Fuzzy Conditional Inference with Twofold Fuzzy Set on Fuzzy Intuitions: An Application to Data Analytics

Venkata Subba Reddy Poli

EasyChair preprints are intended for rapid dissemination of research results and are integrated with the rest of EasyChair.
Fuzzy Conditional Inference with Twofold Fuzzy Set on Fuzzy Intuitions
An Application to Data Analytics

Poli Venkata Subba Reddy

January 14, 2018

Abstract  Fukami, Muzumoto and Tanak (FMK) are studied based on Godel and
Standard sequence definitions. These conditional inference are based on certain con-
ditions. In this paper, fuzzy conditional inference is studied for fuzzy intuitions which
contain “and/or ”, “if · · · then · · · else · · · ” and truth variables. some more fuzzy in-
tuitions are proposed. The proposed methods satisfy many fuzzy intuitions. Zadeh
first defined fuzzy set as a single membership function. The two fold fuzzy sets with
two membership functions will give more evidence than a single membership. The
fuzzy intuitions are studied with two fold fuzzy sets. The fuzzy certainty factor (FCF)
is studied to made as a single membership function. Business intelligence is given as
an example for the fuzzy intuitions.

Keywords  Fuzzy sets · Fuzzy logic · Fuzzy reasoning · Two fold fuzzy set · Fuzzy
conditional inference · Fuzzy certainty factor · Fuzzy truth variables · Business intel-
ligence

1. Introduction

There are many theories proposed to deal with incomplete information. The fuzzy
logic[20] deals with “Belief” rather than “likelihood” (probability). Zadeh [15]. Mam-
FMK [1] proposed fuzzy intuitions and shown that Zadeh fuzzy conditional inference
is not suitable for these intuitions. FMK [6] adapting the Godel definition to prove

Poli Venkata Subba Reddy
Department of Computer Science and Engineering, Sri Venkateswara University, Tirupati-517502, Andhra
Pradesh, India
email: vsrpoli@hotmail.com
smoe fuzzy intuitions. The methods used by FMK are having the certain restrictions. There is a need of method to prove some more fuzzy intuitions. Zadeh defined fuzzy set with a single membership function. The fuzzy set with two membership functions will give more evidence than a single membership function. Here considered the two fold fuzzy set with “True” and “False”. The fuzzy intuition contained “and/or”, “if then else ” and truth variables are studied.

can consider the given fuzzy inference

**Type-1**
If \( x \) is \( \tilde{P} \tilde{Q} \) and \( x \) is \( \tilde{Q} \) or \( x \) is \( \tilde{R} \) then \( y \) is \( \tilde{S} \) is \( \tau \)
\( x \) is \( \tilde{P}_1 \) and \( x \) is \( \tilde{Q}_1 \) or \( x \) is \( \tilde{R}_1 \)

\( y \) is ?

If apple is red and apple is ripe or apple is sweet then apple is good is true
apple is very red and apple is more or less ripe or apple is not sweet

\( y \) is ?

**Type-2**
If \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{Q} \) else \( y \) is \( \tilde{R} \) is \( \tau \)
\( x \) is \( \tilde{P}_1 \)

\( y \) is ?

If Rama is Tall then Sita is Small else Sita is Middle is true
Rama is very Tall

\( Sita \) is ?

2. Fuzzy Logic Based on Truth Functions

Zadeh defined fuzzy set with a single membership function [20]. The fuzzy set with two fuzzy member functions “True” and “False” will give more evidence than the single fuzzy membership function to deal with incomplete information. In the following “two fold fuzzy set” is defined with “True” and “False” fuzzy membership functions. The fuzzy logic and fuzzy reasoning of single membership function is extended to fuzzy logic with two membership functions “True” and “False”.

2.1. The Two Fold Fuzzy Sets

“A two fold fuzzy set” may be defined with two membership functions “True” and “False” for the proposition of type “\( x \) is \( A \)”. The fuzzy set with two membership functions “True” and “False” will give more evidence than the single membership function.

For instance “Rama has Headache ”. 
In this fuzzy proposition, the fuzzy set “Headache” may be defined with “True” and “False”.

**Definition 2.1** The “a two fold fuzzy set” \( \tilde{A} \) in a universe of discourse \( X \) is defined by its membership function \( \mu_{\tilde{A}}(x) \rightarrow [0, 1] \), where \( \tilde{A} = \{\mu_{\text{True}}^{x}(x), \mu_{\text{False}}^{x}(x)\} \) and \( x \in X \).

\( \mu_{\text{True}}^{x}(x) \) and \( \mu_{\text{False}}^{x}(x) \) are the fuzzy membership functions of the “a two fold fuzzy set” \( \tilde{A} \),

\[
\mu_{\text{True}}^{x}(x) = \int \mu_{\text{True}}^{x} / x(x) = \mu_{\text{True}}^{x}(x_1)/x_1 + \cdots + \mu_{\text{True}}^{x}(x_n)/x_n, \\
\mu_{\text{False}}^{x}(x) = \int \mu_{\text{False}}^{x} / x = \mu_{\text{False}}^{x}(x)\mu_{\text{False}}^{x}(x_1)/x_1 + \cdots + \mu_{\text{False}}^{x}(x_n)/x_n, 
\]

where “+” is union.

For example, “young” may be given for the fuzzy proposition “x is young”

\[
young = \{\mu_{\text{True}}^{\text{young}}(x), \mu_{\text{False}}^{\text{young}}(x)\},
\]

\[
\mu_{\text{True}}^{\text{young}}(x) = \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30, \\
+ 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\},
\]

\[
\mu_{\text{False}}^{\text{young}}(x) = \{0.36/10 + 0.31/15 + 0.26/20 + 0.23/25 + 0.2/30 + 0.18/35 \\
+ 0.16/40 + 0.14/45 + 0.12/50\}. 
\]

For instance. “Rama is young” with fuzziness \( [0.8, 0.2] \), where 0.8 is “True” and 0.2 is “False”.

The Graphical representation of “True” and “False” of “young” is shown in Fig. 1.

![Fig.1 Two fold fuzzy set membership functions](image)

**2.2. The Fuzzy Logic with Truth Functions**

The fuzzy logic is combination of fuzzy sets using logical operators. The fuzzy logic with “two fold fuzzy sets” is combination of “two fold fuzzy sets” using logical operators. The fuzzy logic bases on “two fold fuzzy sets” can be studied similar lines of Zadeh’s fuzzy logic.

Some of the logical operations are given below for fuzzy sets with two fold fuzzy
membership functions.

\( \tilde{A}, \tilde{B} \) and \( \tilde{C} \) are fuzzy sets with two fold fuzzy membership functions.

