Pseudospectral method for time-fractional differential equation with boundary conditions

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Abstract

I aim to present a computational method for numerically solving a general form of time-fractional differential equation with boundary conditions. In this method, main problem is converted to a new problem with homogeneous conditions and then an equivalent integro-differential equation by proposing a technique. Next, the shifted Jacobi polynomials are implemented to approximate all the known and unknown functions in the equivalent integro-differential equation. Finally, a system of nonlinear algebraic equations is achieved by utilizing the collocation method which it is solved by Newton's iterative method. The benefits of this method are faster convergence and avoidance of a singular system.
Keywords: Fractional differential equation, Caputo derivative, Jacobi polynomials, Spectral method, Operational matrix.

Main problem

In this article, time-fractional differential equation in general form is considered as [1]

$$\frac{\partial^\alpha}{\partial t^\alpha}u(x,t) = F(x,t,u,u_x,u_{xx}) + h(x,t), \quad t>0, \quad 0<\alpha\le 1,$$
$$u(x,0)=g_0(x), \quad u(1,t)=g_1(t), \quad u_x(1,t)=g_2(t), \quad x\in[0,1],$$

and specification

$$u_x(0,t)=\theta(t).$$

where the fractional derivative is in the Caputo sense, $g_0(x)$, $g_1(t)$ and $g_2(t)$ are considered as known functions and $\theta(t)$ and $u(x,t)$ are unknown.
I propose a method to numerically solve the problem. This method is examined on some numerical tests to demonstrate the its great performance.

References

1. M. Dehghan, S. A. Yousefi, K. Rashedi, Ritz-Galerkin method for solving an inverse heat conduction problem with a nonlinear source term via Bernstein multi-scaling functions and cubic B-spline functions, Taylor & Francis, Inv. Prob. Scis. Eng. 21 (2013) 500-523.