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# CALORIMETRIC METHOD FOR MEASUREMENT OF ELECTRICALLY SMALL ANTENNAS EFFICIENCY

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*Abstract* — To solve the problem of measuring the electrically small antennas efficiency, a calorimetric measurement method is considered. The method is based on the representation of the antenna as a physical system in which electromagnetic radiation is useful, and heat losses are detrimental. Measuring the heat loss of the antenna and its non-radiating equivalent allows you to estimate the efficiency. The description of the developed calorimetric installation for measuring the efficiency is given. Physical and technical features of its work are considered. For testing, a sample of a small-sized spiral antenna was selected, operating at a frequency of 1.9 MHz and matched with a 50 Ohm feedline with VSWR <1.1; the efficiency measurement was carried out, which showed the high efficiency of the antenna being measured.

## **Introduction**

Direct measurements of the parameters of electrically small antennas operating at frequencies from several tens to thousands of kHz are associated with a large expenditure of effort, money and time. However, it is in these ranges that the problem of creating effective antenna devices is most acute.

When designing antennas, developers need a tool that allows them to compare the performance of antennas of various designs with a minimum cost of resources and time. In the VHF and UHF bands, such measurements can be made fairly quickly on a line-of-sight radio path using a gain reference antenna. In the ranges from several tens to thousands of kHz, it is required to develop antennas with minimal dimensions, much less than the wavelength, and their measurement of their parameters is a much more laborious process. Often, due to the complexity of measurements, authors use the effective height (for HF and MF stationary

antennas) or the magnetic moment (for loop antennas) to compare the performance of antennas. In this paper, we are talking about portable and mobile antennas, the use of which is characterized by the inconstancy of the environment. The most informative parameter for comparing their productivity is the efficiency ( $K_{eff}$ ).

Several methods of measuring efficiency are known.

The Q-method [1] is based on the standard definition of the quality factor ( $Q$ ) from the standpoint of the theory of oscillations: the quality factor  $Q_A$  is the ratio of the stored energy in the system (antenna) to the energy dissipated in one period, ie:

$$Q_A = \frac{\omega \cdot W}{P_A} = \frac{\omega \cdot W}{P_\Sigma + P_L}, \quad (1)$$

where  $W$  is the stored energy in the antenna,  $P_A$  is the supply power,  $P_\Sigma$  is the radiation power,  $P_L$  is the loss power.

It is assumed that it is possible to calculate the "ideal quality factor" (radiation quality factor) of the antenna, i.e. lossless antenna. In this case

$$Q_A^{ideal} = \frac{\omega \cdot W}{P_A} = \frac{\omega \cdot W}{P_\Sigma}. \quad (2)$$

Whence it follows that the efficiency can be found from the ratio:

$$K_{eff} = \frac{Q_A}{Q_A^{ideal}} = \frac{P_\Sigma}{P_\Sigma + P_L}. \quad (3)$$

The ideal figure of merit is calculated, and the real one (if it is much greater than unity) is determined from the bandwidth  $\Delta f$  at the points where the active resistance is equal to the reactance, or from the half reflected power (reflection coefficient  $|p_A(\omega)| = 0,5$ ). The first disadvantage of the method is the complexity of calculating the "ideal Q-factor", which requires rather high computational costs for antennas of complex design, with tuning elements, litz wires, dielectric materials, etc., and in an environment that is very different from free space. The second significant drawback should be considered the lack of a clear method for measuring the  $Q_A$ , especially for antennas with a small Q-factor. Another way to measure the Q-factor is to determine it through the derivative of the antenna input impedance, below are some of the proposed formulas [2-4]:

$$Q = \frac{\omega |X'_A(\omega)|}{2R_A(\omega)} \Big|_{\omega=\omega_0} \quad or \quad Q = \frac{\omega |Z'_A(\omega)|}{2R_A(\omega)} \Big|_{\omega=\omega_0}$$

The considered approaches give different results and the question of measuring  $Q_A$  remains open (see [2-7]).