Let tall, weight and more or less weight are two fold fuzzy sets:

- **tall**: \( \tilde{\text{tall}} = [0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5] \)
- **weight**: \( \tilde{\text{weight}} = [0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.3/x_4 + 0.2/x_5, 0.2/x_1 + 0.2/x_2 + 0.1/x_3 + 0.1/x_4 + 0.1/x_5] \)
- **more or less weight**: \( [0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.5/x_4 + 0.4/x_5, 0.4/x_1 + 0.4/x_2 + 0.3/x_3 + 0.3/x_4 + 0.3/x_5] \).

**Negation**

- \( x \) is not \( \tilde{A} \)
  \[ \tilde{A}(x) = \{1 - \mu_{\tilde{A}}^{\text{true}}(x), 1 - \mu_{\tilde{A}}^{\text{false}}(x)\}/x \]
- \( x \) is not tall
  \[ \tilde{\text{tall}} = [0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5] \]
  \[ 1 - \tilde{\text{tall}} = [0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.6/x_4 + 0.8/x_5, 0.5/x_1 + 0.6/x_2 + 0.7/x_3 + 0.8/x_4 + 0.9/x_5] \]

**Disjunction**

- \( x \) is \( \tilde{A} \) or \( y \) is \( \tilde{B} \)
  \[ \tilde{A} \lor \tilde{B} = \{\max(\mu_{\tilde{A}}^{\text{true}}(x), \mu_{\tilde{B}}^{\text{true}}(y)), \max(\mu_{\tilde{A}}^{\text{false}}(x), \mu_{\tilde{B}}^{\text{false}}(y))\}/(x, y) \]
  \[ \text{tall} \lor \text{weight} = [0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.2/x_3 + 0.1/x_4 + 0.1/x_5] \]

**Conjunction**

- \( x \) is \( \tilde{A} \) and \( y \) is \( \tilde{B} \)
  \[ \tilde{A} \land \tilde{B} = \{\min(\mu_{\tilde{A}}^{\text{true}}(x), \mu_{\tilde{B}}^{\text{true}}(y)), \min(\mu_{\tilde{A}}^{\text{false}}(x), \mu_{\tilde{B}}^{\text{false}}(y))\}/(x, y) \]
  \[ \text{tall} \land \text{weight} = [0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.2/x_4 + 0.2/x_5, 0.1/x_1 + 0.1/x_2 + 0.1/x_3 + 0.1/x_4 + 0.1/x_5] \]

**Implication**

Zadeh [18] fuzzy conditional inference is given as

\[ (1) \quad x \text{ is } A \text{ then } y \text{ is } B = \tilde{A} \land \tilde{B} = \text{min}\{1, 1 - (1 - \tilde{A} \lor \tilde{B})\} \]

Mamdani [5] fuzzy conditional inference is given as

\[ (2) \quad \text{if } x \text{ is } A_1 \text{ and } x_2 \text{ is } A_2 \text{ and } \cdots \text{ and } x_n \text{ is } A_n \text{ then } y \text{ is } \tilde{B} = \text{min}\{A_1, A_2, \ldots, A_n, \tilde{B}\} \]

TSK [5] fuzzy conditional inference is given as

\[ \text{if } x \text{ is } A \text{ then } y(t(x)) = \tilde{B} \]

if \( x_1 \) is \( \tilde{A}_1 \) and \( x_2 \) is \( \tilde{A}_2 \) and \( \cdots \) and \( x_n \) is \( \tilde{A}_n \) then \( y = \tilde{B} = t(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \text{min}\{\tilde{A}_1, \ldots, \tilde{A}_n\} \),
Using t-norm, the fuzzy inference is given as
\[ A \ldots, A_n, B \]

if \( x \) is \( \tilde{A} \) then \( y \) is \( \tilde{B} = t(A) = A \)
i.e., \( \mu_B(y) = \int \mu_A(x) \) quad \( (2.3) \)

if \( x_1 \) is \( \tilde{A}_1 \) and \( x_2 \) is \( \tilde{A}_2 \) and \( \ldots \) and \( x_n \) is \( \tilde{A}_n \) then \( y \) is \( \tilde{B} = \tilde{A}_1 \) and \( \tilde{A}_2, \ldots, \tilde{A}_n = \min \{ \tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n \} \)

\( \tilde{B} = \tilde{A}_1 \) and \( \tilde{A}_2, \ldots, \tilde{A}_n \)

The fuzzy conditional inference is given as,
\[ \text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \{\} \]

\( \text{tall } \rightarrow \text{weight } = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \)
\( 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5 \}

Zadeh [18] fuzzy conditional inference is given as

if \( x \) is \( \tilde{A} \) then \( y \) is \( \tilde{B} \) else \( y \) is \( \tilde{C} = (\tilde{A} \times \tilde{B} + \tilde{A}' \times \tilde{C}) \) where “+” is union

Reddy [13] fuzzy conditional inference is given for “if \( x \) is \( \tilde{A} \) then \( y \) is \( \tilde{B} \) else \( y \) is \( \tilde{C} \)” as,

\( \text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \tilde{A} \rightarrow \tilde{B} \)

\( \text{if } x \text{ is not } \tilde{A} \text{ then } y \text{ is } \tilde{C} = \tilde{A}' \rightarrow \tilde{C} \)

**Composition**

if \( x \) is \( \tilde{A} \) then \( y \) is \( \tilde{B} \)

\( x \) is \( \tilde{A}_1 \)

\( y \) is \( \tilde{A}_1 \) o \( (\tilde{A} \rightarrow \tilde{B}) \)

\[ \tilde{A} \circ (\tilde{A} \rightarrow \tilde{B}) = \{ \min[\mu_{\tilde{A}}(x), \min(1, 1 - \mu_{\tilde{A}}^\text{true}(x) + \mu_B^\text{true}(y))], \)
\[ \min[\mu_{\tilde{A}}^\text{false}(x), \min(1, 1 - \mu_{\tilde{A}}^\text{false}(x) + \mu_B^\text{false}(y))] \} / y \]

if \( x = y \)

\[ = \{ \min[\mu_{\tilde{A}}^\text{true}(x), \min(1, 1 - \mu_{\tilde{A}}^\text{true}(x) + \mu_B^\text{true}(y))], \)
\[ \min[\mu_{\tilde{A}}^\text{false}(x), \min(1, 1 - \mu_{\tilde{A}}^\text{false}(x) + \mu_B^\text{false}(x))] \} \]

if \( x \) is \( \text{tall} \) then \( x \) is weight

\( x \) is \( \text{very tall} \)

\( x \) is \( \text{tall} \) o \( \text{weight} \)

**Fuzzy quantifiers**

The fuzzy propositions may contain quantifiers like “very”, “more or less” etc. These fuzzy quantifiers may be eliminated as

**Concentration**

\( \mu_{\text{very } \tilde{A}}(x) = \{ \mu_{\text{very } \tilde{A}}^\text{true}(x)^2, \mu_{\text{very } \tilde{A}}^\text{false}(x)^2 \} \)
$x$ is very tall
$\mu_{\text{very tall}}(x) = \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5, 0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}$

