

# Recent Improvements of Theory Reasoning in Vampire

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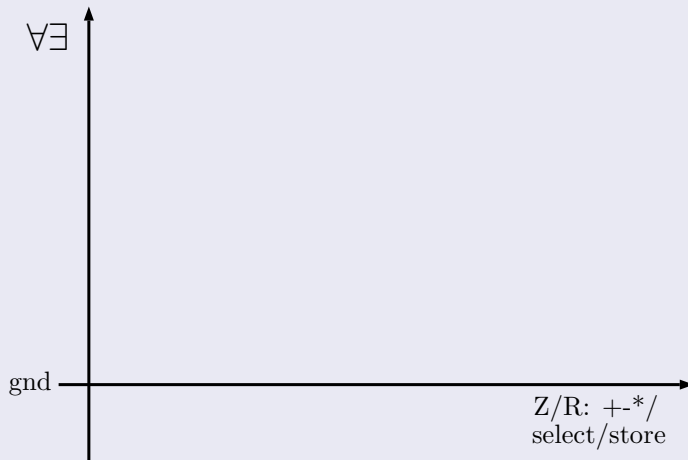
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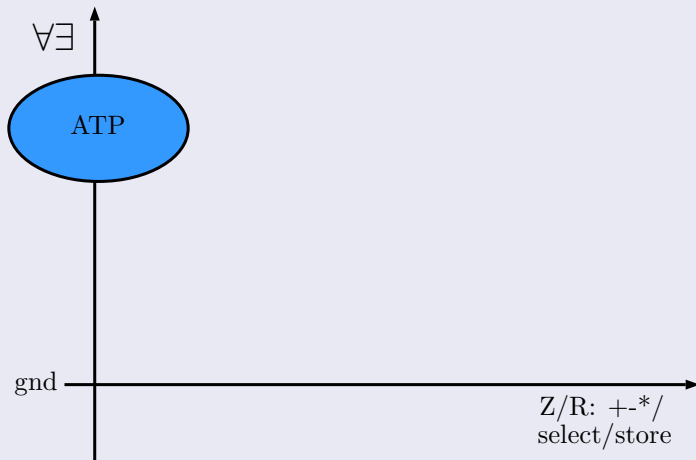
<sup>4</sup>EasyChair