The next method is the Wheeler-cap method [1, 8], in which it is recommended to place the antenna in the center of a metal chamber with dimensions of the order of  $\lambda/3$ . It is based on the assumption that in such a situation the radiation resistance will be bridged. By measuring the input impedance of the antenna inside and outside the chamber, and determining the difference, the efficiency can be measured. It should be noted that the method does not take into account the change in the current distribution along the antenna due to the presence of the Wheeler-cap, is sensitive to the choice of its geometric dimensions and can lead to a significant error when measuring high-Q antennas in the resonance point [9-11].

Methods based on measuring the transmission coefficients  $S_{12}$  between an antenna and a reference placed at opposite ends of a waveguide or reverberation chamber [12-14], which can support only a limited number of radiation modes. Changing the internal space in the chamber or waveguide leads to a change in the transmission coefficient  $S_{12}$ . From the results of measuring the coefficients  $S_{12}$ ,  $S_{11}$  и  $S_{22}$ , the efficiency can be determined.

The last two measurement methods are resource consuming for the frequency range considered in this work.

This paper presents a method called calorimetric, based on measuring the temperature of the antenna released during its operation and proportional to the loss energy. The aim of the work is to analyze the calorimetric method and study the method of its technical implementation.

This paper presents a method called calorimetric, based on measuring the temperature of the antenna, which corresponds to the amount of heat energy released during its operation and is proportional to the loss energy. The *purpose of this work* is to analyze the calorimetric method and study the method of its technical implementation.

## 1. Justification of the calorimetric method for measuring efficiency

According to the approach used in this article, electrically small antenna is represented by an open linear oscillatory system. The work of physical forces in the system leads to the conversion of AC radio frequency energy into heat loss energy and radiation energy, and the system can be viewed from the position of calorimetry [15, 16]. The active power dissipated in an antenna with a volume  $V_A$  uniformly heated to temperature  $T$  is determined by the thermodynamic equation of state of the system

$$P_A = H_A(T_A - T_0) + P_\Sigma + c_A V_A \frac{dT}{dt}, \quad (4)$$

where  $H_A$  is the heat transfer coefficient due to convection, thermal conductivity, thermal radiation,  $c_A$  and  $V_A$  are the specific heat capacity and equivalent volume of

the antenna,  $T_A$  and  $T_0$  are the antenna and ambient temperatures, respectively. The first term on the right-hand side of the equation determines the loss due to heat transfer, the second term determines the part of the supplied power converted into radio frequency radiation, the third term determines the temperature increment under the influence of the supplied power dissipated in it (including in matching elements).

Correct matching of the antenna with the output circuits of the transceiver and the correct tuning of the antenna to the forced resonance mode allow providing a traveling wave mode with  $VSWR < 1.2$ . In this case, the antenna can be considered matched and the input power  $P_{IN}$  (in transmission mode) equal to the dissipated power according to (4):  $P_{IN} = P_A$ . If  $T_A$  and  $T_0$ ,  $H_A$ ,  $c_A$  and  $V_A$ , and the input power  $P_{IN}$  are known, then  $P_\Sigma$  can be calculated using (4). At first glance, the last term in (4) presents difficulties. But, for the case of a stationary state of the system, i.e. in the steady-state mode of receiving and converting energy into radiation and heat at a measurement time  $t \rightarrow \infty$ , the equation of state will take the form:

$$P_A = H_A(T_A - T_0) + P_\Sigma. \quad (5)$$

Further, suppose that the antenna equivalent can be selected in such a way that  $H_A = H_E$ ,  $V_A = V_E$ , while the equivalent radiation power is  $P_{\Sigma_E} \rightarrow 0$ , i.e. it is a structurally similar (in terms of weight, dimensions and materials) resistive load for the transceiver circuits at the selected frequency. Provided that the initial temperatures  $T_0$  are equal, the equation of state for the antenna equivalent is:

$$P_E = H_E(T_E - T_0) = H_A(T_A + \Delta T - T_0),$$

where  $\Delta T = T_E - T_A$ .