**Diffusion**
if $x$ is more or less $\tilde{A}$
$\mu_{\text{more or less } \tilde{A}}(x) = [\mu_{\text{true}}^{\text{more or less } \tilde{A}}(x)^2, \mu_{\text{false}}^{\text{more or less } \tilde{A}}(x)^0.5]$
if $x$ is more or less tall
$\mu_{\text{more or less tall}}(x) = \{0.95/x_1 + 0.89/x_2 + 0.84/x_3 + 0.63/x_4 + 0.45/x_5, 0.70/x_1 + 0.63/x_2 + 0.54/x_3 + 0.44/x_4 + 0.31/x_5\}$

3. Fuzzy Inference for Fuzzy Intuitions

Consider the logical inferences

**Modus Ponens**
$p \Rightarrow q$
P
——
$q$

**Modus Tollens**
$p \Rightarrow q$
$q'$
——
$p'$

**Generalization**
$p \lor q = p$
$P \lor q = q$
$p' \lor p = \text{Contradictory}$

**Specialization**
$p \land q = p$
$p \land q = q$
$p' \land p = \text{Contradictory}$

The inference is given using generalization and specialization

$p \land q \lor r = p \lor r = p$
$p \land q \lor r = q \lor r = q$
$p \land q \lor r = p \lor r = r$

Consider fuzzy inference Type-1
If \( x \) is \( \tilde{P} \) and \( x \) is \( \tilde{Q} \) or \( x \) is \( \tilde{R} \) then \( y \) is \( \tilde{S} \)
\( x \) is \( \tilde{P}_1 \) and \( x \) is \( \tilde{Q}_1 \) or \( x \) is \( \tilde{R}_1 \)

\[ y \text{ is ?} \]

The fuzzy inference is given for Type-1 using generalization and specialization

If \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{S} \)
\( x \) is \( \tilde{P}_1 \)

\[ y \text{ is ?} \]

If \( x \) is \( \tilde{Q} \) then \( y \) is \( \tilde{S} \)
\( x \) is \( \tilde{Q}_1 \)

\[ y \text{ is ?} \]

If \( x \) is \( \tilde{R} \) then \( y \) is \( \tilde{S} \)
\( x \) is \( \tilde{R}_1 \)

\[ y \text{ is ?} \]

Confider fuzzy inference Type-2

If \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{Q} \) else \( y \) is \( \tilde{R} \)
\( x \) is \( \tilde{P}_1 \)

\[ y \text{ is ?} \]

The fuzzy inference is given for Type-2 using generalization and specialization

\texttt{indent (if x is } \tilde{P} \texttt{ then x is } \tilde{Q} \texttt{)}
\( x \) is \( \tilde{P}_1 \)

\[ y \text{ is ?} \]

\texttt{(if x is } P’ \texttt{ then x is } \tilde{R} \texttt{)}
\( x \) is \( \tilde{P}_1 \)

\[ y \text{ is ?} \]

From fuzzy conditional inference Type-1 and Type-2, the two criterions may be given as

\textbf{Criteria-1}
If \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{S} \)
\( x \) is \( P_1 \)
\( y \) is ?

**Criteria-2**

(if \( x \) is \( P' \) then \( x \) is \( \tilde{R} \))
\( x \) is \( P_1 \)
\( y \) is ?

The fuzzy inference is drawing a conclusion from fuzzy propositions.
The fuzzy intuitions for Criteria-1 Based on FMK are given as.

**I-1**

if \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{S} \)
\( x \) is \( \tilde{P} \)
\( y \) is \( \tilde{S} \)

**I-2**

if \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{S} \)
\( x \) is \( \tilde{S} \)
\( y \) is \( \tilde{P} \)

**II-1**

if \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{S} \)
\( x \) is very \( \tilde{P} \)
\( y \) is very \( \tilde{S} \)

**II-2**

if \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{S} \)
\( x \) is very \( \tilde{S} \)
\( y \) is \( \tilde{P} \)

**III-1**

if \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{S} \)
\( x \) is more or less \( \tilde{P} \)
\( y \) is \( \tilde{S} \)
III-2
if $x$ is $\bar{P}$ then $y$ is $\bar{S}$

$x$ is more or less $\bar{S}$

$y$ is $\bar{P}$

IV-1
if $x$ is $\bar{P}$ then $y$ is $\bar{S}$

$x$ is not $\bar{P}$

$y$ is not $\bar{S}$

IV-2
if $x$ is $\bar{P}$ then $y$ is $\bar{S}$

$x$ is not $\bar{S}$

$y$ is not $\bar{P}$

The fuzzy inference is given for Criteria-1 according to fuzzy intuitions.

Table 1: Fuzzy inference for Criteria-1.

<table>
<thead>
<tr>
<th>Intuition</th>
<th>Proposition</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-1</td>
<td>$x$ is $\bar{P}$</td>
<td>$y$ is $\bar{S}$</td>
</tr>
<tr>
<td>I-2</td>
<td>$y$ is $\bar{S}$</td>
<td>$x$ is $\bar{P}$</td>
</tr>
<tr>
<td>II-1</td>
<td>$x$ is very $\bar{P}$</td>
<td>$y$ is very $\bar{S}$</td>
</tr>
<tr>
<td>II-2</td>
<td>$y$ is very $\bar{S}$</td>
<td>$x$ is $\bar{P}$</td>
</tr>
<tr>
<td>III-1</td>
<td>$x$ is more or less $\bar{P}$</td>
<td>$y$ is $\bar{S}$</td>
</tr>
<tr>
<td>III-2</td>
<td>$y$ is more or less $\bar{S}$</td>
<td>$x$ is $\bar{P}$</td>
</tr>
<tr>
<td>IV-1</td>
<td>$x$ is not $\bar{P}$</td>
<td>$y$ is not $\bar{S}$</td>
</tr>
<tr>
<td>IV-2</td>
<td>$y$ is $\bar{S}$</td>
<td>$x$ is not $\bar{P}$</td>
</tr>
</tbody>
</table>

4. Verification of fuzzy intuition using Fuzzy Conditional Inference

Verification of fuzzy intuitions for Criteria-1

4.1.1 In the case of intuition I-1

\[
\tilde{P} \circ (\tilde{P} \rightarrow \tilde{S}) = \tilde{P} \circ (\tilde{P} \times \tilde{S}) = \int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \land \int \mu_{\tilde{S}}(y))
\]

Using (2.4)
10 (2018)

\[ \int \tilde{\mu} \tilde{P}(x) = \int \tilde{\mu} \tilde{S}(y) \]

\( y \) is \( \tilde{S} \)

intuition I-1 satisfied.

**4.1.2 In the case of intuition I-2**

\( (\tilde{P} \rightarrow \tilde{S}) \circ \tilde{S} \)

\( = (\tilde{P} \times \tilde{S}) \circ \tilde{S} \)

\( = (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\tilde{S}}(y) \)

Using (2.4)

\( = \int \mu_{\tilde{P}}(x) \circ \int \mu_{\tilde{S}}(y) \)

Using (2.3)

\( = \int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{P}}(x) \)

\( = \int \mu_{\tilde{P}}(x) \)

\( = y \) is \( \tilde{P} \)

intuition I-2 satisfied.

