Variation of Quantum Speed Limit with Correlated Channels

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Abstract

We study the effect of correlated channels on quantum speed of an open system. This is done with the help of some noise channels like amplitude damping, phase damping channel . Our model serves as a platform for a detail study of the effect of quantum speed limit. It has found that how the quantum speed limit varies by increasing the correlation between consecutive uses of channels.

Keywords

Quantum speed limit, noise channel, Markovian correlated channel.

Introduction

It is arguably impossible to isolate a particular system from surrounding subjected to information loss in the form of dissipation and decoherence. When a quantum system interacts with surroundings the system mixes with the environment. This is defined in terms of open quantum system [1]. There is always loss of information in the form of dissipation and decoherence [2]. There are various method that has been put forward in order to suppress decoherence. One of the method is by using memory [3,4], to retrieve information which was lost due to environment effect. The loss of information are of two types: Markovian and Non-Markovian. The concept of memory arises when the interaction of system with environment is non-Markovian. Many theoretical approaches [5-12] has been put forward to explain non-Markovian memory effect. The concept of memory as non-Markovianity is different from the concept of memory as correlated quantum channels [13,14].

In this paper, we focus on Markov noise to study the quantum speed limit evolution proposed by Macchiavello and Palma [13]. There is a bound to the speed of evolution which was derived from time-energy uncertainty relation for a system undergoing completely positive trace preserving map (CPTP). Two different kind of Markov noise has been taken into consideration one is amplitude damping channel and phase damping channel. We are making an attempt to connect the concept of correlated noise and quantum speed limit. It is well known that there are many applications of quantum speed limit, including quantum metrology [15], computational limits of physical systems [16], quantum optimal control algorithm [17].

We generalise master equation for two qubit atomic system [18], with modelling environment as a thermal radiation field. The dynamics of global system environment. There is a system-environment interaction and we are generalizing decay rate of non-Markovianity in terms of bound of speed of evolution.

In this work, we establish a relation between ratio of correlated speed versus the degree of Markovianity of the paper are as follows. In Sec. II, we introduce the out turn of correlated Markov Noise. In Sec III, we discuss the of effect quantum speed limit on noise channel. In Sec. IV, we introduce the dynamics of correlated channels on quantum speed limit. In Sec. V, we introduce the effect of master equation on quantum speed limit. In Sec. VI we give our conclusions.

II. Correlated Markov Noise

We begin with a brief discussion of correlated Markov noise channels subsequent use of many number of channels generates some correlation. Such kind of channel is called correlated Markov noise channel. Initially we have an input state followed by completely positive trace preserving map (CPTP). Let initial state is ρ followed by CPTP map given as

$$\boldsymbol{\epsilon}\,(\rho) = \sum E_i \rho\, E_i^{\dagger} \tag{1}$$

where E_i are Kraus operators of channels which satisfy CPTP map. Based on Kraus operator approach the state under noise is given by [19]

$$\boldsymbol{\epsilon}\,(\rho) = (1\text{-}\mu) \sum_{ij} E_{ij} \rho E_{ij}^{\dagger} + \mu \sum_k E_{kk} \rho E_{kk}^{\dagger} \tag{2}$$

In above expression, the probability is μ to remain correlated, and the probability is 1-μ for the operation to remain uncorrelated.

A well established model with Markovian noise has been taken into consideration. The time dependent Hamiltonian [20] of a qubit is given by H(t) = k$\boldsymbol{\Gamma}$(t)σ_z.

where $\Gamma(t)$ is an independent random variable. We are dealing with time dependent Karus operator to establish a model for Markov noise channel. The dynamics can be defined in terms of following Kraus operator.

$$K_1(\nu) = \sqrt{\frac{1+\varphi(\upsilon)}{2}} \quad \mathrm{I} \tag{3}$$

$$K_2(\nu) = \sqrt{\frac{1-\varphi(\upsilon)}{2}} \quad \sigma_z \tag{4}$$

Where we have $\varphi(\upsilon) = e^{-\nu}\left[\cos u\nu + \frac{\sin u\nu}{u}\right]$ and u = $\sqrt{(4\tau)^2 - 1}$ with $\boldsymbol{\upsilon} = \frac{t}{2\,\tau}$ being the time scale. Calculations for Kraus operator done for two qubit channel as well as done for correlated channels.