With equal supplied power  $P_{IN} = P_E = P_A$ , provided that the reflectivity from the antenna and the equivalent  $|p_A(\omega)| = |p_E(\omega)|$ , we get

$$H_A(T_A - T_0) + P_\Sigma = H_A(T_A + \Delta T - T_0),$$

from where

$$P_\Sigma = H_A \Delta T. \quad (3)$$

The antenna efficiency, taking into account (3), (5) - (6), takes the form:

$$K_{eff} = \frac{P_\Sigma}{P_A} = \frac{H_A \Delta T}{H_A \Delta T + H_A(T_A - T_0)} = \frac{T_E - T_A}{T_E - T_0}. \quad (4)$$

Thus, measuring the temperature  $T_E$ ,  $T_A$ ,  $T_0$  makes it possible to determine the antenna efficiency.

## 2. Description of the measurement method

Figure 1 shows a block diagram of a calorimetric apparatus that provides a stationary state of an antenna placed in it, on which: G - generator, C - thermal insulation chamber, A - measured antenna (or antenna equivalent), V - ventilator, TS - temperature sensor, HS - sensor humidity, DAB - data acquisition board, PC - personal computer. The antenna (antenna equivalent) is placed in a chamber (C), in which a ventilator (V) provides constant air convection. Air intake and its exit from the chamber are made through technological holes of a given diameter, which are spaced apart, while the temperature and humidity sensors of the internal air are removed at a certain distance from the antenna to eliminate spurious signal that interferes with their work. The chamber is made of expanded polystyrene with a thermal conductivity coefficient of about  $0.03 \text{ W}/(\text{m}\cdot\text{K})$  and a small dielectric loss tangent of the order of  $tg\delta \approx 10^{-4}$ . This choice of material ensures minimal heat transfer between the interior of the chamber and the environment outside the specified process openings [15, 16].

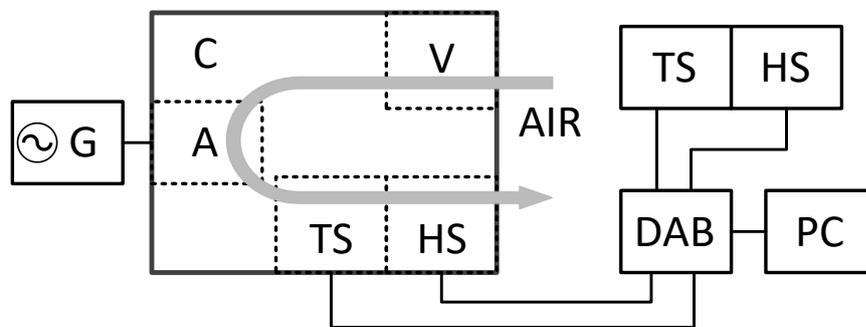


Figure 1 - Block diagram of a calorimetric apparatus for measuring antenna efficiency

An antenna according to a patent for an invention [17], tuned to resonance at a frequency of 1.9 MHz, was selected for measurements. The choice of this frequency is due to the prospects of its use in MF communication systems. The antenna is made on a polyvinyl chloride (PVC) dielectric frame. Figure 2 shows the frequency dependences of the input impedance components:  $R_A(f)$ ,  $X_A(f)$ ,  $Z_A(f)$ . The antenna is matched at a resonance frequency with a 50 Ohm feeder with  $VSWR < 1.1$ .

