

Error bounds for Kronrod extension of generalizations of Micchelli-Rivlin quadrature formula for analytic functions

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Abstract

We consider Kronrod extension of generalizations of the well known Micchelli-Rivlin quadrature formula, of highest algebraic degree of precision, for the Fourier-Chebyshev coefficients. For analytic functions the remainder term of these quadrature formulas can be represented as a contour integral with a complex kernel. We study the kernel, on elliptic contours with foci at the points ∓ 1 and a sum of semi-axes $\rho > 1$, for the quoted quadrature formulas. Starting from the explicit expression of the kernel, we determine the locations on the ellipses where maximum modulus of the kernel is attained. So we derive effective L^∞ -error bounds for these quadrature formulas. Complex-variable methods are used to obtain expansions of the error in these quadrature formulas over the interval $[-1, 1]$. Finally, effective L^1 -error bounds are also derived for these quadrature formulas.

Keywords: Kronrod extension of the generalizations of Micchelli-Rivlin quadrature formula, Chebyshev weight function of the first kind, Error bound, Remainder term for analytic functions, Contour integral representation

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