LPG Fiber Optic Sensor Applied to the Determination of the Flexural Elasticity Modulus of Woods

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Abstract

In this work, we show the feasibility of using CO₂ laser induced long-period gratings (LPGs) for determining the flexural elasticity modules of woods. To do this, we characterized the dynamic response of bars made of different woods put in oscillation. By recovering the bars’ flexural oscillations from the LPG time response and by taking the fast Fourier transform (FFT) of it, the movement vibration frequencies could be obtained. Knowledge of these vibration frequencies for the oscillation lengths of different bars allowed us to calculate the materials’ the flexural elasticity modules.

Keywords: optical fiber sensing; vibration sensor; MOE of woods

1. The flexural elasticity modules determination

In the ‘bar resonance’ method, a bar of the material of interest is arranged in a cantilever configuration and put in oscillation. The resulting movement is studied in order to obtain the bar material’s flexural elasticity modules value [1,3]. We use a curvature sensitive long-period fiber grating to register the displacement of the bar as a function of time. 

If the bar in cantilever configuration is vertically deflected from its equilibrium state and then released, the resulting movement is oscillatory and its amplitude decays as a function of time. The formal treatment of this movement, via the Euler–Bernoulli equation, indicates that the general solution has a series form with infinite frequency components, which are given by equation (1), where ρ and Y are, respectively, the material density and flexural elasticity modules; A and L are, the cross-sectional area and the vibrating length of the bar; and I is the second moment of the cross-section, which, for a rectangular bar of width b and depth h, can be calculated as b h²/12; kv is the viscous damping constant of the system. The value of the constant λn is determined by the boundary conditions of the problem.

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{\rho l}{\pi}} \left(\frac{\pi^2}{L^2} + \frac{k v^2}{2 \rho A} \right)^{\frac{1}{2}}
\]  

As we focus on the determination of the Young’s modulus, one can see, from equation (1), that the knowledge of only one frequency component is enough to attain our goal. In this investigation, we determined the first frequency component, f₁, and then found the flexural elasticity modules of the materials of interest.

2. Long-period gratings and their curvature sensitivity

Long-period fiber gratings (LPGs) consist of a longitudinal periodic perturbation of the refractive index of an optical fiber which is able to couple between core and cladding modes at certain wavelengths. Equation (2) describes the wavelengths λ⁽ᵐ⁾ where the coupling between the referenced modes happen—nc is the effective refractive index of the core mode, n⁽ᵐ⁾cl is the effective refractive index of the mth order cladding mode and Λ is the period of the refractive index perturbation [4].

\[
\lambda⁽ᵐ⁾ = \left( n_{cko} - n_{cl⁽ᵐ⁾} \right) \Lambda
\]  

The curvature causes stretching or compression of the fiber by changing the refractive indices ncko, ncl due to the elasto-optic effect. In addition, the elastic deformation changes the period of the grating (Λ). Changes in refractive indexes of the fiber and in the period of the LPG cause a shift in the wavelength of resonance λ as well as, changes in the depth of the transmittance dip (equation (3)).

\[
T⁽ᵐ⁾ = 1 - \sin^2 (k⁽ᵐ⁾L)
\]  

Where, T⁽ᵐ⁾ is the dip of the m-th mode transmittance and k⁽ᵐ⁾ denotes coupling coefficient between the core mode and the m-th order cladding mode given as
\[ \kappa^{(m)} = \frac{\pi \Delta n_c \ell^{(m)}}{\lambda} \]  

(4)

And, \( J^{(m)} \) is the overlap integral between the fundamental guided mode and the cladding modes in the core area of the fiber [4].

In this research, CO₂ laser-induced LPGs sensors of 500 μm pitch and 2.5 mm long imprinted on standard optical fibers were employed. The experimental setup is shown in figure 1. A super-luminescent LED is used as the light source. A photodetector coupled to an oscilloscope are used for taking measurements. The LPG is glued on a wooden bar, which has one fixed end and the other one is let free. The deflection of the bar (accounted as a vertical displacement \( \Delta y \) of the free bar end), causes the LPG to bend and, thus, its curvature response can be monitored.

\[ \text{Figure 1. (a) Scheme and (b) picture of the experimental setup.} \]

When detecting light using the photodetector, as its voltage response is proportional to the overall optical power from the broadband light source, negative voltage variations are related to negative \( \Delta y \) values and positive voltage variations identify positive \( \Delta y \) values. The dependence of voltage on bar vertical displacement was seen to be linear [1].