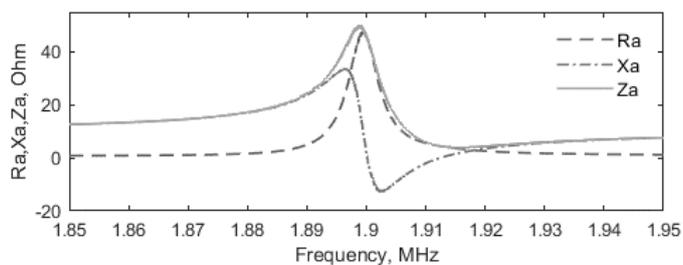


Figure 2 – Dependency plots  $R_A(f)$ ,  $X_A(f)$ ,  $Z_A(f)$

Antenna installed inside the camera is supplied with a power of  $P_{IN} = 10$  W. During the measurement, the ambient temperature  $T_{ext}$  and the air temperature in the  $T_{cam}$  are measured. As a result of the conversion of energy into heat, the air in the chamber is heated. Gradually, the temperature in the chamber increases to a certain level, after which  $T_{cam}$  remains relatively stable, which indicates that the system under study reaches a stationary state. Graphs explaining the processes under consideration are shown in Figure 3-a. The figure shows a smooth quasiperiodic character of temperature change  $T_{ext}$ , while the cooling system in the considered experiment smoothly lowers the air from an average temperature of  $T_{ext} \approx 21$  °C to  $T_{ext} = 20.5$  °C. This nature is associated with the peculiarity of the operation of the air cooling system in the laboratory in which the measurements were carried out. The dependence  $T_{cam}(t)$  has a similar character of change; in this case, a long, steep segment of increasing temperature is seen. In the region of  $t \approx 5000 \div 6000$  s, it is visually visible that a gentle segment  $T_{cam}(t)$  begins, which should be interpreted as the beginning of a stationary state. The average measured temperature in the chamber at this time interval allows you to determine the temperature of the antenna (or its equivalent) placed in it:  $\langle T_{cam} \rangle = T_A$ .

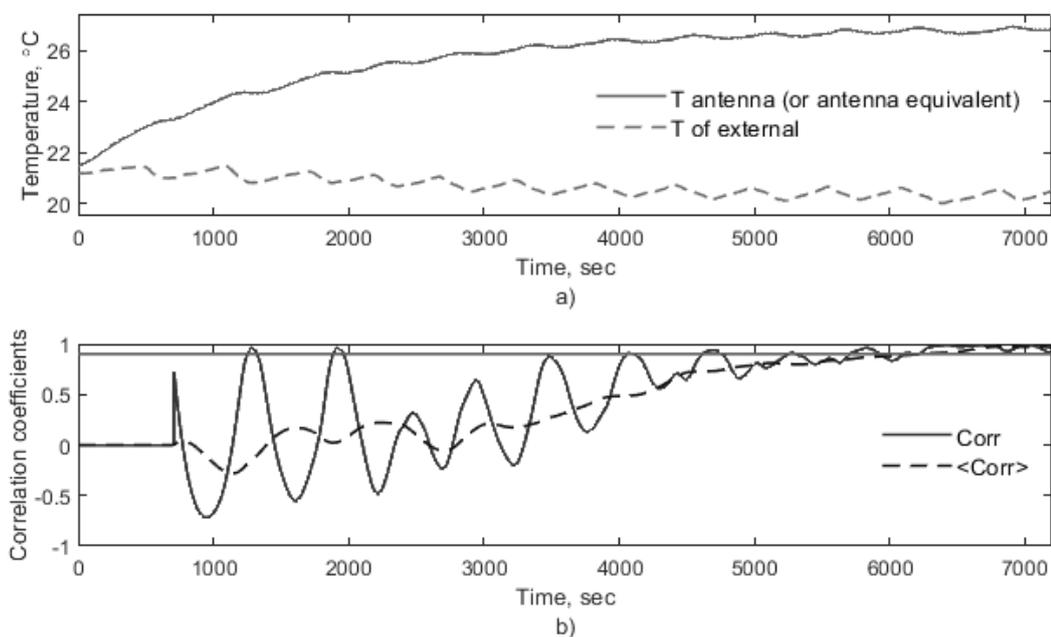


Figure 3 - Graphs of time dependences of temperature (a) and correlation coefficients (b)