**4.1.3 In the case of intuition II-1**

very \( \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S}) \)

\( = \) very \( \tilde{P} \circ (\tilde{P} \times \tilde{S}) \)

\( = \int \mu_{\text{very} \tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \)

Using (2.4)

\( = \int \mu_{\text{very} \tilde{P}}(x) \circ \int \mu_{\tilde{P}}(x) \)

Using (4.1)

\( = \int \mu_{\tilde{S}}(y)^2 \wedge (\int \mu_{\tilde{S}}(y)) \)

\( \int \mu_{\tilde{S}}(y)^2 \)

\( = \int \mu_{\text{very} \tilde{S}}(x) \)

\( = y \) is very \( \tilde{S} \)

Where

\( \int \mu_{\tilde{S}}(y)^2 \subseteq \int \mu_{\tilde{S}}(y) \).

\( \int \mu_{\tilde{S}}(y)^2 \subseteq \int \mu_{\tilde{S}}(y) \).

intuition II-1 satisfied.

**4.1.4 In the case of intuition II-2**

\( (\tilde{P} \rightarrow \tilde{S}) \circ \text{very} \tilde{S} \)

\( = (\tilde{P} \times \tilde{S}) \circ \text{very} \tilde{S} \)

\( = (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\text{very} \tilde{S}}(y) \)

Using (2.4)

\( = (\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\text{very} \tilde{S}}(y) \)

Unlinking (2.3)

\( = (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\text{very} \tilde{P}}(x)^2 \)

\( \int \mu_{\tilde{P}}(x)^2 \)

\( = \int \mu_{\text{very} \tilde{P}}(x) \)

\( = x \) is very \( \tilde{P} \)
Where
\[ \int \mu_{P}(x)^{2} \subseteq \int \mu_{P}(x). \]
\[ \int \mu_{P}(x)^{2} \leq \int \mu_{P}(x). \]

intuition II-2 satisfied.

4.1.5 In the case of intuition III-1

more or less \( \tilde{P} \circ (P \rightarrow \tilde{S}) \)

= more or less \( \tilde{P} \circ (P \times \tilde{S}) \)

= \( \int \mu_{\text{more or less}}P(x) \circ (\int \mu_{P}(x) \land \int \mu_{\tilde{S}}(y)) \)

Using (2.4)

= \( \int \mu_{\text{more or less}}P(x) \circ (\int \mu_{\tilde{S}}(y)) \)

Using (2.3)

= \( \int \mu_{\text{more or less}}(x) \land (\int \mu_{\tilde{S}}(y)) \)

\[ \int \mu_{\tilde{S}}(y)^{0.5} \geq \int \mu_{\tilde{S}}(y). \]

\[ \int \mu_{\tilde{S}}(y)^{0.5} \geq \int \mu_{\tilde{S}}(y). \]

intuition III-1 satisfied.

4.1.6 In the case of intuition III-2

\( (\tilde{P} \rightarrow \tilde{S}) \circ \text{more or less } \tilde{S} \)

= \( (\tilde{P} \times \tilde{S}) \circ \text{very } \tilde{S} \)

= \( (\int \mu_{P}(x) \land \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\text{more or less}}\tilde{S}(x) \)

Using (2.4)

= \( (\int \mu_{P}(x)) \circ \int \mu_{\text{more or less}}\tilde{S}(y) \)

Unlinking (2.3)

= \( (\int \mu_{P}(x)) \land \int \mu_{\text{more or less}}\tilde{P}(x) \)

\[ \int \mu_{\tilde{P}}(x)^{0.5} \geq \int \mu_{\tilde{P}}(x). \]

\[ \int \mu_{\tilde{P}}(x)^{0.5} \geq \int \mu_{\tilde{P}}(x). \]

intuition III-2 satisfied.

4.1.7 In the case of intuition IV-1

not \( \tilde{P} \circ (P \rightarrow \tilde{S}) \)

= \( P^\circ \circ (P \times \tilde{S}) \)

= \( \int \mu_{P}(x) \circ (\int \mu_{P}(x) \land \int \mu_{\tilde{S}}(y)) \)

= \( \int \mu_{P}(x) \circ (\int \mu_{P}(x)) \)

Using (2.4)

= \( \int \mu_{P}(x) \land (\int \mu_{P}(x)) \)
= contradictory
intuition IV-1 not satisfied.

4.1.8 In the case of intuition IV-2

\((\tilde{P} \rightarrow \tilde{S}) \circ \tilde{S}'\)
\((\tilde{P} \times \tilde{S}) \circ \tilde{S}'\)
\((\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\tilde{S}'}(x)\)
Using (2.4)
\((\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\tilde{S}'}(y)\)
Using (2.3)
\((\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\tilde{P}}(x)\)
= Contradictory
intuition IV-2 not satisfied.

Criteria-1 is suitable for I-1, I-2, II-1, II-2, III-1 and III-2.

The fuzzy intuitions are given based on FMK for Criteria-2.

I’-1
if \(x\) is \(\tilde{P}'\) then \(y\) is \(\tilde{R}\)
\(x\) is \(\tilde{P}\)

\(\frac{y}{y}\) is \(\tilde{R}\)

I’-2
if \(x\) is \(P'\) then \(y\) is \(\tilde{R}\)
\(x\) is \(\tilde{R}\)

\(\frac{y}{y}\) is \(\tilde{P}\)

II’-1
if \(x\) is \(P'\) then \(y\) is \(\tilde{R}\)
\(x\) is very \(\tilde{P}\)

\(\frac{y}{y}\) is very \(\tilde{R}\)

II’-2
if \(x\) is \(P'\) then \(y\) is \(\tilde{R}\)
\(x\) is very \(\tilde{R}\)

\(\frac{y}{y}\) is \(\tilde{P}\)

III’-1
if \(x\) is \(P'\) then \(y\) is \(\tilde{R}\)
\(x\) is more or less \(\tilde{P}\)
The inference is given for Criteria-1 according to intuitions.

Table 2: Fuzzy inference for Criteria-2.