III. Effect quantum speed limit on noise channel

Evolution of closed system follows a unitary map. For a dynamical evolution there is a limiting case. The evolution of a quantum state dictates the speed of quantum computation. Quantum physics imposes limit on the speed of evolution of state: this is the quantum speed limit (QSL) [21]. The maximum evolution of a quantum system give rise to the limit of dynamical speed evolution [22,23]. There arises quantum speed limit when there is a finite exchange between system and environment. In this work we present quantum speed limit for noisy dynamics also. The minimum time evolution for closed quantum system is given as

$$\tau = \frac{\pi\hbar}{2\,\Delta E} \tag{5}$$

ΔE is the energy variance, this inequality is known as the Mandelstam-Tamm bound [25]. A bound can be derived for the map represented in terms of time-independent Kraus operator [24].

$$\tau_\theta \geq \frac{2\theta^2}{\pi^2} \frac{\sqrt{tr[\rho]^2}}{\sum_\alpha \left\| K_\alpha(t,0)_\rho\, K_\alpha^\dagger(t,0) \right\|} \tag{6}$$

The time evolution of quantum system $\rho_0$can be written as $\rho_t = \sum_\alpha K_{\alpha\,\rho}\, K_\alpha^\dagger$. Let the map is governed by evolution.

$$\dot{f}(t) = \frac{1}{tr\rho_0^2} \sum_\alpha tr\left[\rho_0 K_\alpha \rho_0 K_\alpha^\dagger\right]$$

On solving above equation by Cauchy- Schwarz inequality a bound can be derived. Parametizing f(t) = cosθ we have

$$\tau_\theta \geq \frac{2\theta^2}{\pi^2} \frac{\sqrt{tr[\rho]^2}}{\sum_\alpha \left\| K_\alpha \rho\, K_\alpha^\dagger \right\|} \tag{7}$$

We exactly compute plot QSL for the following cases:

IV. Dynamics of correlated channel on quantum speed limit

(a) Amplitude noises

Consider the dynamics of amplitude damping channel. Kraus operator for two qubit system are as follows [21].

$$A_1 = \begin{pmatrix} \frac{\sqrt{1+\varphi(t,\tau)}}{2} & 0 \\ 0 & \frac{\sqrt{1+\varphi(t,\tau)}}{2} \end{pmatrix}$$

$$A_{2=} \begin{pmatrix} \frac{\sqrt{1+\varphi(t,\tau)}}{2} & 0 \\ 0 & \frac{\sqrt{1+\varphi(t,\tau)}}{2} \end{pmatrix}$$

where $\varphi(\upsilon) = e^{-\nu}\left[\cos u\nu + \frac{\sin u\nu}{u}\right]$ and u = $\sqrt{(4\tau)^2 - 1}$ with $\boldsymbol{\upsilon} = \frac{t}{2\,\tau}$ being the time scale, τ refers to the degree of non-Markovianity [26]. In this paper we establish a link between ratio of speed limit of uncorrelated and correlated channel and the degree of non-Markovianity. Using equation (6) and doing straight forward calculation ratio of quantum speed limit decreases with τ. Fig 1 demonstrate the decay of speed of evolution for a two qubit amplitude Markov noise. The ratio of quantum speed limit gradually decreases with increase in τ.

