Solitons In Laser-Plasma Interaction: The role of Vasyliunas-Shamel Electrons distribution

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Abstract

In the present paper, we utilize both the Vasyliunas and Shamel models for electrons to explore the formation of compressive and rarefactive solitary waves in plasma laser-created. The distribution called Vasyliunas-Shamel distribution allows observation of the effects of the usual enhanced non-Maxwellian tail with an excess of superthermal particles (Vasyliunas distribution) while including the trapped particles in the low energy part of the distribution (Shamel distribution). We examined the behavior of the K-dV-like equation by using the reductive perturbation technic. A solitary wave solution was presented and its dynamics discussed. It is found that increasing superthermality affects the soliton shape while particle trapping leads to a stronger nonlinearity.

Keywords

Vasyliunas-Shamel distribution— K-dV-like equation —magnetized plasmas.

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1. Introduction

Due to various electron acceleration mechanisms, laser created plasmas are often characterized by the presence of energetic particles in the background [1]. This phenomenon is associated with a power-law dependence at superthermal velocity values, modeled by a kappa-type distribution function, which is more realistic than the standard Maxwellian distribution approach [2]-[5]. Besides, the distribution of superthermal particles is characterized by the spectral index κ, so called kappa-distribution function. The spectral index is a measure of energy spectrum slope of suprathermal particles forming the tail of velocity distribution function. Its smaller value indicates more suprathermal particles in the tail of distribution function, i.e., in the harder side (higher side) of energy spectrum. Kappa distribution approaches the Maxwellian as $\kappa \to \infty$ [6].The general form of the kappa distribution was first suggested by [7] to model space plasma. Rightly, this was sustained by observations made by The Voyager PLS [8]-[9] and the Cassini CAPS (Cassini Plasma Spectrometer) [10].

Very recently, these investigations have attracted the attention of many researchers to study the nonlinear wave structures in plasmas with superthermal tails [11]-[16] (and references therein) and even to investigate the characteristics of solitary structures in different plasma systems.

In this paper, we are going to utilize Vasyliunas and Shamel models for electrons to explore the formation of both compressive and rarefactive solitary waves. The effect of those electrons on the oblique propagation of ion-acoustic wave's propagation in the presence of a uniform external magnetic field is investigated Vasyliuas.

2. Basic equation

In 1968, Vasyliunas introduced an empirical function motivated by the fact that there is many plasma systems where plasma species are not in thermal equilibrium. Vasyliunas examined energy spectra of electrons with the plasma sheet and modeled the velocity distribution of high energy electrons by a non-Maxwelian distribution known as kappa or Vasyliunas distribution given by [2] and [4]

$$f_\kappa(v) = \frac{n_{e0}}{\pi v_\theta^2 \kappa^{3/2}} \frac{\Gamma(\kappa)}{\Gamma(\kappa-1/2)} \left(1 + \frac{v^2}{\kappa v_\theta^2}\right)^{-(\kappa+1)}, \quad \textbf{(1)}$$

Where v_θ is the most probable is speed (effective thermal speed), related to the usual thermal velocity $V_t = (k_B T/m)^{1/2}$ by $\vartheta_\theta = [(2\kappa-3)/\kappa]V_t$, T being the characteristic kinetic temperature, i.e. the temperature of the equivalent Maxwellian with the same average kinetic energy [18], n_{e0} the electron equilibrium density and k_B is the Boltzmann constant. The spectral index $\kappa > 3/2$ measures the slope of the energy spectrum of the superathermal particles forming the tail of the distribution function. The Gamma function arises from the normalization of $f_\kappa(v)$ such that

$$\int f_\kappa(v) d^3v = n_{e0}. \quad \textbf{(2)}$$

Integrating the Kappa distribution over velocity space, the number density for electrons can be obtained as

$$n_e(\phi) = n_{e0}\left(1 - \frac{e\phi}{(\kappa-3/2)k_B T}\right)^{-(\kappa-1/2)}. \quad \textbf{(3)}$$

ϕ is the local electrostatic potential.
In the reductive perturbation method, one uses a small amplitude expansion which is cut off, and it is thus valid only for small ϕ . This is an aspect of Korteweg–de Vries KdV- soliton theory that is sometimes ignored. Using this approach, the electron density is obtained from [3].

$$n_e(\phi) = 1 + \left(\frac{2\kappa-1}{2\kappa-3}\right)\phi + \frac{4\kappa^2-1}{2(2\kappa-3)^2}\phi^2, \quad \textbf{(4)}$$

The density expression given above is only valid for $\kappa > 3/2$.

