Retrograde GEO Orbit Design Method Based on Lunar Gravity Assist for Spacecraft

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RETROGRADE GEO ORBIT DESIGN METHOD BASED ON LUNAR GRAVITY ASSIST FOR SPACECRAFT

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In this paper a design method for changing the inclination of an orbital spacecraft from the ascending orbit to the retrograde orbit is presented. Firstly, revealing the mechanism of the lunar gravity assist, and maximum change capability of the spacecraft orbit parameters will be obtained, based on the Lagrange planetary equation or Hamilton equation; Secondly, studying the flight mechanism to transfer the geocentric orbital spacecraft to the geocentric retrograde orbit; Finally, the orbit technology from the forward GEO orbit to the retrograde GEO orbit is designed, and the technical application method in the actual orbit design project is proposed.

INTRODUCTION

Space debris is called orbital debris. It refers to all man-made objects in the universe except the normal spacecraft. The satellites that have reached end-of-life are scrapped, and the powder produced by the ignition of the engine belongs to the category of space debris [1]. Since October 4, 1975, the first satellite of the former Soviet Union orbited, and human entered the space age. In the past more than half a century, human has conducted more than 4,000 space launches and sent more than 6,000 spacecraft into the space [2]. At present, there are more than 10 countries with independent launch capabilities, and there are more than 50 countries that have space assets and nearly a thousand of them are in use. However, while space activities have brought many benefits, they have also brought a lot of negative influences. The serious negative impact is the continuous increase of space debris. According to estimations by space research departments such as NASA and ESA, the number of usable space objects on the Earth's orbit (both over 10cm in size, mostly debris) has increased from 5,000 in 1981 to more than 17,000 in April 2016. [3], in which the disintegration produces a large part of the debris.

Most of the space debris is flying on the low earth orbit (LEO), and a considerable part of it flies on the geosynchronous orbit (GEO) [4, 5]. Although the density of GEO debris is much lower than that of low orbit, GEO does not have debris removal mechanisms such as atmospheric damping [6]. As a result, GEO debris has a long period of time on this orbit, the number of which grows fast, and it is potentially dangerous to human space activities. Satellites in GEO are extremely expensive. Due to special orbital characteristics, their space resources are very limited and valuable resources. The space debris generated by this orbital zone will threaten all GEO satellites. The geosynchronous orbit is a very precious resource. The space debris problem in the region has attracted the attention of the spacefaring nations.

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This paper will focus on changing the orbital inclination to the retrograde GEO orbit by means of lunar gravity assist. Since the direction of the space debris flying on the GEO orbit is opposite to the retrograde GEO orbital spacecraft, it is possible to quickly observe all spacecraft debris fragments in the GEO zone. This paper simplifies the Earth-Moon system model based on Lagrange's planetary equations. It can quickly solve the optimal initial orbit of the retrograde GEO, and realizes a rapid design method for retrograde GEO.

CONVENTIONAL IMPLEMENTATION OF RETROGRADE GEO ORBITS

This section will first analyze the energy of the retrograde GEO orbit design and give the maneuvering scheme of the spacecraft from the initial GTO orbit to the retrograde GEO orbit. Based on the classical Kepler orbital design method, a two-body dynamics model was used to calculate the velocity impulse Δv needed to achieve retrograde GEO orbital maneuvers from GTO orbits. Analyze the consumption of fuel when GTO orbits under different initial conditions change to retrograde GEO orbits, and give the requirements for orbital maneuvers for rockets and spacecraft under the corresponding conditions.

When the spacecraft separated from the rocket and enters the orbit, it needs change the inclination and eventually reach the retrograde GEO orbit. This process requires the spacecraft's fuel to be consumed. The maneuvering of low orbit to GTO orbits requires the application of a pulse increment Δv_A at the perigee tangent to the GTO on the LEO orbit to lift the orbit to an elliptical GTO orbit with an apogee of 42164.17 km.