Despite the obviousness and clarity of the conclusions above, below are some quantitative indicators that allow, among other things, to make automated measurements. Since  $T_{cam}(t)$  and  $T_{ext}(t)$  are correlated, the stationary state section corresponds to a linear correlation; there is no linear correlation in the section where the temperature  $T_{cam}(t)$  increases. Due to the physical features of the device of the calorimetric installation, there is a time delay  $T_d$  of the temperature change

$T_{cam}(t)$  from  $T_{ext}(t)$ . In our case,  $T_d \approx 140$  s.  $T_{ext}(t)$  oscillates around the average value of  $T_{ext} = 20.5$  °C with an average frequency of  $\tau \approx 600$  s. Taking into account these amendments, we write down the linear correlation coefficient of the processes  $T_{cam}(t)$  and  $T_{ext}(t)$  with a sample of reports for the period  $\tau$  [18-19]:

$$Corr = \frac{\sum_{t-\tau}^t ((T_{ext}(n) - \overline{T_{ext}})(T_{cam}(n - T_d) - \overline{T_{cam}}))}{\sqrt{\sum_{t-\tau}^t (T_{ext}(n) - \overline{T_{ext}})^2 \sum_{t-\tau-T_d}^{t-T_d} (T_{cam}(m - T_d) - \overline{T_{cam}})^2}}, \quad (5)$$

where the mathematical expectations of samples are defined as

$$\overline{T_{ext}} = \frac{1}{\tau} \sum_{n=t-\tau}^t T_{ext}(n),$$

$$\overline{T_{cam}} = \frac{1}{\tau} \sum_{n=t-\tau-T_d}^{t-T_d} T_{cam}(n).$$

The measurements were taken every second, so the sample number corresponds to the measurement time in seconds. The calculation of the correlation coefficients starts after a time  $(\tau + T_d)$  after the start of the measurement process. Figure 2-b shows graphs of the coefficient  $Corr$  according to (5) and its averaged value  $\langle Corr \rangle$  over time  $\tau$ . The horizontal line denotes the threshold level  $Corr = 0.9$  (corresponding to a very strong correlation), chosen as the boundary (beginning) of the linear correlation, and, accordingly, the stationary state of the system under study. The calculation of the average value  $\langle Corr \rangle$  makes sense when working in an automated mode: in some areas, the correlation coefficients can exceed the threshold level (in our case,  $Corr = 0.9$ ), which can lead to the premature end of the experiment.

### 3. Measurement of antenna efficiency

Antenna efficiency is measured by a series of experiments. Humidity control is required as it depends on the thermal conductivity of air. In all experiments of one series, the humidity should be constant. Before carrying out a series of experiments, it is necessary to calibrate the setup in idle operation with a running fan, since during operation, it heats the air and can introduce an error in the measurement results. The graphs obtained as a result of a series of measurements (similar to Figure 3-a) can be normalized relative to the ambient temperature and combined into one.

The antenna equivalent was made on a dielectric frame made of nichrome filament of the required length with a resistance of 50 Ohm. Figure 4 shows graphs of temperature dependences normalized to the average ambient temperature,  $T_{AN}(t)$ ,  $T_{EN}(t)$ ,  $T_{RN}(t)$ , where  $T_{RN}(t)$  is the temperature of the resistors taken to replace the antenna equivalent. This is due to the fact that the production of an equivalent for

each measured antenna is time- and resource-intensive. A resistive load that dissipates the input power can replace the antenna equivalent. In our case, a cascade of six successive links is selected, each link contains four 50 Ohm resistors. In the case of normalized temperatures, the efficiency is determined by the ratio

$$K_{eff} = \frac{T_{EN} - T_{AN}}{T_{EN}}. \quad (6)$$

The graphs show that  $T_{EN}(t)$  and  $T_{RN}(t)$  reach the same temperature level, but at different speeds:  $T_{EN}(t)$  has a lower slope, which is a consequence of its physical construct, which is close to the antenna being measured. The difference in the speed of reaching the equilibrium state is determined by the lower heat capacity of the resistors.