On the other hand, 12 years ago, in 1980, Shamel introduced the concept of a separatrix to the distribution, which separates free electrons from trapped ones. The energy separatrix occurs at the point where the energy of electrons equals to zero.

More recently, in 2014 Williams *et al.* [19] found a new electron distribution called a Shamel-Kappa distribution which reads as:

$$n_e(\phi) = 1 + \left(\frac{2\kappa-1}{2\kappa-3}\right)\phi + \frac{8\sqrt{2/\pi}(\beta-1)\kappa\,\Gamma(\kappa)}{3(2\kappa-3)^{3/2}\Gamma(\kappa-1/2)}\phi^{3/2} + \frac{4\kappa^2-1}{2(2\kappa-3)^2}\phi^2. \quad \textbf{(5)}$$

β is a parameter wich determines the efficiency of trappinfg

Shamel-Kappa equation limits:

For* $\kappa \to \infty$, *(Maxwellian plasma)* ***Eq.(5)* reduces to the Shamel distribution *:*

$$n_e(\phi) \approx 1 + \phi + \frac{4(\beta-1)}{3\sqrt{\pi}}\phi^{3/2} + \phi^2/2, \quad \textbf{(6)}$$

For* $\beta \to 1$ *(Kappa-distributed plasma)* ***Eq.(5)* reduces to Kappa distribution **Eq.(4)**:

$$n_e(\phi) = 1 + \left(\frac{2\kappa-1}{2\kappa-3}\right)\phi + \frac{4\kappa^2-1}{2(2\kappa-3)^2}\phi^2.$$

3. Ion motion modeling

We consider two-component plasma model, whose constituents are inertial cold fluids ions n_i and Vasyliuas- Shamel electrons. We assume that the system is immersed in an external magnetic field $\boldsymbol{B}_0$ $(B_0\hat{z})$ pointing along the z-axis. The direction of wave propagation at an angle θ to $\boldsymbol{B}_0$, together with the magnetic field direction, defines the $x-z$ plane. We also introduce the direction cosines l_x, l_y and $l_z = cos\theta$ with respect to $x-$ and $z-axis$, respectively. The nonlinear dynamics of the ion acoustic solitary waves (IASWs) in such a plasma system is described by continuity, ion momentum and Poisson's equations as follow:

$$\frac{\partial n}{\partial t} + \nabla.(nu) = 0, \quad \textbf{(7)}$$

$$\frac{\partial u}{\partial t} + (u.\nabla)u = -\nabla\phi + \omega_{ci}(u \times \hat{z}), \quad \textbf{(8)}$$

$$\nabla^2\phi = n_e - n. \quad \textbf{(9)}$$

Where n and u represent the ions density and velocity, respectively, and ϕ is the electrostatic potential. The physical quantities $n, u, \phi, x, and\ t$ have been appropriately normalized. Specifically, n is normalized by the unperturbed ion density n_0, u by the sound speed $C_S = \left(\frac{Zk_BT_e}{m}\right)^{1/2}$, ϕ by $\left(\frac{k_BT_e}{e}\right)$, the space and time variables are in units of the Debye length $\lambda_D = C_S/\omega_{pi}$ and the inverse plasma frequency $\omega_{pi} = \left(\frac{4\pi n_0 e^2}{m}\right)^{1/2}$. Here, $\omega_{ci} = \frac{eB_0}{(m\omega_{pi})} = B_0/\sqrt{4\pi n_0 m}$ is the ion cyclotron frequency normalized to the plasma frequency ω_{pi}, and T_e is the electrons temperature.
Using expression of densities n_e and n in Poisson's equation, we obtain

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 1 - n + a\phi + b\phi^2 + c\phi^{3/2}, \textbf{(8)}$$

Where $a = \left(\frac{2\kappa-1}{2\kappa-3}\right)$, $b = \frac{4\kappa^2-1}{2(2\kappa-3)^2}$. $c = \frac{8\sqrt{2/\pi}(\beta-1)\kappa\,\Gamma(\kappa)}{3(2\kappa-3)^{3/2}\Gamma(\kappa-1/2)}$.

