

Numerical evaluations on power series including the numbers of Lyndon words and interpolation functions for the Apostol-type polynomials

Irem Kucukoglu¹ and Yilmaz Simsek²

¹ Department of Software Engineering, Faculty of Engineering and Architecture
Antalya Akev University 07525 Antalya, Turkey

`ikucukoglu@akdeniz.edu.tr`

² Department of Mathematics, Faculty of Science University of Akdeniz 07058
Antalya, Turkey

`ysimsek@akdeniz.edu.tr`

Abstract

Recently, the Lyndon words and their numbers have been investigated by researchers using various methods. Contrary to other studies, in this paper, we use the methods associated with a family of zeta functions interpolating a family of higher-order Apostol-type numbers and polynomials. The main purpose of this paper is not only to define power series including the numbers of Lyndon words and binomial coefficients, but also to construct new computation algorithms in order to simulate these series with numerical analysis and plots. With these algorithms, we provide novel computational methods to the area of the combinatorics on words. Moreover, in order to reduce algorithmic complexity of these algorithms, our other aim is to present an approximation to these series by rational functions of the Apostol-type numbers. Finally, we give some remarks, observations and comments on these polynomials and the numbers of Lyndon words.

Keywords: Lyndon words, Special numbers and polynomials, Apostol-type numbers and polynomials, Arithmetical function, Interpolation function, Riemann zeta function, Hurwitz-Lerch zeta function, Binomial coefficient, Algorithm.

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