Multiple Hermite polynomials and simultaneous quadrature

Walter Van Assche[1]

Department of Mathematics, KU Leuven, Celestijnenlaan 200B box 2400,
BE-3001 Leuven, Belgium
walter@wis.kuleuven.be

Abstract

Multiple Hermite polynomials are an extension of the classical Hermite polynomials for which orthogonality conditions are imposed with respect to $r > 1$ normal (Gaussian) weights with different means c_i, $1 \leq i \leq r$. These polynomials have a number of properties, such as a Rodrigues formula, recurrence relations (connecting polynomials with nearest neighbor multi-indices), a differential equation, etc. The asymptotic distribution of the (scaled) zeros is well understood and an interesting new feature happens: depending on the distance between the means c_i, $1 \leq i \leq r$: the zeros may accumulate on s disjoint intervals, where $1 \leq s \leq r$. We will use the zeros of these multiple Hermite polynomials to approximate integrals of the form $\int_{-\infty}^{\infty} f(x) \exp(-x^2 + c_j x)\, dx$ simultaneously for $1 \leq j \leq r$. The behavior of the quadrature weights depends in an important way on whether or not the zeros are on disjoint intervals or on one interval.

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