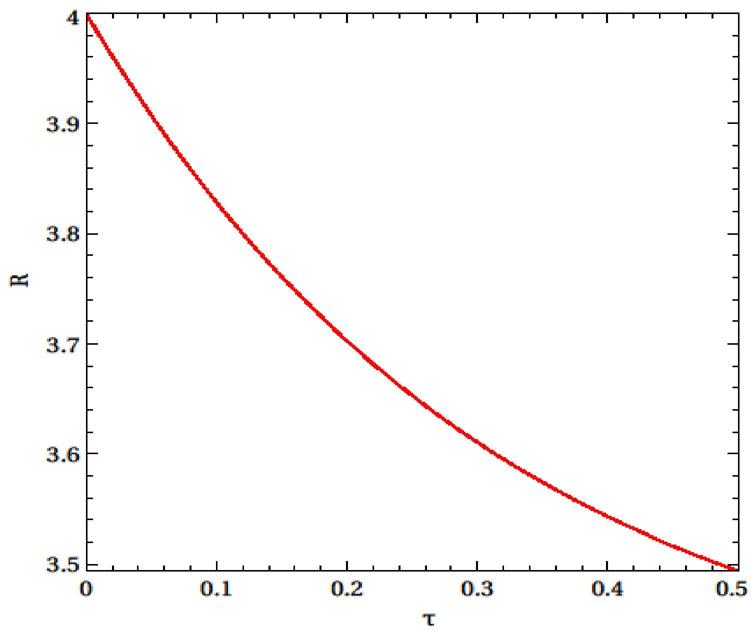

FIG 1 Ratio of uncorrelated-correlated speed decreases with increase in degree of Markovianity.

(b) Phase damping noises:

Phase damping noises describes a quantum noise with decay of off- diagonal element. The Kraus operator for a single qubit can be represented in terms of Pauli operators $\sigma_0 = I$ and σ_3. The Kraus operator for two-qubit system can be represented as [27]. From Fig 2. we see that the evolution of ratio of speed limit for a two-qubit system increases with increase in τ.

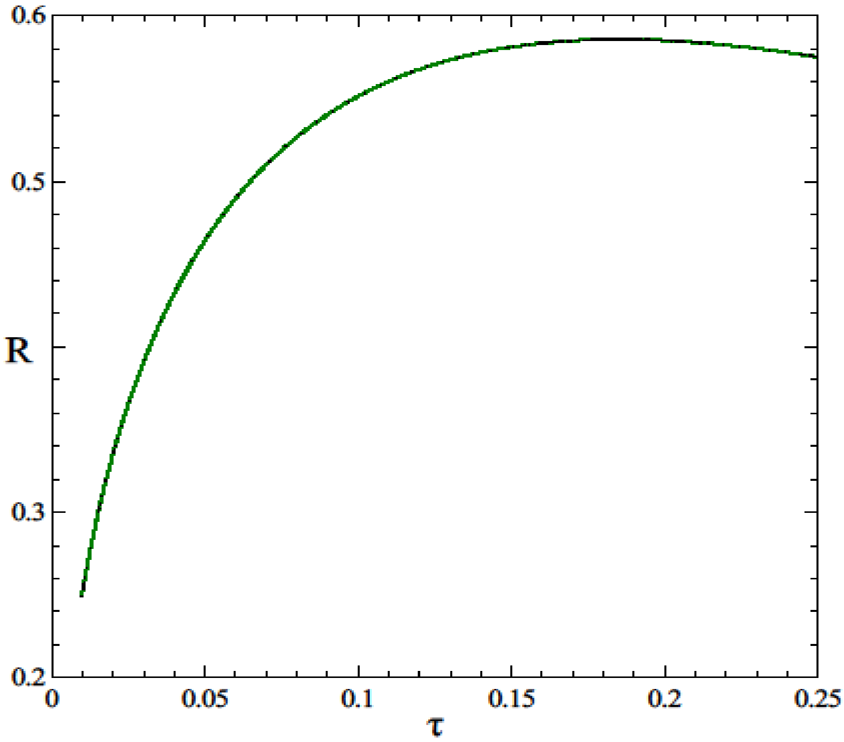

FIG 2. The speed of time evolution increases with increase in degree of non-Markovianity.

V. The effect of Master equation on quantum speed limit.

Let us consider the master equation constructed for a global system-bath interaction. Consider a given system with initial stste coupled to an environment. The global reversible dynamics is governed by unitary evolution and reduced dynamics of system is given by reduced dynamical map. One can assume Markovian dynamics when the time scale of environment is much smaller than that of system [28].

$$\frac{d\rho_t}{dt} = L\rho_t \tag{8}$$

In this section, we compute quantum speed limit (QSL) using master system for two-qubit atomic system [29].

$$\frac{d\rho}{dt} = \mathrm{L_{un}}(\rho) + \mathrm{L_{cor}}(\rho) \qquad (9)$$

Here L_{un} represents uncorrelated Lindbladian operator and L_{co} the correlated operator.

