Relative Localization in Multi-Robot Systems Based on Dead Reckoning and UWB Ranging

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***Abstract*—Combining dead reckoning and Ultra-Wideband (UWB) ranging information to achieve relative localization (RL) becomes prevalent in recent years. However, two main problems, i.e., pose initialization and distributed implementation, are rarely investigated in practical multi-robot systems. In this paper, a novel two-stage RL method is proposed to fill this gap, wherein an initialization strategy, using robot-to-robot measurements acquired at different vantage points during robot motion to determine initial pose, and a consensus-based distributed particle filter (DPF), fusing statistics from local robot and neighbors to realize RL, are designed. In our system, by computing the pairwise relative pose between all robots in a team, the initialization strategy can determine an estimate of the initial pose of all robots with respect to a common reference frame and the corresponding covariance. The consensus-based DPF allows determining the relative pose of all robots with significantly reduced computing costs. Experiments results on a team of differentially driven mobile robots show the effectiveness of the initialization strategy and highlight the low computation cost of the proposed approach.**

I. Introduction

Relative localization (RL), which refers to detect and locate relative configurations of mobile agents with respect to other agents or landmark, is critically important in multi-robot systems since it is the pre-requisite for robot teaming and swarming [1]. Many applications, such as formational control, cooperative transportation, perimeter surveillance, area coverage and situational awareness, are often faced with RL problem, and they are widely investigated in [2]–[5].

Global Positioning System (GPS) is a typical system which is frequently used to address RL problem, where each robot can determine its position in a globally-shared frame and transmit this information to its neighbors [6]. However, equipping each robot with a GPS may not be practical due to volume restrictions and expensive costs, let alone the environments where GPS does not work such as under water deployments [7]. Another convenient method, i.e., extracting range and/or bearing measurements from cameras and visual-makers, suffers from the disadvantages of limited field-of-view, short range, occlusion by other objects and possible demanding computing power capabilities [8]. Alternatively, taking advantage of the distance measurement from sensing devices such us radars, lidars and Ultra-wideband (UWB) to realize RL gains more attention recently. In particular, UWB technology stands out in accurate ranging due to its ability to alleviate multi-path effects [9]–[11]. Nevertheless, UWB does not provide any bearing information and the communication range is limited, thus UWB alone cannot determine the robot localization without any ambiguity. Dead reckoning (DR) determines one's location based on its previous position and speed, which is measured by an IMU sensor or wheel encoder in the case of a mobile robot. If the initial localizations of the robots are known, one can use DR to determine the relative position of a group of users. However, DR may not be accurate due to accumulative error, which must be corrected or eliminated by other sources or information.

In recent years, combining the information from dead reckoning and UWB ranging to achieve RL has been widely studied in [6]–[15], which can be a very promising solution since it has excellent performance in situations that without any given infrastructure (e.g., RL system with fixed anchor). Unfortunately, to the best knowledge of author, existing works rarely considered the initialization (determine the initial robot-to-robot pose without any prior information) and distributed implementation problems in a practical multi-robot system. In [14]–[18], numerous initialization solutions have been investigated, where the basic idea is to compute a coarse estimate for relative pose firstly, and some techniques, such as iterative/non-iterative weighted least square (WLS) are employed for further refinement. In these studies, both noise corrupted distance measurements and noise-free cases are comprehensively considered. While we adopt the same idea, further derivations of [16] is presented in this paper, which makes the coarse estimate can be easily obtained with the linear system we built. Additionally, in most studies, the fusion of DR and UWB ranging information is implemented with a centralized architecture and the fusion center requires strong computational capability.

In this paper, to address the initialization and distributed implementation problems, a novel RL method is developed. Generally, our method includes two main stages, pose initialization and distributed filtering. At the first stage, we use robot-to-robot measurements acquired at different vantage points during robot motion to determine initial pose, where we solve a linear equation to compute a coarse estimate for relative pose firstly, and then we employ an iterative WLS method for further refinement. Secondly, we developed a consensus-based distributed particle filter (DPF) to fuse the data from local robot and neighbors to realize RL. In our system, by computing the pairwise relative pose between all robots in a team, the initialization strategy can determine an estimate of the initial pose of all robots with respect to a common reference frame and the corresponding covariance. The consensus-based DPF allows to determine the relative pose of all robots with significantly reduced computing cost. Experiments results on a team of differentially driven mobile robots show initialization strategy is able to obtain an accurate estimation of relative pose and highlighted that the proposed method has lower computation cost than the centralized approach at each local robot.

