Gauss-Kronrod quadrature:
Recent advances and open questions

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Abstract

Kronrod in 1964, trying to estimate economically the error of the n-point Gauss quadrature formula for the Legendre weight function, developed a new formula by adding to the n Gauss nodes $n + 1$ new ones, which are determined, together with all weights, such that the new formula has maximum degree of exactness. It turns out that the new nodes are zeros of a polynomial orthogonal with respect to a variable-sign weight function, considered by Stieltjes in 1894, without though making any reference to quadrature.

In recent years, Gauss-Kronrod quadrature formulae have attracted considerable attention from both the theoretical and the computational point of view, the former in view of the intriguing mathematical questions they pose and the latter on account of their use in packages of automatic integration; so, these formulae form an active area of research for over 50 years now.

We survey the recent advances on Gauss-Kronrod quadrature, paying particular attention to existence, nonexistence and error term results; at the same time, we point out the important questions that are still open in this area.

Keywords: Gauss quadrature formula, Gauss-Kronrod quadrature formula, Stieltjes polynomials

References

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