Approximation of generalized stochastic processes

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Abstract

Generalized stochastic processes (GSPs) arise as solutions of stochastic partial differential equations (SPDEs) with singularities and represent a good theoretical framework to capture their singular behavior, e.g. if the process possesses infinite variance. The most famous generalized stochastic process is the Gaussian white noise process. We present two theoretical frameworks that provide also the possibility to undertake numerical approximations of generalized stochastic processes:

- Wiener-Itô polynomial chaos expansions,
- Colombeau-type regularizations.

Both theories have the advantage to deal with nonlinear functions of GSPs and therefore they are precious for solving nonlinear SPDEs.

Considering generalized stochastic processes as elements of $S'(\mathbb{R}) \otimes (S)_{-1}$, which is a topological inductive space constructed as the extension of $L^2(\mathbb{R}) \otimes L^2(\Omega, \mathcal{F}, P)$, one can implement the theory of orthogonal polynomials and use a series expansion of GSPs via the Hermite polynomial basis [3]. In this manner the process is uniquely determined by its Fourier coefficients. Truncating the series expansion provides an appropriate approximation of the process. Using this method any SPDE can be transformed into a lower triangular infinite system of PDEs that can be solved recursively. Summing up the obtained coefficients and proving the convergence of the obtained series one arrives to the solution of the initial SPDE.

The other method involves Colombeau algebras [1] of generalized functions denoted by $\mathcal{G}(\mathbb{R}; \mathcal{L}(\Omega, \mathcal{F}, P))$. These are equivalence classes of nets of stochastic processes with smooth sample paths which possess a moderate growth rate and they differ only by a negligible process i.e. a process that is rapidly decreasing to zero. Using this method the sample paths of all input data in a SPDE are now smoothed out by the help of a regularization parameter until they become smoothly differentiable and the SPDE is then solved pathwisely via these smooth sample paths. The final solution corresponds to the Stratonovich-integral solution of the original SPDE.

In this talk we provide a comparison of the two methods and reflect on some recent advances and their applications to solving SPDEs [2, 4–9].

Keywords: Hermite polynomials, Hermite functions, orthogonal expansion, Colombeau algebra, regularization, white noise, generalized stochastic process

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