During the launch of the rocket, due to the existence of a rotation of the earth, the rotation period T_e is 23 hours 56 minutes 04 seconds, which is the earth's rotation velocity of $\omega_e = 2\pi / T_e = 7.2877 \times 10^{-5}$ rad/s. Take the Taiyuan Satellite Launch Center as an example, the earth's rotation velocity is $v_e = \omega_e r_e \cos(\phi) = 363.77$ m/s. From Figure 1, we can see that the rocket needs to increase the velocity of Δv_A at point A at the perigee.

$$\Delta v_A = \sqrt{v_{A_2}^2 + \Delta v_{\text{Earth}}^2 - 2 * v_{A_2} \cdot \Delta v_{\text{Earth}} \cos(i) \frac{\mu}{r_B}} = 9.9567$$

When the spacecraft is separated from the rocket, it needs a velocity increment Δv_B at B, which is

$$\Delta v_B = v_{B_3} - v_{B_2} = 1.4773 \text{ km/s}$$

When the spacecraft completes its maneuver at apogee, it flies along a circular GEO orbit 3. The orbital plane of orbit 3 must transfer to the equatorial plane, and it is necessary to apply an impulse at the intersection of the two orbits, as shown in Figure 3 at points C and D. Assume that the spacecraft applies velocity increment at C. The angle between the circular GTO orbit 3 and the retrograde GEO orbit 4 needs to be changed $\delta = \pi - i$. The velocity of orbit 3 and orbit 4 at C is equal to v_{B_3} . It can be known from the relationship between triangle vectors. The spacecraft needs to apply impulse Δv_C at C.

$$\Delta v_C = 2v_{B_3} \sin\left(\frac{\pi-i}{2}\right) = 5.8055 \text{ km/s}$$

Figure 1. Retrograde GEO orbital maneuvers

Therefore, when the rocket launches a spacecraft from the Taiyuan Satellite Launch Center in the north latitude $38^{\circ}50'55.42''$ and east longitude $111^{\circ}36'28.89''$ in China, when the orbital maneuvering method is used to realize the retrograde GEO orbit, the total velocity impulse required includes the velocity increment Δv_A needed to provide rockets at the A point, the velocity impulse on the GTO apogee Δv_B and the velocity impulse Δv_C of the spacecraft at the point C. The total velocity impulse is $\Delta v_{\text{total}} = 9.9567 + 1.4773 + 5.8055 = 17.2395 \text{ km/s}$.

IMPACT OF THE LUNAR GRAVITATIONAL PERTURBATION ON SATELLITE ORBITS IN A RESTRICTED THREE-BODY PROBLEM

In this section, we will analyze the situation where the launch orbit and the lunar orbital plane are different in the restricted three-body model, and use the lunar gravity as perturbation to analyze from the perspective of orbital perturbation. Firstly, the numerical integration method is used to simulate the change of the orbit of the spacecraft after one cycle of flight under different initial conditions. Since the numerical integration method is very inefficient in calculating strong nonlinear dynamic equations, this paper will use the high-order Legendre polynomial to expand the perturbation of the moon based on the Lagrange planetary equation. The long-term change of the orbital parameter change of the lunar perturbation term is analyzed, that is, analyzing the influence of orbital parameters under multiple lunar gravitational forces, and obtain the maximum changeability of the lunar gravitational force to the inclination of the satellite. Finally, a scheme for implementing a corresponding retrograde GEO space debris orbit design can be derived.

Numerical method to analyze the effect of lunar gravity on orbital parameters

In order to understand the influence of the lunar gravity on the initial orbital parameters of spacecraft, this paper first uses the integral method to simulate and calculate the complete restricted three-body dynamics problem model, and analyzes the results. Many studies have shown that the influence of the lunar gravity on the transfer orbit is closely related to the initial orbital parameters

of the spacecraft. The influence of the lunar gravitational force on the transfer orbit can be controlled by the ascendant right ascension Ω and the perigee argument ω . The main purpose is to understand the change of the lunar gravity to the orbital inclination.

