A product integration rule for hypersingular integrals on the positive semi-axis

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Abstract

The talk deals with the numerical computation of hypersingular integrals of the type

$$\;=\!\!\!\!\!\!\int_0^{+\infty} \frac{f(x)}{(x-t)^{p+1}} u_\gamma(x)dx,$$

where the integral is understood in the Hadamard finite part sense, p is a positive integer, $u_\gamma(x) = e^{-x/2}x^\gamma$ is a Laguerre weight and $t > 0$. Many of the exiting methods make use of the decomposition

$$\int_0^{+\infty} \frac{f(x) - \sum_{k=0}^{p} \frac{f^{(k)}(t)}{k!}(x-t)^k}{(x-t)^{p+1}} u_\gamma(x)dx + \sum_{k=0}^{p} \frac{f^{(k)}(t)}{k!} \;=\!\!\!\!\!\!\int_0^{+\infty} \frac{u_\gamma(x)}{(x-t)^{p+1-k}} dx,$$

requiring the samples of the function's derivatives. Here we propose a product integration rule based on a suitable Lagrange process, with the advantage of avoiding the derivatives computations. We determine conditions under which the rule is stable and convergent in suitable weighted uniform spaces. Finally, we show the performance of the procedure by proposing some numerical tests.

Keywords: Hypersingular integrals, Lagrange interpolation, Orthogonal polynomials, Product integration rules