4. Derivation of KdV equation

Reductive perturbation method [20]-[22] is the best technic to investigate the dynamic of ion acoustic waves for weak nonlinearities. That is why we used it to linearize our equations using whereby, we expand n, u and ϕ in power series of ε. as

$$n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \cdots \quad \textbf{(10)}$$

$$u_x = \varepsilon^{3/2} u_{x1} + \varepsilon^2 u_{x2} + \cdots \quad \textbf{(11)}$$

$$u_y = \varepsilon^{3/2} u_{y1} + \varepsilon^2 u_{y2} + \cdots \quad \textbf{(12)}$$

$$u_z = \varepsilon\ u_{z1} + \varepsilon^2 u_{z2} + \cdots \quad \textbf{(13)}$$

$$\phi = \varepsilon\phi_1 + \varepsilon^{3/2}\phi_2 + \cdots \quad \textbf{(14)}$$

Where ε is a small parameter measuring the weakness of dispersion.
We can rewrite **Eqs. [(7)-(9)]** taking into account **Eq. (4)** and **Eqs. [(10)-(14)]** and the stretched coordinates, $\zeta = \varepsilon^{1/4}(l_x x + l_y y + l_z z - V_0 t)$, $\tau = \varepsilon^{3/4} t$, to get different powers of ε. The direction cosines $x, y, and\ z$ follows the relation, $l_x^2 + l_y^2 + l_z^2 = 1$, and V_0 is the normalized speed. Solving for $n_1, u_1\ and\ \phi_1$, order by order we acquire:
to the lowest order in ε:

$$n_1 = \frac{l_z}{V_0} u_{z1}, \quad \textbf{(15)}$$

$$u_{z1} = \frac{l_x}{V_0} \phi_1, \quad \textbf{(16)}$$

$$n_1 = a\phi_1, \quad \textbf{(17)}$$

$$V_0 = \sqrt{l_z^2 / a}. \quad \textbf{(18)}$$

To the higher order of ε:

$$\frac{\partial n_1}{\partial \tau} - V_0 \frac{\partial n_2}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left(l_x u_{x2} + l_y u_{y2} + l_z u_{z2} + l_z n_1 u_{z1}\right) = 0, \quad \textbf{(19)}$$

$$\frac{\partial u z_1}{\partial \tau} - V_0 \frac{\partial u_{z2}}{\partial \zeta} + l_z u_{z1} \frac{\partial u_{z1}}{\partial \zeta} + l_z \frac{\partial \Phi_2}{\partial \zeta} = 0, \quad \textbf{(20)}$$

$$\frac{\partial^2 \phi_1}{\partial \zeta^2} = a\phi_1 + b\phi_1^2 - n_2. \quad \textbf{(21)}$$

Using **Eqs. [(19)-(21)]** we finally derived the following equation,

$$\frac{\partial \phi_1}{\partial \tau} + A\, {\phi_1}^{1/2} \frac{\partial \phi_1}{\partial \zeta} + B \frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \quad \textbf{(22)}$$

$$A = -\frac{3}{4} \frac{l_z\, c}{a^{3/2}}.\ B = \frac{l_z}{2a^{3/2}}\left(1 + \frac{1 - l_z^2}{\omega_{ci}^2}\right). \quad \textbf{(23)}$$

Equation **(22)** is known as the Korteweg-de Vries-like equation (KdV-like equation) describing the nonlinear propagation of the ion acoustic solitary waves in plasma with Vasyliunas-Shamel distributed electrons. Here, the wave steepening is represented by the nonlinear coefficient, and the wave broadening by dispersion coefficient. One can note that the balance between the nonlinear steepening and dispersive stretch of the wave gives rise to soliton occurrence.

