Orthogonal polynomials, Geronimus transformations and quadrature rules

Francisco Marcellán

Instituto de Ciencias Matemáticas (ICMAT), Calle Nicolás Cabrera 13-15, Campus Cantoblanco UAM, 28049 Madrid, Spain,
and
Departamento de Matemáticas, Universidad Carlos III de Madrid, Avenida de la Universidad 30, 28911 Leganés, Spain
pacomarc@ing.uc3m.es

1. Abstract

Given a sequence $\{P_n\}_{n\geq 0}$ of monic orthogonal polynomials with respect to a linear functional u and a fixed integer k, we establish necessary and sufficient conditions so that the quasi-orthogonal polynomials Q_n, defined by $Q_n(x) = P_n(x) + \sum_{i=1}^{k} b_{i,n} P_{n-i}(x), n \geq 0$, with $b_{i,n} \in R$, and $b_{k,n} \neq 0$ for $n \geq k$, also constitute a sequence of orthogonal polynomials with respect to a linear functional v which is called the Geronimus transformation of u ([1]). Indeed, there exists a polynomial $h(x)$ of degree k such that $h(x)v = u$. This is an inverse problem of one analyzed in [4]. The characterization turns out to be equivalent to recurrence formulas for the coefficients $b_{i,n}$. An algorithm to find the polynomial $h(x)$ is presented. Some illustrative examples are provided.

These results are used to state explicit relations between various type quadrature rules. The classical situation, when the algebraic degree of precision is the highest possible, is well-known and the quadrature formulas are the Gaussian ones whose nodes coincide with the zeros of the corresponding orthogonal polynomials and the Christoffel numbers are expressed in terms of the so-called kernel polynomials [5]. In our case, we can relax the requirement for the highest possible degree of precision in order to gain the possibility to either approximate integrals of more specific continuous functions containing a polynomial factor or including additional fixed nodes. The construction of such quadrature processes is related to quasi-orthogonal polynomials (see [2]) and [3]).

This is a joint work with C. F. Bracciali (UNESP, Brazil) and S. Varma (University of Ankara, Turkey).

Keywords: Quasi-orthogonality, Geronimus transformations, orthogonal polynomials, zeros and Christoffel numbers. LaTeX

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