In general, the spacecraft's orbit depends on five factors: the semi-major axis a , the eccentricity e , the inclination i , the argument of perigee ω , and the longitude of ascending node Ω . When it is necessary to determine the spacecraft's orbit change during a period of time under the action of the camera, it is necessary to integrate the equation of motion. In order to facilitate the analysis, the numerical integration method is first used to simulate a restricted three-body problem model with strong nonlinearity. It is possible to initially consider the amount of change in orbital gravitational perturbations after a period of time T . Multiple simulations will be performed on various initial conditions.

The present simulation example considers the case where the initial inclination i is 28.5 degrees, the dimensionless perigee height $R_p = 0.001$ (384 km), the apogee height $R_a = 0.6$ (230640 km), the semi-major axis $a = 0.3005$ (115510 km), and the eccentricity $e = 0.9967$. In order to analyze the effects of the lunar gravity on the spacecraft orbital parameters, it is necessary to traverse the argument of perigees ω and the longitude of ascending node Ω , both in the range of $[0, 360]$ degrees with an interval of 1 degree. Integration time is from the moment of the perigee of the initial orbit t_0 to the moment of the next perigee $t_0 + T$.

The simulation result contour map is shown in Fig. 2. The abscissa is the argument of the perigee ω and the range is $[0, 360]$ degrees. The vertical axis is ascendant ascension Ω . The left side shows the change in the inclination i , and the right side shows the change in the semi-major axis a . It can be seen from the calculation results in the figure that the horizontal axis and the vertical axis have symmetry, and the inclination i and the semi-major axis a increase and decrease are symmetric, which means that the lunar gravity is within one orbital period. The influence of spacecraft orbits can not only increase the semi-major axis or inclination, but also make it reduce the same equivalent weight. The magnitude of the change is related to the initial orbital parameters ω and Ω .

The results show that under the current initial conditions, the maximum positive and negative changes of the lunar inclination i in one cycle is 10.7015 degrees. Similarly, the maximum positive and negative directions of the spacecraft semi-major axis a are the change amount is 0.0913 (35096.17km). The main frequency of the computer used is 3.6 GHz and the memory is 16 G. The calculation time required to obtain the simulation result through integration is 125 minutes.

In this paper, simulations are performed on the orbits under different initial conditions, and the effect of the lunar gravity on different initial orbits of the spacecraft during one orbital period is calculated. The calculations show that the change of the inclination due to the lunar gravity increases with the eccentricity of the orbit. Increased, the maximum inclination increases by more than 70 degrees.

(Orbit parameters: Perigee height $R_p=0.001$ (384km) , Apogee height $R_a=0.6$ (230640km) , semi-major axis $a=0.3005$ (115510km) , eccentricity $e=0.9967$, Initial inclination $i=28.5\text{deg}$)

Figure 2. Impact of Lunar gravity on spacecraft orbits in one orbital cycle

Dynamic model based on Legendre series expansion

In the previous section, a numerical integration method was used to simulate the orbital changes of the spacecraft after one period under different initial conditions. Through simulation results analysis, in the restricted three-body dynamics model, the lunar gravity can produce a certain amount of change for large elliptical orbits. However, since the numerical integration method is very inefficient when calculating strong nonlinear dynamic equations, it will take a long time to integrate one orbital period, and it is impossible to search for an ideal retrograde orbit. This article will give a fast orbit calculation method. Based on the Lagrange planetary equation, the high-order Legendre polynomials will be used to expand the perturbation of the Moon. Analyze the long-term change of the orbital parameter change of the lunar perturbation term, that is, analyze the influence of orbital parameters under multiple lunar gravitational forces, and obtain the maximum changeability of the lunar gravitational force to the inclination of the satellite. Finally, we can come up with a plan for implementing a corresponding retrograde GEO orbital design. The velocity is much faster than the integral method in calculating the restricted three-body problem model.