5.Solitonic solution

We seek a stationary wave solution of Eq. (21), and introduce independent variables $\zeta\ and\ \tau$ to $\xi = \zeta - U_0\tau$ and $\tau = \tau$, where U_0 is the wave speed (in the reference frame) normalized by the sound speed C_S , and by setting suitable boundary conditions, viz., $\Phi \to 0, d\Phi_1/d\xi \to 0$, $d^2\Phi_1/d\xi^2 \to 0$ as $\xi \to \pm\infty$. Thus, we obtain the stationary wave solution **(22)** as;

$$\Phi_1 = \Phi_m sech^4\left(\frac{\zeta - U_0\tau}{\Delta}\right). \quad \textbf{(24)}$$

where $\Phi_1 \equiv \Phi$, $\Phi_m = (15U_0/8A)^2$ is the soliton amplitude and $\Delta = 4\sqrt{B/U_0}$ its width.
It is meaningful to indicate that the nature of solitary wave can be determined by the sign of parameter "A". Accordingly, if we obtain a negative (positive) potential corresponding to A<0(A>0), the system can support rarefactive (compressive) solitary structures respectively.
Note that Eq. (23) shows that "A" can be either negative or positive. Therefore both rarefactice and compressive solitons are able to propagate in this plasma. We will see further that in our model that it is the superthermality (via κ) which determines the regime of rarefactive and compressive waves solutions.

6. Results

The basic features of ion acoustic solitary waves in a two component magnetized plasma system consisting of cold ions and electrons with Vasyliunas-Shamel distribution are analyzed and graphically examined how the profile of the potential perturbation and the electric field are affected bythe plasma configuration parameters (viz., the superthermality via κ, the propagation angle θ , the magnetic field via ω_{ci} and the Mach number M) affect The results are displayed in figures [1-4] where the effects of each plasma parameters are examined.

6.1. Effect of Electron spectral indices κ

Figure **1** shows how the amplitude varies with the electron spectral index κ when the angle between the direction of propagation and the magnetic field and the ion cyclotron frequency $(\theta = 10°,\ \omega_{ci} = 0.3)$ are held constant. The superthermal electrons lead to reduction of the compressive wave's amplitude until change its polarity. Thus, the electron spectral index κ determines the regime of compressive and rarefactive wave solutions as shown in **figure 2**.
We found that for specific values of κ, namely $(1.6 < \kappa < 1.74\ )$ we have rarefactive waves corresponding to negative potential. While above the value of 1.74 the plasma system can support only compressive solitary structures.