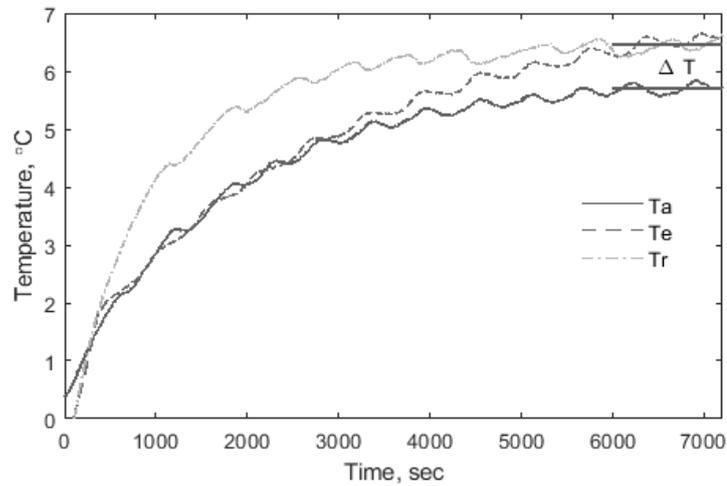


Figure 4 - Temperature dependence  $T_{AN}(t)$ ,  $T_{EN}(t)$ ,  $T_{RN}(t)$

Table 1 summarizes the resulting characteristics of the antenna under test.

Resonance frequency $f_p$ , MHz	1.9
VSWR	1.1
Relative bandwidth according to $VSWR = 2: f_p/\Delta f_{2VSWR}$	~ 250
Overall dimension in wavelengths, $\lambda$	~ 0,002
Efficiency, %	11,6

#### 4. Discussion of the features of the method.

A. As it was noted, before carrying out the tests, it is necessary to calibrate the installation in idle operation with a working fan; in our case, the heating of the internal space by the fan during the measurement reached 0.4 ° C. If this factor is not taken into account, then, taking into account the existing temperature

difference  $T_{EN}(t) - T_{AN}(t) = 0.75$  ° C, the error in measuring the efficiency will be significant.

B. For antennas operating at wavelengths of hundreds of meters, the free space condition is not met under any real conditions, and the real conditions of the Fresnel zone, depending on the installation site, can differ significantly. Therefore,  $P_{\Sigma}$  means the radiation power of the entire electromagnetic field of the antenna, and the loss power means only heat losses in the antenna itself.

C. Taking into account the range of air temperature variation (less than 10 degrees), the measurement error of the antenna efficiency can be less than 1% even without taking into account the change in the heat capacity of the air during the measurement, the main factor determining the measurement error is the accuracy of the temperature sensors.

D. To compare the dependences in Figure 3-a, linear correlation coefficients were used. The authors do not set the task of performing a deep statistical analysis of the obtained temperature dependences in this work. It only shows the availability of a convenient working tool for empirical research.

## **Conclusion**

The considered method provides researchers and developers with an affordable and effective tool for sufficiently accurate measurement of the efficiency of small antennas, the dimensions of which are one to two orders of magnitude smaller than the wavelength. This method allows you to measure the efficiency of low-frequency antennas, the measurement of the efficiency of which by other methods is extremely resource-intensive or completely impossible. The method allows you to measure high-resonance antennas. The antenna considered in the work at a frequency of 1.9 MHz has dimensions of  $\sim 0.002 \cdot \lambda$  and a measured efficiency of 11.6%, which is a very good result for an antenna of such small dimensions.

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