<table>
<thead>
<tr>
<th>Intuition</th>
<th>Proposition</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>I'-1</td>
<td>x is ( \tilde{P} )</td>
<td>y is ( \tilde{R} )</td>
</tr>
<tr>
<td>I'-2</td>
<td>y is ( \tilde{R} )</td>
<td>x is ( \tilde{P} )</td>
</tr>
<tr>
<td>II'-1</td>
<td>x is very ( \tilde{P} )</td>
<td>y is very ( \tilde{R} )</td>
</tr>
<tr>
<td>II'-2</td>
<td>y is very ( \tilde{R} )</td>
<td>x is ( \tilde{P} )</td>
</tr>
<tr>
<td>III'-1</td>
<td>x is more or less ( \tilde{P} )</td>
<td>y is ( \tilde{R} )</td>
</tr>
<tr>
<td>III'-2</td>
<td>y is mor or less ( \tilde{R} )</td>
<td>x is ( \tilde{P} )</td>
</tr>
<tr>
<td>IV'-1</td>
<td>x is not ( \tilde{P} )</td>
<td>y is not ( \tilde{R} )</td>
</tr>
<tr>
<td>IV'-2</td>
<td>y is ( \tilde{R} )</td>
<td>x is not ( \tilde{P} )</td>
</tr>
</tbody>
</table>

Verification of fuzzy intuitions for Criteria-2

4.2.1 In the case of intuition I'-1

\[
\tilde{P} \circ (\tilde{P} \rightarrow \tilde{R}) \\
=\tilde{P} \circ (\tilde{P} \times \tilde{R}) \\
= \int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{R}}(y))
\]

Using (2.4)
= \int \mu_P(x) \circ (\int \mu_P(x))
\text{indent} = \int \mu_P(x) \wedge (\int \mu_P(x))
= \text{Contradictory}
\text{intuition \textit{I}'-1 not satisfied.}

\subsection*{4.2.2 In the case of intuition \textit{I}'-2}

\begin{align*}
(\hat{P'} \rightarrow \hat{R}) \circ \hat{R} \\
= (\hat{P'} \times \hat{R}) \circ \hat{R} \\
= (\int \mu_P(x) \wedge \int \mu_R(y)) \circ \mu_R(y)
\end{align*}

\text{Using (2.4)}

\begin{align*}
&= (\int \mu_P(x)) \circ \mu_R(y) \\
&= (\int \mu_P(x)) \wedge \mu_R(y)
\end{align*}

\text{Using (2.3)}

\begin{align*}
&= \min(\int \mu_R(x), \int \mu_R(y)) \\
&= \text{Contradictory}
\text{intuition \textit{I}'-2 not satisfied.}

\subsection*{4.2.3 In the case of intuition \textit{II}'-1}

\begin{align*}
\text{very } \hat{P} \circ (\hat{P'} \rightarrow \hat{R}) \\
= \text{very } \hat{P} \circ (\hat{P'} \times \hat{R}) \\
= \int \mu_P(x)^2 \circ (\int \mu_P(x) \wedge \int \mu_R(y))
\end{align*}

\text{Using (2.4)}

\begin{align*}
&= \int \mu_P(x)^2 \circ (\int \mu_P(x)) \\
&= \int \mu_P(x)^2 \wedge (\int \mu_P(x))
\end{align*}

\text{indent} = \int \mu_P(x)^2 \wedge (\int \mu_P(x))
= \text{Contradictory}
\text{intuition \textit{II}'-1 not satisfied.}

\subsection*{4.2.4 In the case of intuition \textit{II}'-2}

\begin{align*}
(\hat{P'} \rightarrow \hat{R}) \circ \text{very } \hat{R} \\
= (\hat{P'} \times \hat{R}) \circ \text{very } \hat{R} \\
= (\int \mu_P(x) \wedge \int \mu_R(y)) \circ \mu_R(y)^2
\end{align*}

\text{Using (2.4)}

\begin{align*}
&= (\int \mu_P(x)) \circ \mu_R(y)^2 \\
&= (\int \mu_P(x)) \wedge \mu_R(y)^2
\end{align*}

\text{Using (2.3)}

\begin{align*}
&= \min(\int \mu_R(x), \int \mu_R(y)^2) \\
&= \text{Contradictory}
\text{intuition \textit{II}'-2 not satisfied.}

\subsection*{4.2.5 In the case of intuition \textit{III}'-1}

\begin{align*}
\text{more or less } \hat{P} \circ (\hat{P'} \rightarrow \hat{R}) \\
= \text{more or less } \hat{P} \circ (\hat{P'} \times \hat{R}) \\
= \int \mu_P(x)^{0.5} \circ (\int \mu_P(x) \wedge \int \mu_R(y))
\end{align*}

\text{Using (2.4)}

\begin{align*}
&= \int \mu_P(x)^{0.5} \circ (\int \mu_P(x)) \\
&= \int \mu_P(x)^{0.5} \wedge (\int \mu_P(x))
\end{align*}
4.2.6 In the case of intuition III’-2

\((\tilde{P} \rightarrow \tilde{R}) \circ \text{more or less } \tilde{R}\)

= \((\tilde{P} \times \tilde{R}) \circ \text{more or less } \tilde{R}\)

= \(\int \mu_{\tilde{P}}(x) \land \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\tilde{R}}(y)^{0.5}\)

Using (2.4)

= \(\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\tilde{R}}(y)^{0.5}\)

= \(\int \mu_{\tilde{P}}(x)) \land \int \mu_{\tilde{R}}(y)^{0.5}\)

Using (2.3)

= \(\mu_{\tilde{P}}(y), \int \mu_{\tilde{R}}(y))\)

= \(\text{Contradictory}\)

intuition II’-2 not satisfied.

4.2.7 In the case of intuition IV’-1

\((\tilde{P} \rightarrow \tilde{R}) \circ \text{not } \tilde{R}\)

= \(\mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \land \int \mu_{\tilde{R}}(y))\)

Using (2.4)

= \(\int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x)\)

using (2.3)

= \(\int \mu_{\tilde{R}}(x) \land (\int \mu_{\tilde{R}}(x))\)

= \(\int \mu_{\tilde{R}}(y)\)

= \(\text{y is not } \tilde{R}\)

intuition IV’-1 satisfied.

4.2.8 In the case of intuition IV’-2

\((\tilde{P} \rightarrow \tilde{R}) \circ \text{not } \tilde{R}\)

= \(\mu_{\tilde{P}}(x) \circ \text{not } \tilde{R}\)

= \(\int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \land \int \mu_{\tilde{R}}(y))\)

Using (2.4)

= \(\int \mu_{\tilde{P}}(x) \circ \int \mu_{\tilde{R}}(y)\)

= \(\int \mu_{\tilde{P}}(x) \land \int \mu_{\tilde{R}}(y)\)

Using (2.3)

= \(\mu_{\tilde{P}}(x), \int \mu_{\tilde{R}}(y))\)

= \(\text{y is not } \tilde{P}\)

intuition IV’-2 satisfied.

Criteria-2 is suitable for IV-1 and IV-2.

5. Fuzzy Truth Variables

Zadeh [16] defined quantification of truth variables as composition of fuzzy set and truth variables.
Definition 5.1 The quantification of fuzzy truth variables for fuzzy set of fuzzy proposition of the type “x is A is τ” is defined as µ^τ_A(x) = µ_A(x)^τ, where µ_A(x) is inverse of comparability function of A, “o” is composition and τ is fuzzy truth variable like true, false, very true etc.

