Generalized Fuzzy Logic and Fuzzy Decision Set for Incomplete Information

Venkata Subba Reddy Poli
Abstract—The information available to the system is incomplete in many applications particularly in Decision Support Systems. The fuzzy logic deals with incomplete information with belief rather than likelihood (probability). Some times decision has to be taken with the fuzzy information. In this paper, fuzzy Decision set is defined with two fold fuzzy set. The fuzzy inference and reasoning are studied for fuzzy Decision sets. Business and Medical applications are given.

I. INTRODUCTION

The information available to many applications like Business, Medical, Geological, Control Systems etc. are incomplete or uncertain. The fuzzy logic will deal incomplete information with belief rather than likelihood (probable). Zadeh formulated uncertain information as fuzzy set with single membership function. The fuzzy set with two membership function will give more evidence than single membership function. The two fold fuzzy set is with fuzzy membership functions “Belief” and “false”. Usually, in Medical and Business applications, there are two opinions like “Belief” and “Disbelief” about the information and decision has to be taken under risk. For instance, in Mycin[1], The medical information is defined with belief and disbelief i.e. $CF[h,e]=MB[h,e] - MD[h,e]$, whe “e” is the evidence for given hypothesis “h”. The fuzzy set is used instead of Probability to define fuzzy certainty factor.

II. FUZZY LOGIC

Various theories are studied to deal with imprecise, inconsistent and inexact information and these theories deal with likelihood where as fuzzy logic will deals with belief. Zadeh[23] has introduced fuzzy set as a model to deal with uncertain information as single membership function. The fuzzy set is a class of objects with a continuum of grades of membership. The set $A$ of $X$ is characterized by its membership function $\mu_A(x)$ and ranging values in the unit interval $[0, 1]$. $\mu_A(x): X \rightarrow [0, 1]$, $x \in X$, where $X$ is Universe of discourse. $A = \mu_A(x1)/x1 + \mu_A(x2)/x2 + ... + \mu_A(xn)/xn$, “+” is union. For example, the fuzzy proposition “x is young” Young = \{0.95/10 + 0.9/20 + 0.8/30 + 0.6/40 + 0.4/50 + 0.3/60 + 0.2/70 + 0.15/80 + 0.1/90\} not young = \{0.05/10 + 0.1/20 + 0.2/30 + 0.4/40 + 0.6/50 + 0.8/60 + 0.7/70 + 0.95/80 + 0.9/90\}

For instance “Rama is young” and the fuzziness of “young” is 0.8 The Graphical representation of young and Not young is shown in fig.1
For example, consider the fuzzy proposition “x has mild Headache”.

The fuzzy logic is defined as combination of the fuzzy sets using logical operators[21]. Some of the logical operators are given below:

\[
\text{If } x \text{ is not } A \Rightarrow A' = \mu_A(x)/x \\
\text{Conjunction} \Rightarrow x \text{ and } y \text{ is } B \Rightarrow (x, y) \text{ is } A \times B \\
A \times B = \min(\mu_A(x)), \ \mu_B(y)\} \ (x, y) \\
\text{If } x = y \\
x \text{ is } A \text{ and } y \text{ is } B \Rightarrow (x, y) \text{ is } A \times B \\
\text{AAB = } \min(\mu_A(x)), \ \mu_B(y)\}\ (x, y) \text{ is } A \text{ or } B \Rightarrow (x, y) \text{ is } A' \times B' \\
A' \times B' = \max(\mu_A(x)), \ \mu_B(y)\} \ (x, y) \\
\text{Disjunction} \Rightarrow A \cup B = \mu_A(x) + \mu_B(y)\} /x \\
\text{Implication} \Rightarrow A \Rightarrow B = \mu_A(x) \Rightarrow \mu_B(y)\} \ (x, y) \\
\text{Composition} \Rightarrow A \circ B = \mu_A(x), \ \mu_B(y)\} / (x, y) \\
\text{Concentration} \Rightarrow x \text{ is very } A \\
\mu_{\text{very } A}(x) = \mu_A(x)^2 \\
\text{Difussion} \Rightarrow x \text{ is very } A \\
\mu_{\text{more or less } A}(x) = \mu_A(x)^{0.5} \\
\text{III. GENERALIZED FUZZY LOGIC WITH TWO FOLD FUZZY SET}
\]

Suppose A, B and C are fuzzy sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Since formation of the generalized fuzzy set simply as two fold fuzzy set, Zadeh fuzzy logic is extended to these generalized fuzzy sets.