$$L_{un} = \sum_{i=1,2} \gamma_i(\mathrm{N}+1)\left(\sigma_i^{-}\rho\sigma_i^{+} - \frac{1}{2}(\sigma_i^{-}\sigma_i^{+}\rho + \rho\sigma_i^{+}\sigma_i^{-})\right)$$

and $L_{cor} = \sum_{i=1,2} \gamma_i \mathrm{N}\left(\sigma_i^{-}\rho\sigma_i^{+} - \frac{1}{2}(\sigma_i^{-}\sigma_i^{+}\rho + \rho\sigma_i^{-}\sigma_i^{+})\right)$

where N is Planck's distribution function.

$\sigma_1^{-} = \sigma^{-} \otimes I$, $\sigma_1^{+} = \sigma^{+} \otimes I$, $\sigma_2^{-} = I \otimes \sigma^{-}$, $\sigma_2^{+} = I \otimes \sigma^{+}$

The γ_i are called decay parameter. The bound for the speed of evolution is calculated for the correlated channel having generator in the form of Eq(9). Using Eq. (6), we determine ratio of uncorrelated-correlated bound on speed of evolution for this model. We establish a link between ratio of bound of evolution as a function of a. Here, a is a measure of the degree of non-Markovianity[29].The generalization of time-dependent Lidbladian for uncorrelated channel can be calculated in a straightforward way using Eq.6. Similar calculation can be done for the sum of correlated-uncorrelated noise. The coupling depends on the qubit position r_n, and the interaction Hamiltonian is proportional to $\sqrt{\gamma_{ij}}$. We studied this model in two cases:

Case 1: Consider the case when $\gamma_1 = \gamma_2 = \gamma_3 = \gamma$; we generalise Lindbladian form of master equation for uncorrelated channel.

Case 2: Consider the case when $\gamma_{12} = \gamma_{21} = \gamma a(k_0 r_{12})$ where γ_{ij} is the multi qubit interaction of composite system with bath. We determine the bound of evolution as upper and lower bound using triangle inequality.

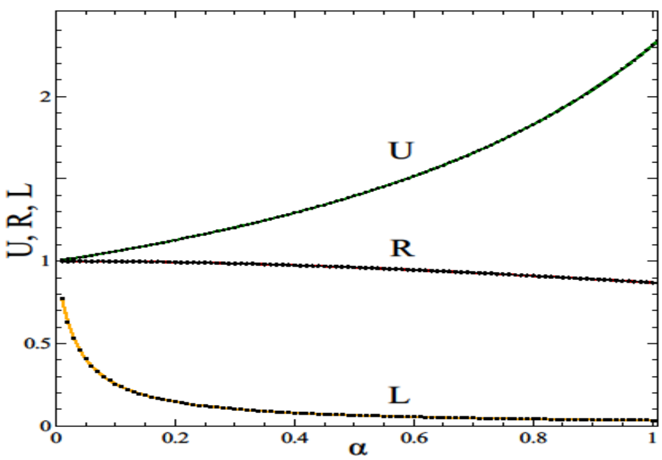

FIG 3 : Figure shows that, as the degree of non-Markovianity a is increased, the speed limit for correlated Markov noise increases for upper bound and for lower bound the speed limit decreases.

VI. Conclusions

In conclusion, we have proposed a scheme for detailed study of correlated channel under Markov noise. Different types of Markov noise channel have been taken into account, such as amplitude damping channel and phase damping channel. The effect of Markov noise on correlated channels has been discussed in detail. We summarize the results as follows. Firstly, the ratio of QSL for correlated/uncorrelated channels generated for amplitude damping channel and phase damping channel was calculated. The speed of evolution for correlated channel under Markov noise decreases for amplitude damping channel and increases for phase damping channel. Secondly, global system environment interaction is taken into consideration. We considered a two qubit atomic model and the master equation for a two qubit atomic system consists of Lindbladian operator for correlated and uncorrelated noise. We studied the detailed effect of Markov noise for this two qubit atomic model. We have further extended our case for the ratio of bound of evolution. We have shown that there are two bounds of evolution for this model. The speed of upper bound for the model increases whereas the speed of lower bound decreases.

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