Definition 5.2 The composition of fuzzy truth variables for “a two fold fuzzy set” of fuzzy proposition of the type “x is A is τ” may be defined as

\[ \hat{A}(x)^{\tau_1} = \mu_A(x)^{\tau_1}, \mu_A^{\text{false}}(x) \] o τ

where quantification of truth variable applied on respective truth functions, i.e.,

\[ \hat{A}(x)^{\tau_2} = \mu_A(x)^{\tau_2}, \mu_A^{\text{false}}(x) \]

For instance, τ_1 = not true, very true, more or less true e.t.c.

For instance, τ_2 = not false, very false, more or less false e.t.c.

The truth functional modification of fuzzy proposition “x is A is τ” is given as

\[ \mu_A^{\text{true}}(x), \mu_A^{\text{false}}(x) \] of very true = \[ \mu_A^{\text{true}}(x), \mu_A^{\text{false}}(x) \]

The truth functional modification of fuzzy proposition “x is A is very false” is given as

\[ \mu_A^{\text{true}}(x), \mu_A^{\text{false}}(x) \] of very false = \[ \mu_A^{\text{true}}(x), \mu_A^{\text{false}}(x) \]

The truth functional modification of fuzzy proposition “x is tall is very true” is given as

\[ \text{tall} = [0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\
0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5] \]

\[ \text{very tall} = [0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5, \\
0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5] \]

The truth functional modification of fuzzy proposition “x is tall is very false” is given as

\[ \text{tall} = [0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\
0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5] \]

\[ \text{very tall} = [0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\
0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5] \]

The nested fuzzy propositions of the form

x is A is (τ_1 is (τ_2 is... is τ_n)) = x is A o τ_1 o τ_2 o ... o τ_n.

Consider quantification of truth variables for fuzzy inference Type-1

If \( \hat{P} \) and \( \hat{Q} \) or \( \hat{R} \) then \( \hat{S} \) is \( \tau \)

x is \( P_1 \) and x is \( Q_1 \) or x is \( R_1 \)

y is ?

The fuzzy inference is given for Type-1 using generalization and specialization.
If \( x \) is \( \tilde{P} \) then \( y \) is \( \bar{S} \) is \( \tau \)
\[ x \equiv \tilde{P}_1 \]
\[ \text{y is ?} \]

If \( x \) is \( \tilde{Q} \) then \( y \) is \( \bar{S} \) is \( \tau \)
\[ x \equiv \tilde{Q}_1 \]
\[ \text{y is ?} \]

If \( x \) is \( \tilde{R} \) then \( y \) is \( \bar{S} \) is \( \tau \)
\[ x \equiv \tilde{R}_1 \]
\[ \text{y is ?} \]

Confider fuzzy inference Type-2

If \( x \) is \( \tilde{P} \) then \( y \) is \( \tilde{Q} \) else \( y \) is \( \tilde{R} \) is \( \tau \)
\[ x \equiv \tilde{P}_1 \]
\[ \text{y is ?} \]

The fuzzy inference is given for Type-2 using generalization and specialization

indent (if \( x \) is \( \tilde{P} \) then \( x \) is \( \tilde{Q} \)) is \( \tau \)
\[ x \equiv \tilde{P}_1 \]
\[ \text{y is ?} \]

(if \( x \) is \( P' \) then \( x \) is \( \tilde{R} \)) is \( \tau \)
\[ x \equiv \tilde{P}_1 \]
\[ \text{y is ?} \]

From fuzzy conditional inference Type-1 and Type-2, two criteria may be given as

**Criteria-1**
If \( x \) is \( \tilde{P} \) then \( y \) is \( \bar{S} \) is \( \tau \)
\[ x \equiv \tilde{P}_1 \]
\[ \text{y is ?} \]

**Criteria-2**
(if \( x \) is \( P' \) then \( x \) is \( \tilde{R} \)) is \( \tau \)
\[ x \text{ is } \tilde{P}_1 \]

\[ \therefore y \text{ is ?} \]

The fuzzy inference is drawing a conclusion from fuzzy propositions. The fuzzy intuitions are defined based on FMK for Criteria-1.

\[ I_\tau - 1 \]
if \( x \text{ is } \tilde{P} \) then \( y \text{ is } \tilde{S} \text{ is } \tau \)
\[ x \text{ is } \tilde{P} \]
\[ \therefore y \text{ is } \tilde{S}^\tau \]

\[ I_\tau - 2 \]
if \( x \text{ is } \tilde{P} \) then \( y \text{ is } \tilde{S} \text{ is } \tau \)
\[ x \text{ is } \tilde{S} \]
\[ \therefore y \text{ is } \tilde{P}^\tau \]

\[ II_\tau - 1 \]
if \( x \text{ is } \tilde{P} \) then \( y \text{ is } \tilde{S} \text{ is } \tau \)
\[ x \text{ is very } \tilde{P} \]
\[ \therefore y \text{ is very } \tilde{S}^\tau \]

\[ II_\tau - 2 \]
if \( x \text{ is } \tilde{P} \) then \( y \text{ is } \tilde{S} \text{ is } \tau \)
\[ x \text{ is very } \tilde{S} \]
\[ \therefore y \text{ is } \tilde{P}^\tau \]

\[ III_\tau - 1 \]
if \( x \text{ is } \tilde{P} \) then \( y \text{ is } \tilde{S} \text{ is } \tau \)
\[ x \text{ is more or less } \tilde{P} \]
\[ \therefore y \text{ is } \tilde{S}^\tau \]

\[ III_\tau - 2 \]
if \( x \text{ is } \tilde{P} \) then \( y \text{ is } \tilde{S} \text{ is } \tau \)
\[ x \text{ is more or less } \tilde{S} \]
\[ \therefore y \text{ is } \tilde{P}^\tau \]

\[ IV_\tau - 1 \]
if $x$ is $\tilde{P}$ then $y$ is $\tilde{S}$ is $\tau$

$x$ is not $\tilde{P}$

---

$y$ is not $\tilde{S}^\tau$

$\textsc{Iv}_\tau - 2$

if $x$ is $\tilde{P}$ then $y$ is $\tilde{S}$ is $\tau$

$x$ is not $\tilde{S}$

---

$y$ is not $\tilde{P}^\tau$

The inference is given for Criteria-1 according to intuitions.