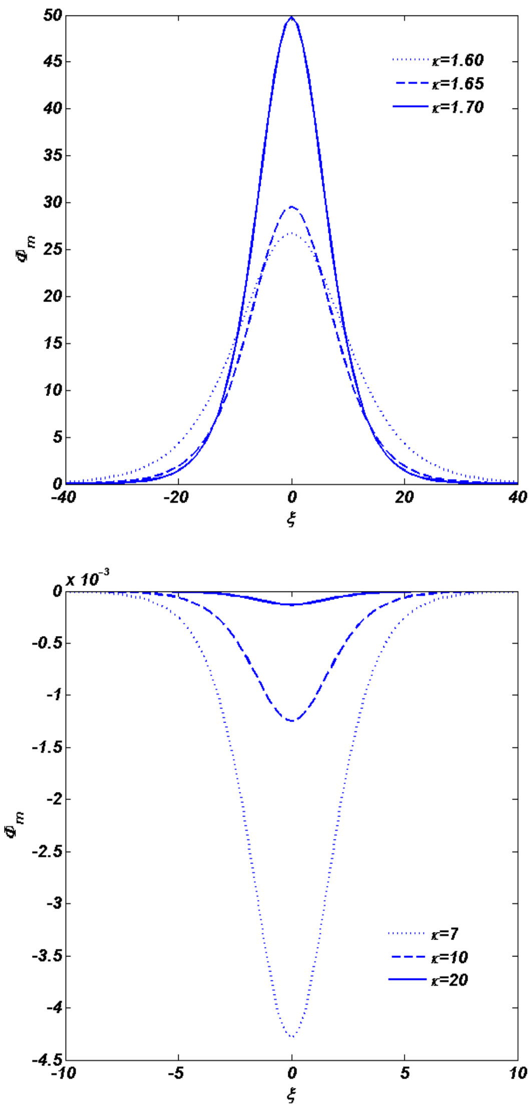


Figure 1. Plot of the solitary wave solution for different Values of κ for $\theta = 10°, \omega_{pi} = 0.3, \beta = 0.7\ and\ M = 1.5.$

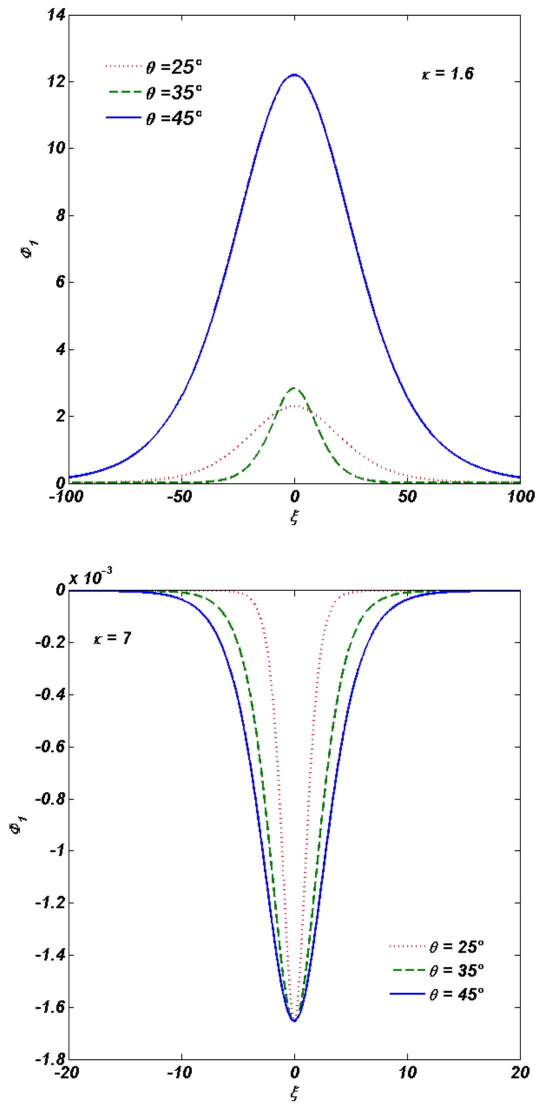


Figure 2. Plot of the solitary wave solution for different Values of θ for $\omega_{pi} = 0.3,\ M = 1.5, \beta = 0.7\ and\ \kappa = 1.6\ for\ positive\ potential\ and \kappa = 7\ \ for\ negative\ potential.$

6.2. Effect of angle of propagation θ

Figure 2 depict the effect of the propagation angle θ on compressive and rarefactive solitary waves with fixed plasma parameters, $\kappa = 1.6$, $\omega_{ci} = 0.1$, (for positive potential) and $\kappa = 7$, $\omega_{ci} = 0.1\ and\ M = 1.50$, (for negative potential). It is seen that the angle θ changes slightly the width of the rarefactive waves (negatives solitary structures) while it has no real effect on their amplitude. By cons, the amplitude and width of compressive waves increase significantly with θ.

6.3. Effect of the magnetic field via ω_{pi}

We have analyzed the effect of magnetic field via ω_{pi} on electrostatic solitary waves, for given values of the other plasma parameters. It is seen that the magnitude of the external magnetic field has no effect on the SWs amplitude. But, it has an effect on their width. Indeed, Figures 3 show that as the ω_{pi} increases, the width of the wave increases too, i.e., the magnetic field makes the solitary waves more spiky.

