A Nyström method for Fredholm integral equations with exponential weights on $(0, +\infty)$

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Abstract

We propose a Nyström-type method to approximate the solution of integral equations of the form

$$f(x) - \mu \int_0^{+\infty} k(x,y) f(y) w(y)\, \mathrm{d}y = g(x)\,, \quad x \in (0, +\infty),$$

where μ is a real parameter,

$$w(y) = \mathrm{e}^{-y^{-\alpha} - y^{\beta}}\,, \quad \alpha > 0\,,\ \beta > 1\,,$$

the given functions k and g can grow exponentially with respect to their arguments, when they approach to 0^+ and/or $+\infty$ [5].

Since the solution of this kind of equations can increase exponentially for $x \to 0^+$, the methods based on the weighted polynomial approximation with Laguerre-type weights are not suitable in this case. So, a first difficulty is to choose proper function spaces where these equations can be studied. To this aim, we introduce another exponential weight u and new function spaces C_u with weighted uniform metric. We prove that the proposed method is stable and convergent in this metric, using our recent results on polynomial approximation with the weight u [1–3] and Gaussian rules with the weight w [4].

Finally, we give a priori error estimates and show some numerical examples, including a comparison with other Nyström methods.

Keywords: Fredholm integral equation, Nyström method, Gaussian rule, exponential weights, real semiaxis.

References

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