<table>
<thead>
<tr>
<th>Intuition</th>
<th>Proposition</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_\tau - 1$</td>
<td>$x$ is $\tilde{P}$</td>
<td>$y$ is $\tilde{S}^\tau$</td>
</tr>
<tr>
<td>$I_\tau - 2$</td>
<td>$y$ is $\tilde{S}$</td>
<td>$x$ is $\tilde{P}^\tau$</td>
</tr>
<tr>
<td>$\textsc{Ii}_\tau - 1$</td>
<td>$x$ is very $\tilde{P}$</td>
<td>$y$ is very $\tilde{S}^\tau$</td>
</tr>
<tr>
<td>$\textsc{Ii}_\tau - 2$</td>
<td>$y$ is very $\tilde{S}$</td>
<td>$x$ is $\tilde{P}^\tau$</td>
</tr>
<tr>
<td>$\textsc{III}_\tau - 1$</td>
<td>$x$ is more or less $\tilde{P}$</td>
<td>$y$ is $\tilde{S}^\tau$</td>
</tr>
<tr>
<td>$\textsc{III}_\tau - 2$</td>
<td>$y$ is more or less $\tilde{S}$</td>
<td>$x$ is $\tilde{P}^\tau$</td>
</tr>
<tr>
<td>$\textsc{IV}_\tau - 1$</td>
<td>$x$ is not $\tilde{P}$</td>
<td>$y$ is not $\tilde{S}^\tau$</td>
</tr>
<tr>
<td>$\textsc{IV}_\tau - 2$</td>
<td>$y$ is $\tilde{S}$</td>
<td>$x$ is not $\tilde{P}^\tau$</td>
</tr>
</tbody>
</table>

Where $\tau = \tau_1$ or $\tau_2$.

Fuzzy Conditional Inference is straight forward based on verification of fuzzy intuitions for Criteria-1

Criteria-1 is suitable for $I_\tau - 1, I_\tau - 2, II_\tau - 11, II_\tau - 2, \textsc{III}_\tau - 1$ and $\textsc{III}_\tau - 2$.

The fuzzy intuitions are defined based on FMK for Criteria-2.

$I'_\tau - 1$

if $x$ is $\tilde{P}'$ then $y$ is $\tilde{R}$ is $\tau$

$x$ is $\tilde{P}$

---

$y$ is $\tilde{R}^\tau$

$I'_\tau - 2$

if $x$ is $\tilde{P}'$ then $y$ is $\tilde{R}$ is $\tau$

$x$ is $\tilde{R}$
y is $P^\tau$

$II'_\tau - 1$
if $x$ is $P'$ then $y$ is $\bar{R}$ is $\tau$
x is very $\bar{P}$

y is very $\bar{R}^\tau$

$II'_\tau - 2$
if $x$ is $P'$ then $y$ is $\bar{R}$ is $\tau$
x is very $\bar{R}$

y is $P^\tau$

$III'_\tau - 1$
if $x$ is $P'$ then $y$ is $\bar{R}$ is $\tau$
x is more or less $\bar{P}$

y is $\bar{R}^\tau$

$III'_\tau - 2$
if $x$ is $P'$ then $y$ is $\bar{R}$ is $\tau$
x is more or less $\bar{R}$

y is $\bar{S}^\tau$

$IV'_\tau - 1$
if $x$ is $P'$ then $y$ is $\bar{R}$ is $\tau$
x is not $\bar{P}$

y is not $\bar{R}^\tau$

$IV'_\tau - 2$
if $x$ is $P'$ then $y$ is $\bar{R}$ is $\tau$
x is not $RJS$

y is not $\bar{P}^\tau$

The inference is given for Criteria-2 according to intuitions.
### Table 2: Fuzzy inference for Criteria-2.

<table>
<thead>
<tr>
<th>Intuition</th>
<th>Proposition</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1'$ - 1</td>
<td>$x$ is $\tilde{P}$ o is $\tau$</td>
<td>$y$ is $\tilde{R}$</td>
</tr>
<tr>
<td>$I_2'$ - 2</td>
<td>$y$ is $\tilde{R}$ o is $\tau$</td>
<td>$x$ is $\tilde{P}$</td>
</tr>
<tr>
<td>$II_1'$ - 1</td>
<td>$x$ is very $\tilde{P}$ o is $\tau$</td>
<td>$y$ is very $\tilde{R}$</td>
</tr>
<tr>
<td>$II_2'$ - 2</td>
<td>$y$ is very $\tilde{R}$ o is $\tau$</td>
<td>$x$ is $\tilde{P}$</td>
</tr>
<tr>
<td>$III_1'$ - 1</td>
<td>$x$ is more or less $\tilde{P}$ o is $\tau$</td>
<td>$y$ is $\tilde{R}$</td>
</tr>
<tr>
<td>$III_2'$ - 2</td>
<td>$y$ is more or less $\tilde{R}$ o is $\tau$</td>
<td>$x$ is $\tilde{P}$</td>
</tr>
<tr>
<td>$IV_1'$ - 1</td>
<td>$x$ is not $\tilde{P}$ o is $\tau$</td>
<td>$y$ is not $\tilde{R}$</td>
</tr>
<tr>
<td>$IV_2'$ - 1</td>
<td>$y$ is $\tilde{R}$ o is $\tau$</td>
<td>$x$ is not $\tilde{P}$</td>
</tr>
</tbody>
</table>

Where $\tau = \tau_1$ or $\tau_2$

Fuzzy Conditional Inference is straight forward based on verification of fuzzy intuitions for Criteria-2

Criteria-2 is suitable for $IV_1'$ - 1 and $IV_2'$ - 2.

### 4. Fuzzy Certainty Factor

The fuzzy certainty factor (FCF) shall made as single fuzzy membership functions with two fuzzy membership functions to eliminate the conflict of evidence between “True” and “False”.

**Definition 4.1** The FCF of $\mu_{\tilde{A}}$ for propositions “$x$ is $\tilde{A}$” is characterized by its membership function $\mu^\text{FCF}_{\tilde{A}}(x) \rightarrow [0, 1]$, where

$$
\mu^\text{FCF}_{\tilde{A}}(x) = \frac{\mu^\text{True}_{\tilde{A}}(x) - \mu^\text{False}_{\tilde{A}}(x)}{x},
$$

where $\mu^\text{True}_{\tilde{A}}(x) < 0, \mu^\text{CF}(x) = 0$ and $\mu^\text{False}_{\tilde{A}}(x) > 0$ are the redundant, insufficient and sufficient respectively.

The FCF will compute the conflict of evidence of the incomplete information.

For Example

\[
\begin{align*}
\mu^\text{True}_{\text{young}}(x) &= [0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30 + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50] \\
\mu^\text{False}_{\text{young}}(x) &= [0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30 + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50]
\end{align*}
\]

The Graphical representation of FCF is shown in Fig. 3.
6. Application to New Fuzzy Conditional Inference for Fuzzy Intuitions

The Business intelligence needs reasoning. The Business data is defied with fuzziness with linguistic variables.

If $x$ is Production and $x$ is Supply or $x$ is Demand then $y$ is Profit
$x$ is less Production and $x$ is less Supply or $x$ is more Demand
y is ?
If $x$ is Production then $y$ is Profit
$x$ is less Production
y is ?
If $x$ is Supply then $y$ is Profit
$x$ is less Supply
y is ?
If $x$ is Demand then $y$ is Profit
$x$ is more Demand
y is ?