IWIL 2017 – Maun, May 7, 2017

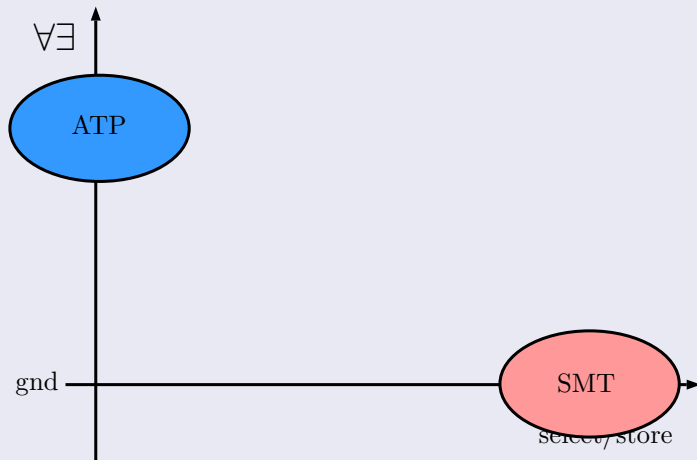
## Two Dimensions of Complexity



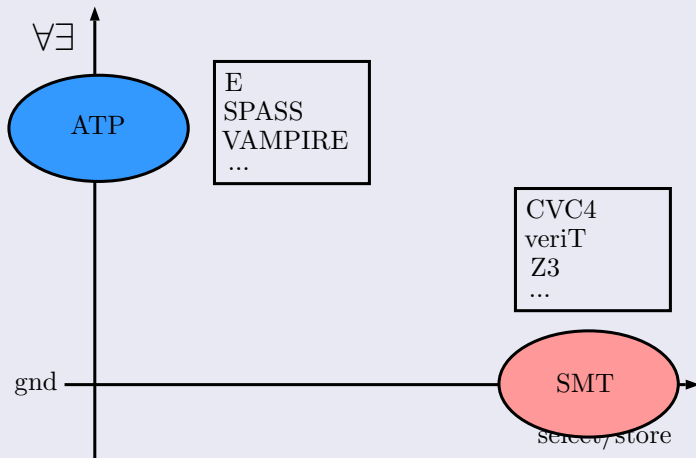
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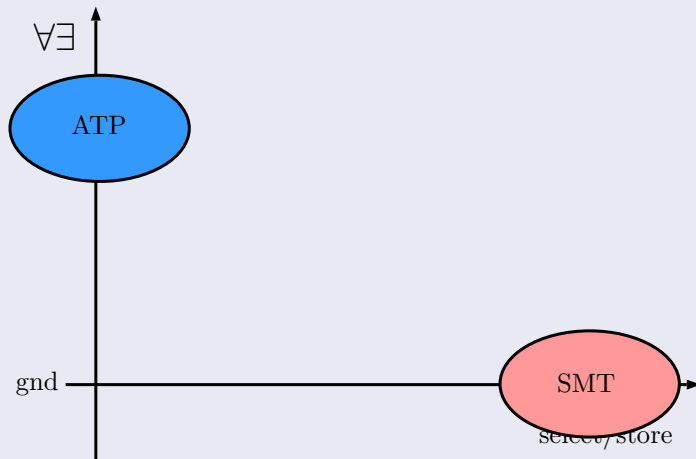
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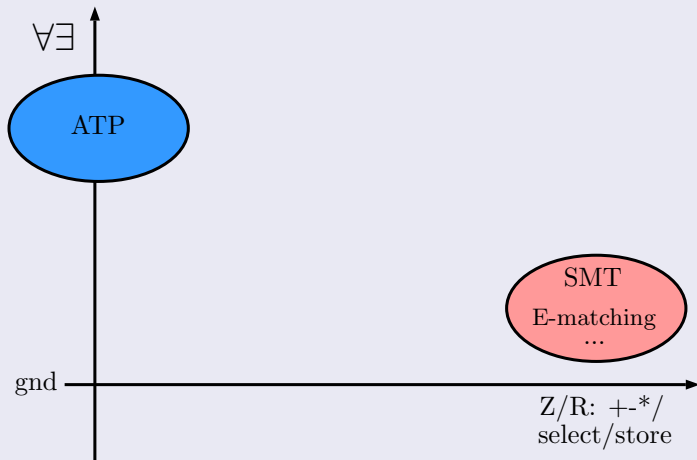
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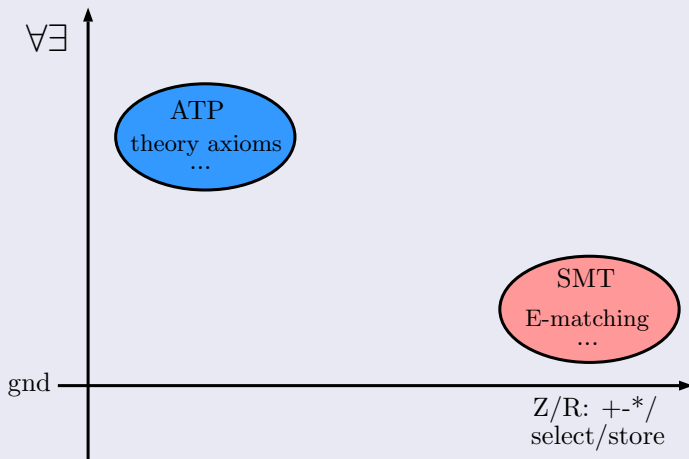
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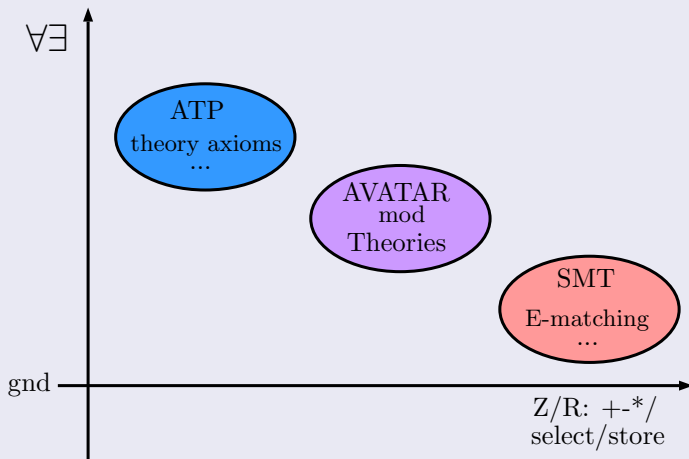