This section will give a simplified dynamic model. Assuming that the Earth is in the center of the reference coordinate system, the quality is m_0 , the quality of the mass of the observing celestial body is m' , which make a uniform circular motion around the earth, with semi-major axis a' , and the average velocity is n' ($n'^2 a'^2 = G[m_0 + m']$). The orbit is a spatial three-dimensional elliptical orbit. Its semi-major axis is a , the eccentricity e , the inclination is i , the argument of perigee is ω , and the ascending node Ω , and the average angular velocity is obtained by the formula $n^2 a^2 = Gm_0$.

Figure 3. Schematic diagram of the impact orbit of spacecraft

Therefore, the spacecraft's gravitational perturbation of the lunar can be described as:

$$R = \frac{\mu' G(m_0 + m')}{\sqrt{r^2 + r'^2 - 2rr' \cos(S)}} \quad (1)$$

Where, $\mu' = m' / (m_0 + m')$, G is the universal gravitational constant, S is the angle between Earth's connection to the lunar and the connection between Earth and spacecraft, as shown in Figure 3.

Equation (1) can be expressed as a classical Legendre polynomials as

$$R = \frac{\mu' G(m_0 + m')}{r'} \sum_{n=2}^{\infty} \left(\frac{r}{r'} \right)^n P_n[\cos(S)] \quad (2)$$

Where, P_n is the Legendre polynomial. In order to obtain high-precision approximate analysis, the higher-order Legendre polynomials are taken and the orders are taken to the fourth order, which are respectively P_1 , P_2 , P_3 . Bringing P_1 , P_2 , P_3 and P_4 into (2) can get each Legendre polynomial, as

$$\begin{aligned} R_2 &= \frac{\mu' G(m_0 + m')}{r'} \left\{ \left(\frac{r}{r'} \right)^2 P_2[\cos(S)] \right\} \\ &= \frac{\mu' n'^2 a^2}{2} \left\{ \left(\frac{a'}{r'} \right)^3 \left(\frac{r}{a} \right)^2 [3\cos^2(S) - 1] \right\} \end{aligned} \quad (3)$$

$$\begin{aligned}
R_3 &= \frac{\mu' G(m_0 + m')}{r'} \left\{ \left(\frac{r}{r'} \right)^3 P_2[\cos(S)] \right\} \\
&= \frac{\mu' n'^2 a^2}{2r'} \left(\frac{a'}{r'} \right)^3 \left(\frac{r}{a} \right)^3 [5\cos^3(S) - 3\cos(S)]
\end{aligned} \tag{4}$$

$$\begin{aligned}
R_4 &= \frac{\mu' G(m_0 + m')}{r'} \left\{ \left(\frac{r}{r'} \right)^4 P_4[\cos(S)] \right\} \\
&= \frac{\mu' n'^2 a^4 a'^2}{8r'^5} \left(\frac{r}{a} \right)^4 [35\cos^4(S) - 30\cos^2(S) + 3]
\end{aligned} \tag{5}$$

The above formula can substitute the relation $n'^2 a'^2 = G[m_0 + m']$.

Because the spacecraft perturbation is generally divided into short period and long period, in order to analyze the influence of the lunar perturbation on the spacecraft many times, it is necessary to analyze the change of the orbital parameters when the spacecraft is subjected to perturbation power for a long time. Therefore, the elimination of short-cycle perturbation terms requires the following basic definitions:

$$\langle F \rangle = \frac{1}{2\pi} \int_0^{2\pi} (F) dM \tag{6}$$

Where, M is the flat point angle of the spacecraft, which is related to time, whit the symbol “ $\langle \rangle$ ”.