\[\text{Negation}\Rightarrow A' = \{1 - \mu_A(x), 1 - \mu_A(x)\} /x \]
\[\text{Disjunction}\Rightarrow A \cup B = \{\mu_A(x), \mu_B(y)\} \ (x, y),\ \max(\mu_A(x), \mu_B(y)\} \ (x, y)\} / (x, y) \]
\[\text{Conjunction}\Rightarrow A \cap B = \{\min(\mu_A(x), \mu_B(y)\} \ (x, y),\ \min(\mu_B(x), \mu_B(y)\} \ (x, y)\} / (x, y) \]
\[\text{Implication}\Rightarrow A \Rightarrow B = \{\min(1, 1 - \mu_A(x), 1 - \mu_B(y)\} \ (x, y)\} / (x, y) \]
\[\text{Composition}\Rightarrow A \circ B = \{\mu_A(x), \mu_B(y)\} \ (x, y) \\
\text{The fuzzy propositions may contain quantifiers like “very”, “more or less”. These fuzzy quantifiers may be eliminated as}
\]
\[\text{Concentration}\Rightarrow x \text{ is very } A \\
\mu_{\text{very } A}(x) = \mu_A(x)^2 \\
\text{Difussion}\Rightarrow x \text{ is very } A \\
\mu_{\text{more or less } A}(x) = \mu_A(x)^{0.5} \\
\text{For instance, let A, B and C are} \\
A = \{0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 + 0.6/x_5, 0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0.5/x_4 + 0.6/x_5, 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.8/x_5, 0.9/x_1 + 0.6/x_2 + 0.8/x_3 + 0.7/x_4 + 0.8/x_5, 0.8/x_1 + 0.7/x_2 + 0.6/x_3 + 0.7/x_4 + 0.8/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\} \\
A \cup B = \{0.8/x_1 + 0.7/x_2 + 0.8/x_3 + 0.6/x_4 + 0.6/x_5, 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.8/x_5, 0.9/x_1 + 0.6/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.4/x_4 + 0.5/x_5\} \\
A \cap B = \{0.8/x_1 + 0.7/x_2 + 0.6/x_3 + 0.6/x_4 + 0.7/x_5, 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.8/x_5\} \\
A' = \text{not } A = \{0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 + 0.5/x_5, 0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 + 0.4/x_5\} \\
A \Rightarrow B = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\} \\
\mu_{\text{more or less } A}(x) = \{0.5, 0.6\} \\
\mu_{\text{more or less } A}(x) = \{0.5, 0.6\} \\
\text{For instance, let A, B and C are} \\
A = \{0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\} \\
A \cup B = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\} \\
A \cap B = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\} \\
A' = \text{not } A = \{0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 + 0.5/x_5, 0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 + 0.4/x_5\} \\
A \Rightarrow B = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\} \\
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\]
IV. FUZZY DECISION SET

Zadeh[22] proposed fuzzy set to deal with incomplete information. Generalized fuzzy set with two fold membership function $\mu_A(x) = \{ \mu_A^{\text{Belief}}(x), \mu_A^{\text{Disbelief}}(x) \}$ is studied [18].

The fuzzy Certainty Factor may be defined as (FCF)

$$\mu_A^{\text{FCF}}(x) = \mu_A^{\text{Belief}}(x) - \mu_A^{\text{Disbelief}}(x),$$

where

$$\mu_A^{\text{FCF}}(x) = \mu_A^{\text{Belief}}(x) - \mu_A^{\text{Disbelief}}(x)$$

fuzzy Decision set $R$ is defined based on convex fuzzy set [10]

$$R = A \mu_A^{\text{FCF}}(x) \geq \alpha, \text{ where } \alpha \in [0,1]$$

For instance,

Demand = $\{ 0.8/x_1+0.7/x_2+0.86/x_3+0.75/x_4+0.88/x_5, 0.2/x_1+0.3/x_2+0.25/x_3+0.3/x_4+0.2/x_5 \}$

$$\mu_{\text{Demand}}^{\text{FCF}}(x) = 0.6/x_1+0.4/x_2+0.61/x_3+0.45/x_4+0.68/x_5$$

The Generalized fuzzy set for Demand for the Items and fuzzy certainty factor is ahown in Fig2.