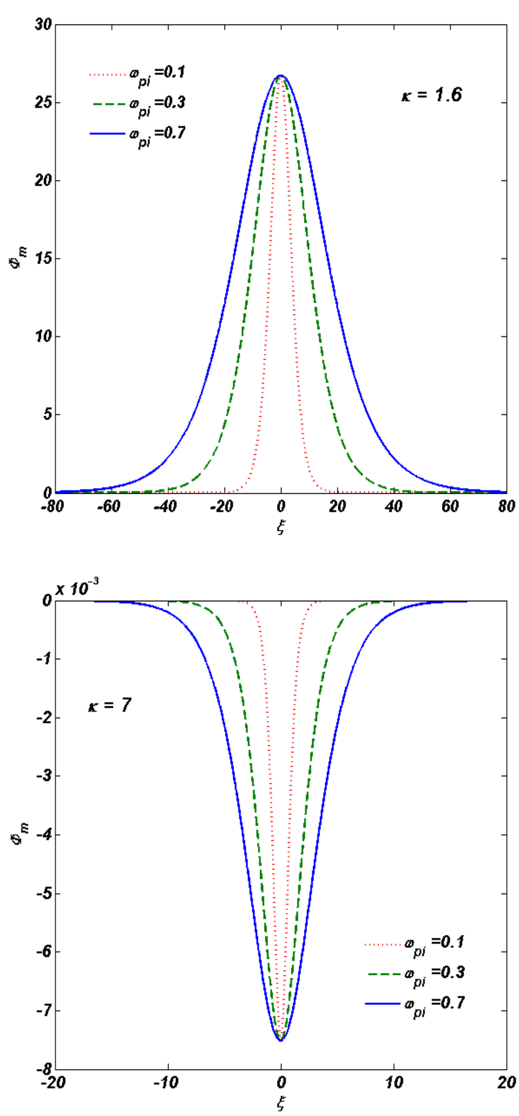


Figure 3. Plot of the solitary wave solution for different Values of ω_{pi} for $\theta = 10°,\ M = 1.5, \beta = 0.7\ and\ \kappa = 1.6\ for\ positive\ potential\ and \kappa = 7\ \ for\ negative\ potential.$

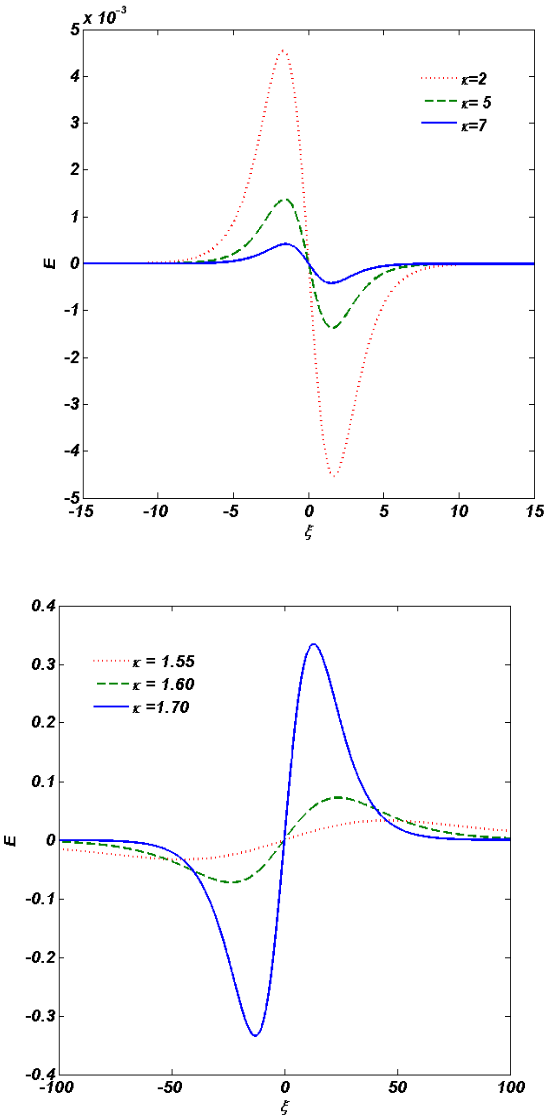


Figure 4. Solitary wave electric field magnitude for different values of κ .