In order to obtain the average of the Legendre polynomials R_2 , R_3 and R_4 , this paper will proceed as follows. First introduce the parameters α and β , define $\alpha = \hat{P} \cdot \hat{r}'$, and $\beta = \hat{Q} \cdot \hat{r}'$, where $\hat{r}'$ is the unit vector from the Earth to the direction of the moon, $\hat{P}$ and $\hat{Q}$ are the normal cartesian components. They have a function of the orbital surface parameters i , ω and Ω of the spacecraft. $\hat{P}$ points toward the perigee direction and $\hat{Q}$ perpendicular to the $\hat{P}$ direction. When perturbation celestial orbit is considered as circular orbit, the following relation can be obtained:

$$\alpha = \cos(\omega) \cos(\Omega - M') - \cos(i) \cos(\omega) \cos(\Omega - M') \tag{7}$$

$$\beta = -\sin(\omega) \cos(\Omega - M') - \cos(i) \cos(\omega) \cos(\Omega - M') \tag{8}$$

When the parameters α and β are defined, it can be seen that there is a functional relationship between the two parameters and the parameter S related to the position of the moon and the body of the subject. Can be described by the following relation

$$\cos(S) = \alpha \cos(f) + \beta \sin(f) \tag{9}$$

Combining Equation (9) with Equations (3), (4), and (5), the relationship equation between orbital parameters and perturbation terms of spacecraft can be obtained. The true anomaly f of the

spacecraft in equation (9) needs to be replaced by the eccentric anomaly E . It can be obtained by the following transformation of the classic orbital element relationship.

In order to obtain the average perturbation of a short period, it is necessary to integrate the eccentric anomaly E instead of the eccentric anomaly M . Therefore, the relationship $dM = [1 - e \cos(E)]dE$ between the eccentric anomaly E and the mean anomaly M can be used. The corresponding relationship is brought into the average perturbation equation (6) to obtain the corresponding average term.

Therefore, the next step is to perform a quadratic averaging to eliminate the variable M' that is related to the perturbation object, that is, the flat point angle of the moon. For this reason, it is necessary to assume that the Kepler orbital element is constant during the averaging process. Since the spacecraft orbital period is much smaller than the lunar cycle, the orbital parameters slowly oscillate with the lunar cycle. The next step is to bring the fourth-order Legendre polynomials into the Lagrange planetary equations to obtain the average spacecraft's equation of motion, which can be described by orbital elements:

$$\frac{\partial a}{\partial t} = 0 \quad (10)$$

$$\frac{\partial e}{\partial t} = \frac{35\mu'n'^2\sqrt{1-e^2}}{n}\sin^2(i)\sin(4\omega) + f_1(a, e, i, \omega) \quad (11)$$

$$\frac{\partial i}{\partial t} = \frac{-\mu'n'^2}{8n\sqrt{1-e^2}}\sin(2i)\sin(2\omega) + f_2(a, e, i, \omega) \quad (12)$$

$$\frac{\partial \omega}{\partial t} = \frac{\mu'n'^2}{n\sqrt{1-e^2}}[(15\cos^2 i - 1 + e^2) + (1 - e^2 - \cos^2 i)\cos(2\omega)] + f_3(a, e, i, \omega) \quad (13)$$

$$\frac{\partial \Omega}{\partial t} = \frac{\mu'n'^2 \cos i}{12n\sqrt{1-e^2}}[(e^2 \cos(2\omega) - 3e^2 - 2)] + f_4(a, e, i, \omega) \quad (14)$$

$$\frac{\partial M_0}{\partial t} = \frac{-\mu'n'^2}{8n}[(e^2 + 17)(3\cos^2 i - 1) + 5(1 + e^2)\sin^2 i \cos^2 \omega] + f_5(a, e, i, \omega) \quad (15)$$

The function $f_i(a, e, i, \omega)$ is a fourth-order term of a polynomial. At this point, we have obtained the Lagrange planetary equation after the second average. Some conclusions can be drawn from this equation:

(1) The parameter μ' is a constant, $\mu' = m' / (m_0 + m')$. In addition to the Earth-Moon system, it is also applicable to other perturbed three-body problem systems.

(2) Coefficient $K_1 / K_2 = 4096a'^2 / a^2$ gives the influence of the second-order and fourth-order terms on the system. When the semi-major axis of the spacecraft (the body to be perturbed) increases, the influence of the fourth-order term on the system will also increase. At the same time,

when the eccentricity of the spacecraft increases, the impact of the related items on the system also increases.

