Analysis of Rope-Sheave Contact Theories Using an Arbitrary Lagrangian-Eulerian Approach and a Bristle Contact Model

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Abstract

In this paper three different rope-sheave contact theories are compared with a new computational simulation method based on the Arbitrary Lagrangian-Eulerian approach [1]. There are two theories that provide simple close-form solutions to this contact problem, namely:

1. *The Creep Theory*, developed after the seminal works of Euler (capstan formula) and Eytelwin, later refined by Reynolds, Grashof and Swift [2].
2. *Firbank's Theory*, also called *Shear Theory*, developed by Firbank [3].

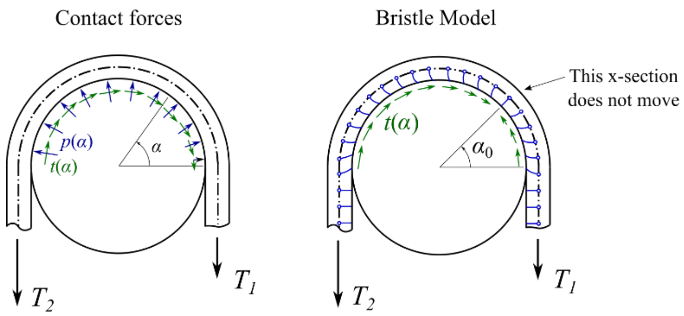

Figure 2. Normal and tangential rope-sheave contact forces

Creep Theory accounts for axial extension but does not account for shear deformation. Firbank's Theory accounts for shear theory but does not account for axial extension. There is a simple method that considers both axial extension and shear deformation. Besides, it accounts also for the flattening of the cross section of the ropes due to the normal contact forces. This method, that is based in the bristle contact method, has been recently developed by the author [4]. The *Bristle Method* provides a simple analytical solution of the rope-sheave contact in static analysis.

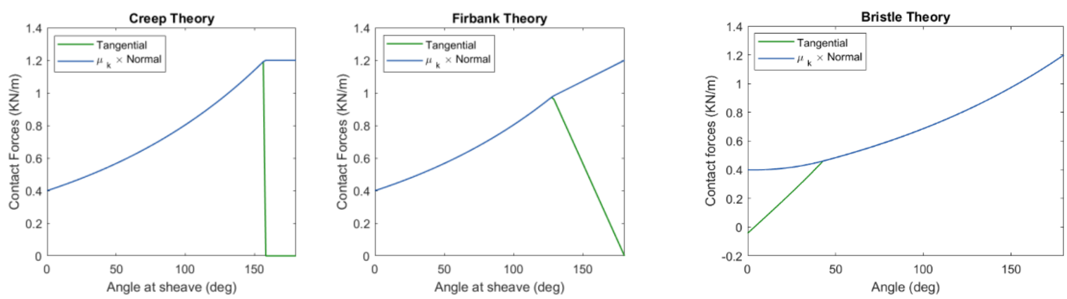

Figure 2. Normal and tangential rope-sheave contact force distributions

The comparison of the results of these three theories shows that:

1. The contact forces obtained with the Creep Theory and Firbank's Theory only depend on the tensions at the free spans of the rope (T_1 and T_2, see Fig. 3), the radius of the sheave, and the coefficient of friction. However, the contact forces obtained with the Bristle Theory also

depend on the axial, shear and flattening stiffness of the rope.

2. The Creep Theory and Firbank's Theory locate the slip arc at the low-tension end of the sheave. However, the Bristle Theory locates the slip arc at the high-tension end of the sheave.
3. In the adherence arc, the Creep Theory and Firbank's Theory result in constant or linear normal contact forces. The Bristle Theory results in an exponential evolution of the normal contact force in the adherence arc, being the exponential coefficient *r* a function of the different stiffness constants of the slender body.
4. In contrast to the results of the Creep Theory and Firbank's Theory, Bristle theory predicts tangential contact forces that can change sign in the adherence arc.

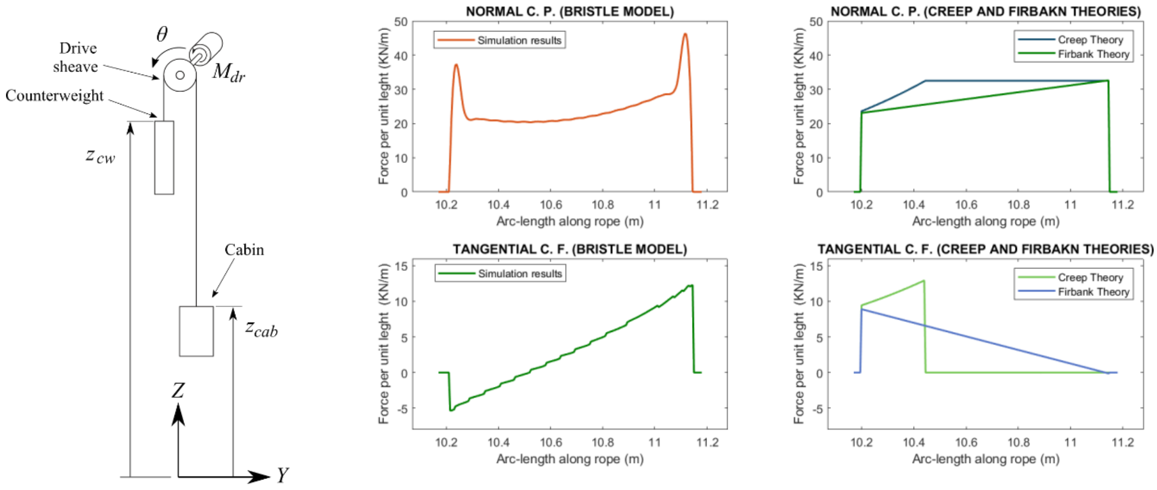

Figure 3. Comparison of simulation results with contact theories in elevator's ride

Figure 3 shows the contact forces during the steady-state period in the simulated ride of a simple elevator. Plots on the left are simulation results. Plots on the right are the result of the application of Creep Theory and Firbank's Theory. The main difference in the normal contact forces is due to the edge effects introduced by the bending stiffness of the rope. Tangential contact forces are totally different. It is important highlight that in Europe the safety calculations of elevators that are related to the rope-sheave contact are based on the Creep Theory.

References

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