7. Electric field

The magnitude of the electric field is found by taking the negative gradient of the solution in **Eq.24** so that;

$$E = -\frac{2\Phi_m}{\Delta} sech^4\left(\frac{\zeta - U_0\tau}{\Delta}\right) tanh\left(\frac{\zeta - U_0\tau}{\Delta}\right). \quad \textbf{(25)}$$

The dependence of the electric field E on $\kappa, \omega_{pi}, M\ and\ \theta$ is depicted in **figures [(4)-(7)]**. It is clearly shown that the population of superthermal electrons have a great impact on the electric field , . Indeed, "E" becomes more localized with higher amplitude with increase κ (decreased superthermality).

8. Conclusion

Properties of solitary wave propagating in magnetized plasmas of cold ions and electrons with a Vasyliunas-Shamel distribution have been investigated in this paper. Indeed, we studied the combined effect of Vasyliunas-Shamel electrons, external magnetic field and obliqueness on the basic features of ion acoustic waves (IAW). Elsewhere, we derived a K-dV equation and its corresponding solitary wave by using the reductive perturbation theory. The parameters of superthermality (via κ) and trapping (via β) play a predominant and crucial role in determining the polarity of the solitary wave and its shape as well as the magnitude of electric field. It is also shown that the properties of solitons are influenced significantly by the plasma parameters. It is useful to point out that there are many linear and nonlinear phenomena observed in space and laser-created plasma that cannot be explicated by usual Vasyliunas or Shamel distribution of plasma particles. Accordingly, one need a more generalized distribution function to explain such phenomena. A combination of those distributions called Vasyliunas-Shamel distribution was used in this paper to study more precisely the ion acoustic wave properties. Inasmuch as Vasyliunas-Shamel distribution allows observation of the effects of the usual enhanced non-Maxwellian tail with an excess of superthermal particles (Vasyliunas distribution) while including the trapped particles in the low energy part of the distribution (Shamel distribution).

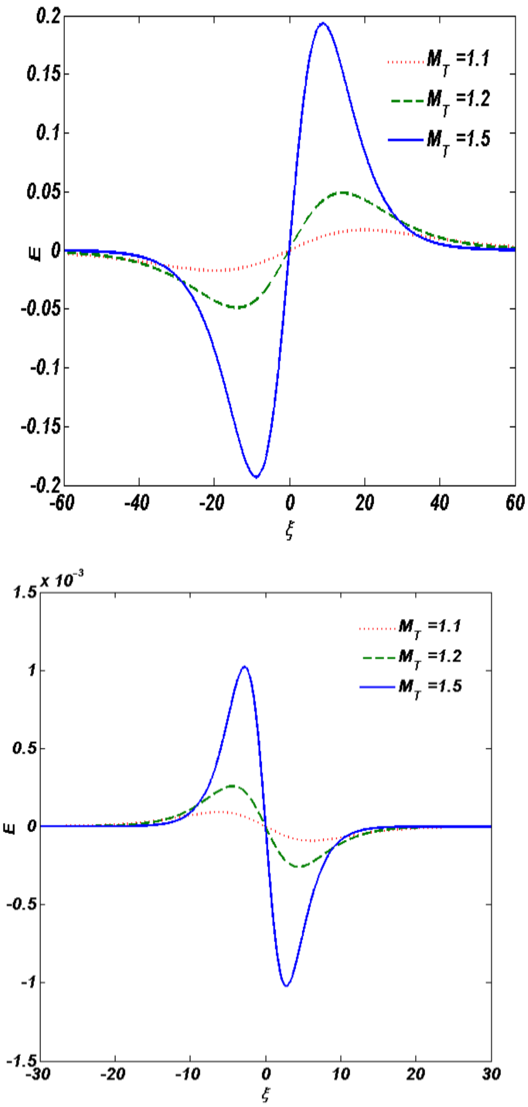


Figure 5. Solitary wave electric field magnitude for different values of M.