(3) The difference between the simplified system model after the quadratic averaging and the strict restricted three-body model will vary with the initial orbital parameters of the spacecraft.

(4) The semi-major axis a of the orbit is a constant, and the size of a remains unchanged during long-period disturbance.

(5) It can be seen from the differential equations that longitude of ascending node Ω has no effect on the movement of the spacecraft.

(6) When the spacecraft's initial inclination $i = 0$ or initial eccentricity $e = 0$, the eccentricity e and the inclination i are not affected by the perturbation, that is, the spacecraft remains in a circular orbit or in the earth-moon-coplanar orbit. The occurrence of this phenomenon is the result of the second-average perturbation equation. In the strict restricted three-body problem, the orbit of the spacecraft is always in a circular orbit. Therefore, the size of the initial eccentricity of the spacecraft has a close relationship with the magnitude of the influence of the perturbation force on the spacecraft orbit.

Influence of lunar gravity on orbital parameters

Because the numerical integration method is very inefficient when calculating strong nonlinear dynamic equations, integrating one orbital period will also take a long time. However, to achieve retrograde GEO orbits, the spacecraft is required to use the lunar gravity several times to maximize the change in its inclination. Therefore, multi-period simulations and calculations are needed. However, due to the low computational efficiency of simulations, simulation calculations based on the restricted three-body problem model will require too much computation time to be achieved. Therefore, simulation calculations need to be performed based on the simplified Lagrange planetary equations (10)-(15). This section will simulate and verify the initial conditions under different conditions.

In order to analyze the effectiveness and accuracy of the gravitational model described by the second-averaged Lagrange planetary equation, the following equations will be simulated and analyzed. The simplified quadratic average Lagrange planetary equation and the restricted three-body problem model (which can be considered as the exact model) need to be compared and analyzed. The numerical simulation was performed based on two models for different initial conditions.

By simulating different initial conditions, the initial conditions are: perigee height R_a is 68,878km, apogee altitude R_e is 266,378km, semi-major axis a is 136,617km, and eccentricity e is 0.9497. Three scenarios were simulated and analyzed: 1) Three argument of perigees ω (0 degrees, 45 degrees, and 90 degrees); 2) Three initial inclination i (30 degrees, 45 degrees, and 90 degrees); 3) The two ascending nodes of right ascension Ω are (0 degrees and 45 degrees). The simulation results are shown in Figure 4. From the simulation results in the figure, it can be seen that the change of the argument of perigee ω affects the eccentricity e and the inclination i . The change of the initial inclination i also has an obvious effect on the eccentricity e and inclination i ; The curves of the eccentricity e , the inclination i , and the semi-major axis a of the right ascension point for the right ascension Ω coincide completely with each other, indicating that the change in the right ascension Ω at the ascending node has no effect on the orbital eccentricity e and the inclination i .

Figure 4. Fitting of orbital parameters with different initial conditions

From the simulation results above, we can see that using the gravitational model described by the Lagrange planetary equation after the second averaging, the effect of the lunar gravity on the orbital parameters mainly depends on the initial inclination i and the argument of perigee ω . At the same time, it should be noted that when the initial orbital inclination is 90 degrees, the lunar gravity has no effect on the orbital parameters of the space bridge. Therefore, in order to make the greatest possible change of the orbital inclination by means of the lunar gravity, it is only necessary to perform traversal calculations on the value of initial inclination i and the perigee accessory ω to find the maximum value of the inclination change.

Next we need to analyze the validity and accuracy of the model. We need to compare the quadratic averaged Lagrange planetary equation with the restricted three-body problem model. As shown in Figure 9, the initial conditions are: perigee height R_a is 6996km, apogee height R_e is 230643km, orbital eccentricity e is 0.9476, perigee accessory ω is 0 degree, and inclination i is typically 28.5 degrees, due to initial The ascendant right ascension Ω and the true anomaly θ have no influence on the evolution of the orbit and can be chosen at will. By calculating the evolution of the orbit over a long period of time under this initial condition, the deviation of the Lagrange planetary equation after the quadratic averaging and the complete restricted three-body problem model is analyzed after 1300 days of orbit integration.

