On a quadrature method for Prandtl's integro-differential equations in weighted Zygmund spaces with uniform norm

Maria Carmela De Bonis and Donatella Occorsio

Department of Mathematics, Computer Science and Economics, University of Basilicata, Via dell'Ateneo Lucano 10, Potenza, Italy
{mariacarmela.debonis,donatella.occorsio}@unibas.it

Abstract

This talk deals with a quadrature method for approximating the solutions of singular integro-differential equations of Prandtl's type

$$u(x)f(x) + af'(x) + \frac{b}{\pi}\int_{-1}^{1}\frac{f'(y)}{y-x}dy + \frac{1}{\pi}\int_{-1}^{1}k(x,y)f(y)dy = g(x), \quad |x| \le 1,$$

where the unknown solution f satisfies the additional conditions

$$f(-1) = f(1) = 0,$$

$a, b \in \mathbb{R}$ are given constants and u, g, k are given functions.

Several authors have studied this type of integro-differential equations and related numerical methods (see, for example, [4, Ch. 3, Section I], [3, Section 3], [5, Section 9.53], [1],[2]).

We prove that the proposed method is stable and convergent. We give error estimates in weighted spaces of continuous functions equipped with uniform norms. Moreover we show some numerical tests that confirm the theoretical estimates.

Keywords: Cauchy singular integral equations, Singular integro-differential equations, Gaussian quadrature rules, Orthogonal polynomials

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