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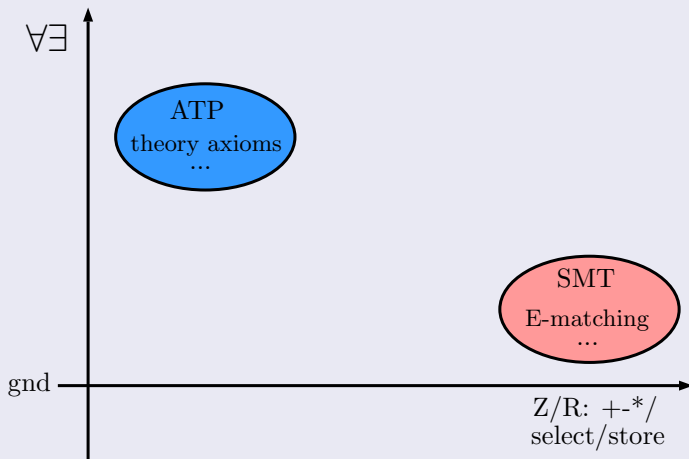




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- new constrains can be often “discharged” by 1.

- 1 Short preliminaries
- 2 Theory instantiation
- 3 Abstraction through unification
- 4 Experiments
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## Main Arsenal for Theory reasoning in Vampire

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## Theory abstraction rule

$$L[t] \vee C \implies x \not\approx t \vee L[x] \vee C,$$

where  $L$  is a theory literal,  $t$  a non-theory term, and  $x$  fresh.

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NB: abstraction can be “undone” by the equality factoring rule

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Consider the conjecture  $(\exists x)(x + x \simeq 2)$  negated and classified to

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It takes Vampire 15 seconds to solve using theory axioms deriving lemmas such as

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immediately resolve to

$$x * x \not\simeq 4 \vee 2 \simeq x,$$

but this cannot be solved with axioms only in reasonable time.

As an inference rule

$$\frac{C}{(D[x])\theta} \textit{TheoryInst}$$

where  $\mathcal{A}_{(P)}(C) = T[x] \rightarrow D[x]$  is a (partial) abstraction of  $C$ ,  
and  $\theta$  a subst. such that  $T[x]\theta$  is valid in the underlying theory.

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Implementation:

- Abstract relevant literals
- Collect relevant pure theory literals  $L_1, \dots, L_n$
- Run an SMT solver on  $T[\mathbf{x}] = \neg L_1 \wedge \dots \wedge \neg L_n$
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Consider a unit clause  $p(1, 5)$  abstracted as

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But we can obtain a “more general” instance

$$p(y + 1, y)$$

using equality resolution.

Example (some literals constrain less/more than others)

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Three options for thi:

- **strong**: Only select strong literals where a literal is strong if it is a negative equality or an interpreted literal
- **overlap**: Select all strong literals and additionally those theory literals whose variables overlap with a strong literal
- **all**: Select all non-trivial pure theory literals

Recall that we collect relevant pure theory literals  $L_1, \dots, L_n$  to run an SMT solver on  $T[\mathbf{x}] = \neg L_1 \wedge \dots \wedge \neg L_n$

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## Example (The Division by zero catch!)

The following two clauses are satisfiable:

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# Interacting with the SMT solver

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Evaluation may fail:

- result out of Vampire's internal range
- result is a proper algebraic number

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Be careful about:

- the interaction with theory axiom support
- handling of division by zero

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Finally, Theory Instantiation could produce

$$p(7).$$

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- 3 incompatible with theory axiom reasoning (theory part requires special treatment)
- 4 inferences need to be protected from undoing abstraction (recall equality resolution)

Instead of full abstraction ...

