	0	0	0
CBMC benchmarks (35 safe, 7 unsafe)	JayHorn (Eldarica)	29	7	3	0	0	3
	JayHorn (Spacer)	29	7	3	0	0	3
	CPAChecker	6	0	0	0	0	36
Total (119 safe, 76 unsafe)	JayHorn (Eldarica)	94	69	17	0	5	3
	JayHorn (Spacer)	98	67	14	0	6	3
	CPAChecker	54	26	0	0	0	108



Download, read development blog and papers,
contribute!

<http://jayhorn.github.io/jayhorn/>