Error estimations of Turán formulas with Gori-Micchelli and generalized Chebyshev weight functions

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Abstract

S. Li in [Studia Sci. Math. Hungar. 29 (1994) 71–83] proposed a Kronrod type extension to the well-known Turán formula. He showed that such an extension exists for any weight function. For the classical Chebyshev weight function of the first kind, Li found the Kronrod extension of Turán formula that has all its nodes real and belonging to the interval of integration, $[-1, 1]$. In this paper we show the existence and the uniqueness of the additional two cases – the Kronrod exstensions of corresponding Gauss-Turán quadrature formulas for special case of Gori-Micchelli weight function and for generalized Chebyshev weight function of the second kind, that have all their nodes real and belonging to the integration interval $[-1, 1]$. Numerical results for the weight coefficients in these cases are presented, while the analytic formulas of the nodes are known.

Keywords: Gauss-Turán quadrature formulas, Kronrod extension, Gori-Micchelli weight function, Generalized Chebyshev weight function of the second kind, Error estimation

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