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and  $\theta$  is the most general such substitution given  $\Gamma$ .

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## Example (strive for generality)

Unifying  $a + b$  with  $c + d$  should produce  $(\{\}, a + b = c + d)$  and not  $(\{\}, a = c \wedge b = d)$ .

```
function mguAbs( $s, t$ )  
  if  $t$  is a variable and not occurs( $t, s$ ) then  
    return ( $\{t \mapsto s\}, true$ )  
  if  $s$  is a variable and not occurs( $s, t$ ) then  
    return ( $\{s \mapsto t\}, true$ )  
  if  $s$  and  $t$  have different top-level symbols then  
    if canAbstract( $s, t$ ) then  
      return ( $\{\}, s = t$ )  
    return ( $\perp, \perp$ )  
  if  $s$  and  $t$  are constants then  
    return ( $\{\}, true$ )  
  ...
```

...

**let**  $s = f(s_1, \dots, s_n)$  **and**  $t = f(t_1, \dots, t_n)$  **in**

$\theta = \{\}$  **and**  $\Gamma = true$

**for**  $i = 1$  **to**  $n$  **do**

$(\theta_i, \Gamma_i) = \text{mgU}_{\text{Abs}}((s_i\theta), (t_i\theta))$

**if**  $(\theta_i, \Gamma_i) = (\perp, \perp)$  **or**  $\text{canAbstract}(s, t)$  **and**  $\Gamma_i \neq true$

**then**

**if**  $\text{canAbstract}(s, t)$  **then**

**return**  $(\{\}, s = t)$

**return**  $(\perp, \perp)$

$\theta = \theta \circ \theta_i$  **and**  $\Gamma = \Gamma \wedge \Gamma_i$

**return**  $(\theta, \Gamma)$

# When do we abstract?

## Example (do not produce unsatisfiable constraints)

Allowing  $p(1)$  and  $p(2)$  to unify under the constraint that  $1 \simeq 2$  is not useful in any context.

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## For option to choose from:

- `interpreted_only`: only produce a constraint if the top-level symbol of both terms is a theory-symbol,
- `one_side_interpreted`: only produce a constraint if the top-level symbol of at least one term is a theory symbol,
- `one_side_constant`: as `one_side_interpreted` but if the other side is uninterpreted it must be a constant,
- `all`: allow all terms of theory sort to unify and produce constraints.

New inference rule: **Resolution-wA**

$$\frac{A \vee C_1 \quad \neg A' \vee C_2}{(\Gamma \rightarrow (C_1 \vee C_2))\theta},$$

where  $(\theta, \Gamma) = \text{mgu}_{\text{Abs}}(A, A')$  and  $A$  is not an equality literal.

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When starting from an obvious conflict:

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eager evaluation destroys this by simplifying

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But **Resolution-wA** allows us to still derive

$$4 \neq x + 3$$

and Theory Instantiation could derive the empty clause.

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For this experiment:

- 24 reasonable combinations of option values: `fta`, `uwa`, `thi`
- approx. 100 000 runs in total

# Comparison of Three Options

fta	uwa	thi	solutions
on	off	all	252
on	off	overlap	265
on	off	strong	266
on	off	off	276
off	all	all	333
off	all	overlap	351
off	all	strong	354
off	one_side_interpreted	all	364
off	all	off	364
off	one_side_constant	all	374
off	interpreted_only	all	379
off	one_side_interpreted	overlap	385
off	one_side_interpreted	strong	387
off	off	all	392
off	one_side_constant	strong	397
off	one_side_constant	overlap	401
off	interpreted_only	overlap	407
off	one_side_interpreted	off	407
off	interpreted_only	strong	409
off	one_side_constant	off	417
off	off	overlap	428
off	interpreted_only	off	430
off	off	strong	431
off	off	off	450

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Problems newly solved thanks to thi and uwa:

- ALIA (arrays and linear integer arithmetic): 2
- AUFNIRA: 3
- LIA : 10
- LRA : 1
- UFLIA : 1
- UFNIA : 1

Two new techniques for reasoning with theories and quantifiers



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Thank you for your attention!