3. ‘Bar resonance’ method results

Initially, the bar put in oscillation in a cantilever setup, is slightly deflected and then released. The signal measured in the oscilloscope takes a sinusoidal form whose amplitude decays as a function of time (figure 2a). By taking the Fourier transform of the measured signal, one can identify the frequency components of the oscillating bar movement. The figure 2b shows the amplitude of the fast Fourier transform (FFT), calculated from figure 2a data as a function of the frequency.

\[ \text{Figure 2. a) Photodetector time response of LPG sensor. b) The fast Fourier transform (FFT) amplitude as a function of frequency.} \]

The value found for \( K_v \) (in all cases) shows that \( \left( \frac{k_v}{\ell} \right)^2 \ll \frac{Y I}{\rho A} \left( \frac{\lambda n}{\ell} \right)^4 \). The flexural elasticity modules value of the material of the cantilever (Y) is determined from the experimental data fitting frequency of the first normal mode (\( f_1 \)) versus the overhang length (\( L \)) of the cantilever.

Figure 3 shows the frequency setting of the first normal mode (\( f_1 \)) against the cantilevered length (\( L \)) for bars of five different woods (cachimbo, mahogany, cedar, pine and tornillo). The values determined for the flexural elasticity
moduli \( Y \) of the woods, based on the adjustment of equation (1) with their respective experimental data, are shown in table 2.

**Figure 3.** The first harmonic frequencies \( f_1 \) (blue circles) versus the length of the cantilevered. The solid curve represents the fitting of equation (1) with the experimental data of the woods. The determined values of the flexural elasticity moduli of the woods are shown in each curve.

Table 1 shows the values of the dimensions, densities, humidity percentage and moment of inertia \( I \) of the woods studied.

<table>
<thead>
<tr>
<th>Material</th>
<th>( h ) (m)</th>
<th>( b ) (m)</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( I ) (Kgm(^2)).10(^8)</th>
<th>Moisture content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cariniana domesticata Martius (Cachimbo)</td>
<td>0.0158</td>
<td>0.0513</td>
<td>590</td>
<td>1.6862</td>
<td>12.42</td>
</tr>
<tr>
<td>Swietenia mahagoni (Caoba)</td>
<td>0.0148</td>
<td>0.05155</td>
<td>469</td>
<td>1.3926</td>
<td>12.50</td>
</tr>
<tr>
<td>Carapa guianensis Aubl. (Cedro)</td>
<td>0.01615</td>
<td>0.05101</td>
<td>451</td>
<td>1.7906</td>
<td>11.59</td>
</tr>
<tr>
<td>Pinus patula (Pino)</td>
<td>0.0149</td>
<td>0.04925</td>
<td>465</td>
<td>1.3576</td>
<td>10.91</td>
</tr>
<tr>
<td>Cedrelinga cateniformis (Tornillo)</td>
<td>0.01574</td>
<td>0.05175</td>
<td>470</td>
<td>1.6817</td>
<td>11.39</td>
</tr>
</tbody>
</table>
The flexural elasticity modules, determined by this technique, for the five woods studied, they were in good agreement with the order of magnitude reported for these modules, using different techniques \[5\text{-}10\].

**Table 2: flexural elasticity moduli of the woods**

<table>
<thead>
<tr>
<th>Material</th>
<th>Y (GPa)</th>
<th>Y(reference) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cariniana Domesticata Martius (Cachimbo)</td>
<td>11.53</td>
<td>9.29-12.84</td>
</tr>
<tr>
<td>Swietenia mahagoni (Caoba)</td>
<td>7.05</td>
<td>7.2-10.3</td>
</tr>
<tr>
<td>Carapa guianensis Aubl. (Cedro)</td>
<td>6.07</td>
<td>6.0-7.25</td>
</tr>
<tr>
<td>Pinus patula (Pino)</td>
<td>10.1</td>
<td>8.0-13.1</td>
</tr>
<tr>
<td>Cedrelinga cateniformis (Tornillo)</td>
<td>9.86</td>
<td>8.2-10.9</td>
</tr>
</tbody>
</table>

**Conclusions**

This paper has reported the application of fiber long-period gratings to vibration monitoring and flexural elasticity modules determination. To the best of our knowledge, this is the first article to deal with the determination of wood material’s flexural elasticity modules using long-period fiber gratings. Cachimbo, Mahagoni, Cedar, Pinus and Tornillo samples were tested. The values of the flexural elasticity moduli of the woods, determined by us, are in good agreement with the values reported in the literature using other techniques. This indicates that long-period fiber gratings can be straightforwardly employed in the dynamic characterization of a material’s elastic properties.

**Acknowledgments**

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**References**