Fig. 5 shows the simulation results. The red curve is the orbital parameter of the restricted three-body problem model integral, and the blue curve is the orbital parameter of the integral of the Lagrange planetary equation after the second-average. The simulation results show that under the influence of the gravitational force of the lunar, the semi-major axis a shows a periodical change with a short period, and the mean value of the semi-major axis does not change. The eccentricity e also exists short-term periodic fluctuations change, but their average value decreases over time, and is almost identical to the change trend of the Lagrange planetary equation after the second-average; the inclination i changes similar to the eccentricity. There are short-term periodic fluctuations. With the passage of time, the inclination of the orbit increases continuously, and the trend of the inclination of the orbit in the integration result of the Lagrange planetary equation after quadratic averaging is almost exactly the same, when the eccentricity increases. The relative accuracy of the fitting will be weakened.

Figure 5. The variation of orbital parameters with different initial eccentricity e

Through the above simulation analysis, we can see that the orbital evolution of the long-range flight of the Lagrange planetary equation after the second-average of the Earth-Moon system is due to the gravitational attraction of the Moon. The orbital parameters of the spacecraft are mainly related to the initial eccentricity e , perigee secondary angle ω and inclination i . The smaller the eccentricity e , the more accurate the description of the Lagrange planetary equation after the second averaging, but when the eccentricity decreases, the spacecraft is less affected by the lunar gravitation, resulting in a more limited change in the inclination i . Therefore, the initial eccentricity e needs to be as large as possible while satisfying the accuracy requirement. When the eccentricity is given, the orbital parameters of the spacecraft change mainly depending on the initial argument of perigee ω and the initial inclination i . Therefore, in order to maximize the change of the orbit inclination angle, the values of the initial ω and i need to be reasonably designed. In this paper, we first traverse the initial conditions of ω and i , and observe the evolution of the orbital parameters of the initial ω and i .

Fig.6 The initial argument of perigee ω is in the range $[0^\circ, 180^\circ]$, the initial inclination i is in the range $[10^\circ, 120^\circ]$, the perigee R_a is still 6996 km, the apogee height R_e is increased to 320,000 km, and the eccentricity e increases to 0.9584. The values of right ascension Ω and true anomaly θ are both 0 degrees. Fig. 6 is a contour plot of the orbital maximum i obtained on the basis of the integration of the Lagrange planetary equation after the second-average for 200 days given initial conditions. Figure 6 shows the contour plot of the maximum change in orbit, which is the difference between the maximum value of the inclination i and the initial inclination i during the evolution of the orbit.

The simulation results show that the maximum value of the inclination i can be increased to 170 degrees. When the given initial orbital inclination angle is greater than 90 degrees, the increase in the orbital inclination angle is the largest, indicating that when the initial orbital inclination angle is greater than 90 degrees, the lunar gravity is the highest. It is easy to change the inclination of the orbit.

$$Ra=6996\text{km}, Re=209083\text{km}, i=[10^\circ, 120^\circ], om=[0^\circ, 180^\circ]$$

Figure 6. The maximum change of the inclination angle of the initial argument of perigee ω and inclination i

Retrograde GEO orbit design scheme

The analysis of this article shows that the lunar gravitation can exert a great influence on the spacecraft orbit, so it can be effectively used to launch the spacecraft from Earth orbit into the retrograde GEO orbit. First use the lunar gravity to change the inclination to maximum, and then use the Lunar own maneuverability to change the inclination i by means of the lunar gravity. Through the above simulation design and analysis, the following retrograde GEO orbit design solutions can be given. The specific program steps are as follows:

Step 1: The satellite is launched by the rocket to a vertical large elliptical orbit (GTO) orbit with an inclination greater than 90 degrees. At this time, the velocity impulse applied by the rocket is Δv_0 ;