$I_t - 1$
if $x$ is Demand then $y$ is Profit
$x$ is Profit
y is Demand

$I_t - 2$
if $x$ is Demand then $y$ is Profit
$x$ is Demand
y is Profit
Consider the fuzzy data sets for production.
Table 3: Fuzzy data sets.

<table>
<thead>
<tr>
<th>Item No.</th>
<th>Demand</th>
<th>FCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item1</td>
<td>{0.5,0.1}</td>
<td>0.4</td>
</tr>
<tr>
<td>Item2</td>
<td>{0.6,0.1}</td>
<td>0.5</td>
</tr>
<tr>
<td>Item3</td>
<td>{0.9,0.2}</td>
<td>0.7</td>
</tr>
<tr>
<td>Item4</td>
<td>{0.9,0.1}</td>
<td>0.8</td>
</tr>
<tr>
<td>Item5</td>
<td>{1.0,0.0}</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The fuzzy conditional inference using is given by

Profit = Demand

Table 3: Fuzzy data sets.

<table>
<thead>
<tr>
<th>Item No.</th>
<th>Profit</th>
<th>FCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item1</td>
<td>{0.5,0.1}</td>
<td>0.4</td>
</tr>
<tr>
<td>Item2</td>
<td>{0.6,0.1}</td>
<td>0.5</td>
</tr>
<tr>
<td>Item3</td>
<td>{0.9,0.2}</td>
<td>0.7</td>
</tr>
<tr>
<td>Item4</td>
<td>{0.9,0.1}</td>
<td>0.8</td>
</tr>
<tr>
<td>Item5</td>
<td>{1.0,0.0}</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The fuzzy conditional inference for Criteria-1 and Criteria-2 is given as

Table - 4: Fuzzy inference.

<table>
<thead>
<tr>
<th>Item No.</th>
<th>I-1</th>
<th>I-2</th>
<th>II-1</th>
<th>II-2</th>
<th>III-1</th>
<th>III-2</th>
<th>IV'-1</th>
<th>IV'-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item1</td>
<td>0.3</td>
<td>0.2</td>
<td>0.09</td>
<td>0.09</td>
<td>0.55</td>
<td>0.55</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Item2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.71</td>
<td>0.71</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Item3</td>
<td>0.7</td>
<td>0.7</td>
<td>0.49</td>
<td>0.49</td>
<td>0.84</td>
<td>0.84</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Item4</td>
<td>0.8</td>
<td>0.8</td>
<td>0.64</td>
<td>0.64</td>
<td>0.89</td>
<td>0.89</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Item5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The fuzzy intuitions are suitable for I-1, I-2, II-1, II-2, III-1, III-2, IV'-1 and IV'-2.
The Business intelligence needs reasoning. The Business data is defined with fuzziness with linguistic truth variables.

If \( x \) is Production and \( x \) is Supply or \( x \) is Demand then \( y \) is Profit is very true
\( x \) is less Production and \( x \) is less Supply or \( x \) is more Demand

\( y \) is ?
If \( x \) is Production then \( y \) is Profit is very true
\( x \) is less Production

\( y \) is ?
If \( x \) is Supply then \( y \) is Profit is very true
\( x \) is less Supply

\( y \) is ?
If \( x \) is Demand then \( y \) is Profit is very true
\( x \) is more Demand

\( y \) is ?

\( I_\tau - 1 \) if \( x \) is Demand then \( y \) is Profit is very true
\( x \) is Profit

\( y \) is Demand very true
\( = [0.4, 0.1] \) very true
\( = [0.25, 0.1] \)
FCF = [0.15]

\( I_\tau - 2 \) if \( x \) is Demand then \( y \) is Profit is very false
\( x \) is Demand

\( y \) is Profit very false
\( = [0.4, 0.1] \) very false
\( = [0.5, 0.01] \)
FCF = [0.49]

\( II_\tau - 1 \) if \( x \) is Demand then \( y \) is Profit is very true
\( x \) is very Profit

\( y \) is very Demand very true
\( = [0.25, 0.01] \) very true
\( = [0.35, 0.01] \)
FCF = [0.34]

\( II_\tau - 2 \) if \( x \) is Demand then \( y \) is Profit is very false
The fuzzy conditional inference using is given by

\[ \text{Profit} = \text{Demand} \]

The fuzzy conditional inference for Criteria-1 and Criteria-2 is given as
Table 5: Fuzzy inference.

<table>
<thead>
<tr>
<th>Item No.</th>
<th>$I_τ - 1$</th>
<th>$I_τ - 2$</th>
<th>$II_τ - 1$</th>
<th>$II_τ - 2$</th>
<th>$III_τ - 1$</th>
<th>$III_τ - 2$</th>
<th>$IV'_τ - 1$</th>
<th>$IV'_τ - 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item1</td>
<td>0.15</td>
<td>0.49</td>
<td>0.34</td>
<td>0.25</td>
<td>0.08</td>
<td>0.08</td>
<td>0.94</td>
<td>0.61</td>
</tr>
<tr>
<td>Item2</td>
<td>0.26</td>
<td>0.59</td>
<td>0.45</td>
<td>0.77</td>
<td>0.28</td>
<td>0.22</td>
<td>0.65</td>
<td>0.41</td>
</tr>
<tr>
<td>Item3</td>
<td>0.61</td>
<td>0.76</td>
<td>0.81</td>
<td>0.94</td>
<td>0.59</td>
<td>0.74</td>
<td>0.21</td>
<td>0.14</td>
</tr>
<tr>
<td>Item4</td>
<td>0.71</td>
<td>0.89</td>
<td>0.84</td>
<td>0.94</td>
<td>0.62</td>
<td>0.85</td>
<td>0.20</td>
<td>0.11</td>
</tr>
<tr>
<td>Item5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The fuzzy intuitions are suitable for $I_τ - 1$, $I_τ - 2$, $II_τ - 1$, $II_τ - 2$, $III_τ - 1$, $III_τ - 2$, $IV'_τ - 1$ and $IV'_τ - 2$.

7. Conclusion

FMK studied fuzzy intuitions based on Godel and Standard sequence methods. Some more intuitions are proposed. These methods satisfy some fuzzy intuitions. The proposed methods satisfy some more fuzzy intuitions. The fuzzy set with the two fold fuzzy membership function will give more evidence than a single fuzzy membership one. The fuzzy logic with two fold fuzzy membership function is discussed based on “True” and “False”. The fuzzy intuitions are discussed using to fold fuzzy sets. The fuzzy Inference and fuzzy reasoning are studied for “a two fold fuzzy sets”. The FCF is studied as the difference between the two fuzzy membership functions “True” and “False”. The fuzzy Certainty Factor is made as a single fuzzy membership function to compute the conflict of evidence of the Incomplete Information. The fuzzy intuition with truth variables are studied for “a two fold fuzzy set”. The business intelligence is discussed as application for “a two fold fuzzy set” for fuzzy intuitions.

Acknowledgments

The Author would like to thank Sri Venkateswara University, Tirupati authorities for providing facilities to carry out this work.

References