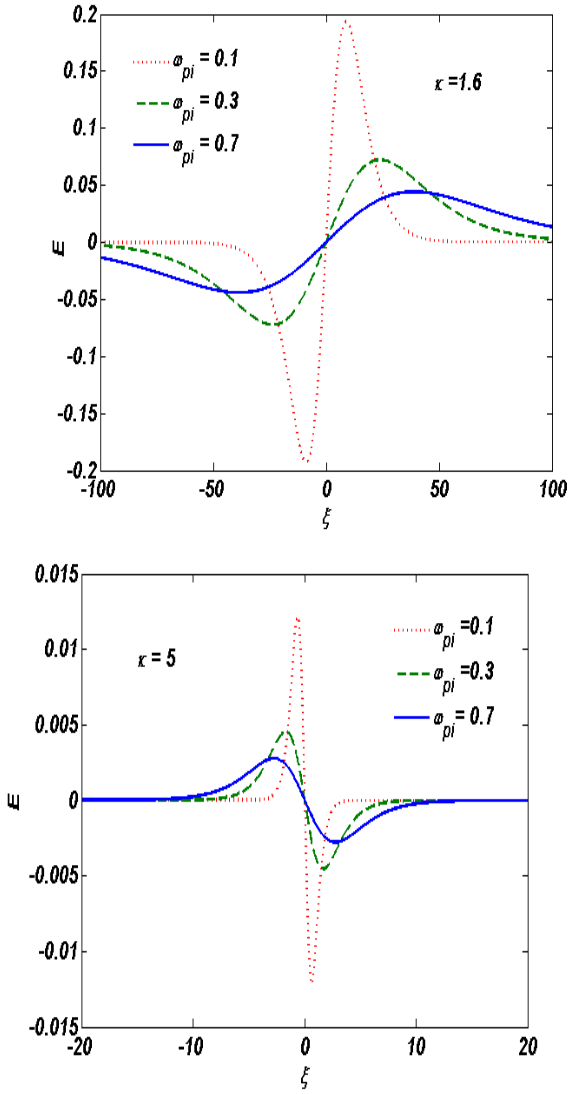
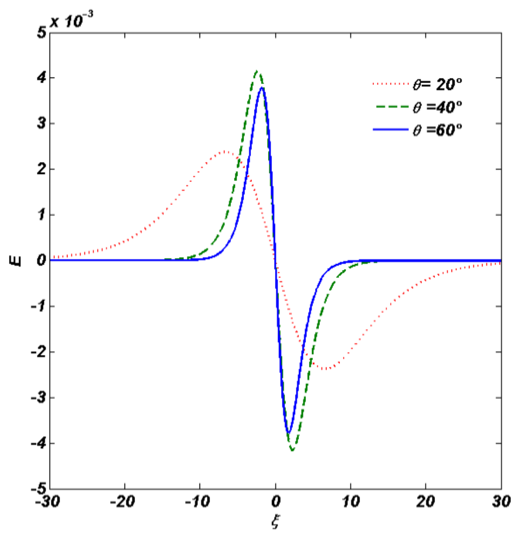


Figure 6. Solitary wave electric field magnitude for different values of ω_{pi}.



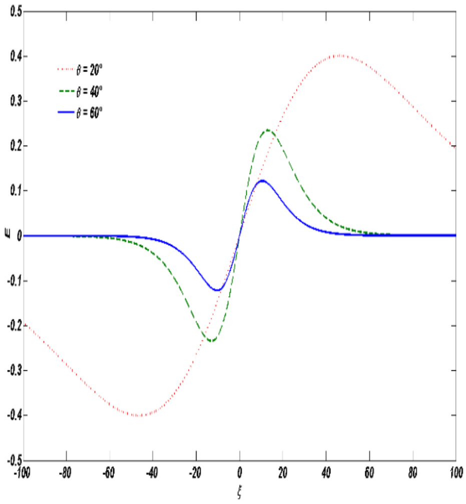


Figure 7 Solitary wave electric field magnitude for different values of θ.

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