Non-linear multi-point flux approximation in the near-well region

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Abstract

In reservoir engineering, accurate well modeling is crucial for reliable fluid flow simulations. Flow in the entire reservoir is induced mainly by wells, therefore poor near-well modeling results in accuracy loss throughout the model. Groundwater flow equation (Richards equation) is obtained from the conservation law, Boussinesq approximation and Darcy's [5]

$$\frac{\partial \theta}{\partial t} = \nabla \cdot \left(k_{\mathrm{r}}(s) \mathbb{K}(\mathbf{x}) \nabla h \right),$$

where θ is the water content, k_{r} is relative conductivity, $\mathbb{K}(\mathbf{x})$ is symmetric and positive definite hydraulic conductivity tensor, and h is the hydraulic head.

Hydraulic head varies logarithmically and its gradient changes sharply in the well viscinity. Thus, linear approximation of hydraulic head is inappropriate and numerical methods based on it are inaccurate in the near-well region. Local grid refinement can alleviate the problem, but this comes at a computational cost.

Non-linear multi-point flux approximation [3] is obtained as a combination of two one-sided linear fluxes. Non-linear multi-point scheme is second order accurate and preserves the elliptic local maximum principle. Nevertheless, linear approximation is employed and therefore the accuracy is lost if a well is present. Not only that the hydraulic head is inconsistent, but also the well extraction rate is wrong.

Two correction methods presented in [1, 2] for non-linear two-point scheme are also applicable for multi-point scheme. The WFC scheme modifies flux on the well faces, while the fluxes through other faces are approximated using the unmodified non-linear multi-point scheme. The NWC scheme modifies fluxes in the specified near well region. The NWC scheme is further generalized in [4] for polyhedral grids and arbitrary wells. Both of these corrections change only the approximation of one-sided linear fluxes, but use the same logic for their combining as the non-linear multi-point scheme.

Obtained results indicate that WFC scheme greatly improves the well extraction rate compared to the uncorrected scheme, but the hydraulic head is still inconsistent even though it is improved. On the other hand, NWC scheme gives not only improved well extraction rate, but also obtained hydraulic head is second order accurate.

Both correction methods are implemented in WODA [6] (Well Outline and Design Aid), an open-source tool for simulation of unsaturated groundwater flows in discontinous and anisotropic enviroment.

Keywords: Multi-point nonlinear finite volume method, Near-well modeling, Groundwater flow simulations

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