Step 2: The satellite is separated from the rocket after launch, and enters a long-term ground-month system flight mode. During this flight, the satellite is subject to the impact of the lunar gravitation many times, which makes the satellite's orbital inclination obtain the maximum increment;

Step 3: When the satellite's inclination change amount reaches the maximum, it is generally obtained at the GTO orbital apogee. At this time, the inclination is close to the retrograde flight. The satellite applies the forward maneuvering impulse Δv_1 by itself to make the retrograde GTO orbit perigee, elevated to the same height as GEO orbits;

Step 4: When the satellite flies to the perigee along the retrograde GTO orbit, the altitude reaches the GEO orbital altitude, and the reverse velocity impulse Δv_2 is applied at the perigee. This causes

the satellite apogee to decrease and turns into a circular orbit. At this time, the satellite orbit is at a close angle of 180 degree reverse circular orbit;

Step 5: The satellites fly along circular orbits. When flying at the intersection of the reverse GEO orbits, a velocity impulse Δv_3 is applied to change the orbital inclination so that the orbital inclination of the orbit becomes 180 degrees, and finally a retrograde GEO orbital design is achieved.

Retrograde GEO orbit optimization design results

This paper adopts the above design retrograde GEO orbital design scheme. Through simulation analysis, it can be seen that the orbital evolution of the long-range flight of Lagrange's planetary equation after the second-average of the Earth-Moon system follows the gravitational attraction of the moon. The orbital parameters of the spacecraft are mainly related to the initial orbit. The eccentricity e , the initial argument of perigee ω , and the initial inclination i are related. Based on the simulation analysis, this paper traverses the initial argument of perigee ω and the initial inclination i through the twice-averaged Lagrange planetary equation, and obtains the initial range of maximum value of the dip change, and obtains the initial range of the smaller range. Simulation condition value interval. Then based on the Earth-moon restricted three-body problem model, the value interval is again traversed to obtain a lunar gravity assist orbit.

The optimal initial conditions were obtained by traversing the solution: the height of the perigee of the GTO orbit was 500 km, that is, the height of the perigee of the orbit $R_a = 6996$ km. The apogee height is $R_e = 326,378$ km. The eccentricity is $e = 0.9587$ and the initial inclination $i = 90.19$ degrees. The right ascension Ω and true anomaly θ are 0 degrees, and the argument of perigee ω is 0.00012 degrees. After the GTO orbiting satellite flew 232.8 days in the terrestrial-restricted three-body system, the maximum change in inclination was $\Delta i = 82.93$ degrees, and the inclination $i = 173.12$ degrees. The satellites then maneuver according to the above mentioned retrograde GEO orbital design scheme. The results of the optimal retrograde GEO orbital maneuvering parameters are shown in Table 1. The orbit design result is shown in Figure 7.

Table 1 Optimal design results for retrograde GEO orbits

Maneuver	Provider	Velocity impulse (km/s)
Δv_0	Rocket	10.6647
Δv_1	Satellite	0.3944
Δv_2	Satellite	1.0239
Δv_3	Satellite	0.3690
Total velocity impulse	12.4520 (km/s)	
Total satellite velocity impulse	1.7873 (km/s)

Figure 7. The optimal retrograde GEO orbit in the lunar and earth system

CONCLUSION

This paper achieves the design method of retrograde GEO orbit by means of lunar gravity. If use the direct launch method, such as launching from the Taiyuan Satellite Launch Center, requires a minimum maneuvering capability of 3.5020 km/s, a minimum of 2.5023 km/s velocity pulse is emitted from the Wenchang Satellite Launch Center. No matter where it is launched, the maneuverability of the satellites is too high and it cannot be achieved at present. By effectively using the lunar gravitational force and using orbital design methods that combine lunar gravity perturbation with satellite maneuvers, the design of retrograde GEO space debris monitoring trajectory is well achieved. The total energy consumption is 12.4520 km/s. The energy provided by satellites is a velocity pulse of 1.7873 km/s.

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