![Generalized Fuzzy Set](image)

Fig2: Generalized fuzzy set

Suppose fuzzy Decision set is defined

$$\mu_{\text{Demand}}^{\text{FCF}}(x) \geq 0.5$$

$$= 1/x_1+0/x_2+1/x_3+0/x_4+1/x_5$$

![Fuzzy Decision Set](image)

Fig3: fuzzy Decision set

The fuzzy logic is combination of logical operators. Consider the logical operations on fuzzy Decision sets $r_1, r_2$ and $r_3$

**Negation**

If $x$ is not $R_1$
R1’=1-µR1(x)/x

Conjunction
x is R1 and y is R2↔ (x, y) is R1 x R2
R1 x R2= min(µR1(x), µR2(y))(x, y)
If x=y
x is R1 and y is R2↔ (x, y) is R1& R2
R1& R2= min(µR1(x), µR2(y))(x, y) x is R1 or y is R2↔ (x, y) is R1’ x R2
R1’ x R2’= max(µR1(x), µR2(y))(x, y)
If x=y
x is R1 and x is R2↔ (x, x) is R1 V R2
R1 V R2= max(µR1(x), µR2(y))/x

Implication
If x is R1 then y is R2 = µR1(x)+µR2(y)/x
If x=y
R1→ R2= min{ 1, 1-µR1(x)} +µR2(y)/x
If x is R1 then y is R2 else y is R3= R1 x R2 + R1’ x R3
The fuzzy proposition “If x is R1 then y is R2 else y is R3” may be divided into two clause “If x is R1 then y is R2 “ and “If x is not R1 then y is R3” [15]
If x is R1 then y is R2 else y is R3 = R1→ R2= min{ 1, 1-µR1(x)+µR2(y))/x, y)
If x is not R1 then y is R2 else y is R3 = R1’→ R3 = min { 1, 1-µR1(x)+µR2(y))/x, y)

Composition
R1 o R2= R1 x R2= min{ µR1(x), µR2(y)(x, y)
If x=y
R1 o R2= min{ µR1(x), µR2(y))/x
R1 o R2= min{ µR1(x), µR2(y))/x
The fuzzy propositions may contain quantifiers like “Very”, “More or Less”. These fuzzy quantifiers may be eliminated as

Concentration
x is very R1
µvery R1(x), =µR1(x) ^ 2

Difusion
x is very R1
µmore or less R1(x) =µR1(x) .5

VI. FUZZY INFERENCE IN DECISION MAKING
Decision management is usually happens in Decision Support Systems.
EXAMPLE 1
Consider Business rule
If x is Demand of the product then x is High Price
Let x1, x2, x3, x4, x5 be the Items.
The Generalized fuzzy set
Demand ={ 0.56/x1+0.48/x2+0.86/x3+0.36/x4+0.88/x5, 0.06/x1+0.04/x2+0.07/x3+0.03/x4+0.2/x5 }  
µDemand (x) =
0.5/x1+0.44/x2+0.79/x3+0.33/x4+0.68/x5
High Price = 0.49/x1+0.52/x2+0.35/x3+0.4/x4+0.3/x5

EXAMPLE 2
Consider Medical Diagnosis
If x has infection in the leg then surgery
Let x1, x2, x3, x4, x5 are the Patients.
The fuzzy set
µInfection (x) = 0.76/x1+0.78/x2+0.46/x3+0.86/x4+0.58/x5, 0.16/x1+0.12/x2+0.06/x3+0.14/x4+0.05/x5
µSurgery (x ) = 0.5/x1+0.26/x2+0.55/x3+0.24/x4+0.35/x5, 0.09/x1+0.06/x2+0.05/x3+0.04/x4+0.03/x5
µInfection & Surgery (x) = 0.5/x1+0.2/x2+0.5/x3+0.2/x4+0.32/x5
Using inference rule A→B= min{ 1, 1-µA(x) + µB(x)}
µInfection & Surgery (x) = 0.9/x1+0.56/x2+0.9/x3+1/x4+1/x5
µInfection & Surgery (x) ≥6
µInfection & Surgery (x) <6
The fuzzy Decision set is

VI. CONCLUSION
The decision has to be taken under incomplete information in many applications like Business, Medicine etc. The fuzzy logic is used to deal with incomplete information. The fuzzy Decision set is defined with two fold fuzzy set. The fuzzy logic is discussed with two fold fuzzy set. The fuzzy Decision set, inference and reasoning are studied. The Business and Medical applications are discussed for fuzzy Decision set.
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