II. Models and Problem Statement

Consider a multi-robot system comprising of M moving robots (M is not known and may vary during operation) and they are denoted by $\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_M$. Each robot $\mathcal{R}_i, i \in \{1, 2, ..., M\}$ has its own sensing, communication and local information processing capabilities. Without losing generality, we define $\mathcal{R}_j, j = 1, ..., M, j \neq i$ as the remaining robots. As shown in Fig. 1, an illustration of the considered localization setup is given.

A. System Model

We place an IMU sensor and a wheel encoder on each robot for DR, the motion of $\mathcal{R}_i$ is described by the following equation

$$\begin{aligned} \mathbf{x}_{i,k} &= \mathbf{x}_{i,k-1} + \mathbf{u}_{i,k} \\ &= \begin{bmatrix} x_{i,k-1} \\ y_{i,k-1} \\ \theta_{i,k-1} \end{bmatrix} + \mathbf{u}_{i,k} \end{aligned} \tag{1}$$

where $\mathbf{x}_{i,k} = [x_{i,k}, y_{i,k}, \theta_{i,k}]^T$ denotes the pose of robot i at time instant k, $x_{i,k}$ and $y_{i,k}$ denote the position of $\mathcal{R}_i$ at time k, θ_k is the orientation, $\mathbf{u}_k$ is the displacement of robot i between $k-1$ and k, and we model

$$\begin{aligned} \mathbf{u}_{i,k} &= \mathbf{d}_{i,k} + \mathbf{w}_{i,k} \\ &= \begin{bmatrix} \triangle\, x_{i,k} \\ \triangle\, y_{i,k} \\ \triangle\, \theta_{i,k} \end{bmatrix} + \mathbf{w}_{i,k}, \end{aligned} \tag{2}$$

where $\mathbf{d}_{i,k}$ denotes the actual displacement of $\mathcal{R}_i$, $\mathbf{w}_{i,k}$ is a (by assumption) white error of $\mathbf{u}_k$ with covariance $\mathbf{P}_{i,k}$. At time k, the measurements of UWB are used for robot-to-robot ranging (see Fig. 1), and the measurement of $\mathcal{R}_i$, i.e., $\mathbf{z}_{i,k}^{i,j}$, is produced according to the measurement model[1]

$$\begin{aligned} z_{i,k}^{i,j} &= \| \mathbf{g}(\mathbf{x}_{j,k}) - \mathbf{x}_{i,k} \|_2 + v_k \\ &= \sqrt{(x_{i,k} - x_{j,k}^{\mathrm{T}})^2 + (y_{i,k} - y_{j,k}^{\mathrm{T}})^2} + v_k \end{aligned} \tag{3}$$

where $\mathbf{z}_{i,k}^{i,j}$ represents the measured distance between $\mathcal{R}_i$ and $\mathcal{R}_j$, $\mathbf{g}(\cdot)$ is a transformation function, which is used to ensure $\mathbf{x}_{i,k}$ and $\mathbf{x}_{j,k}$ are expressed with respect to a common frame of reference, $(x_{j,k}^{\mathrm{T}}, y_{j,k}^{\mathrm{T}})$ denotes the transformed coordinate of $\mathcal{R}_j$, v_k is the measurement noise of UWB, we assume it follows a Gaussian distribution with mean zero and covariance $\sigma_{i,k}^2$, and we define $\mathbf{R}_k = \mathrm{diag}(\sigma_{1,k}^2, ..., \sigma_{M,k}^2)$.

B. Problem Statement

As shown in Fig. 1, for each robot $\mathcal{R}_i$, our goal is to obtain the relative configuration of its neighboring robots $\mathcal{R}_j$ at each time instant k, i.e.,

$$\mathbf{x}_{i,k}^{i,j} = \mathbf{g}(\mathbf{x}_{j,k}) - \mathbf{x}_{i,k}.$$

[1]Note that the variation of orientation is not used.

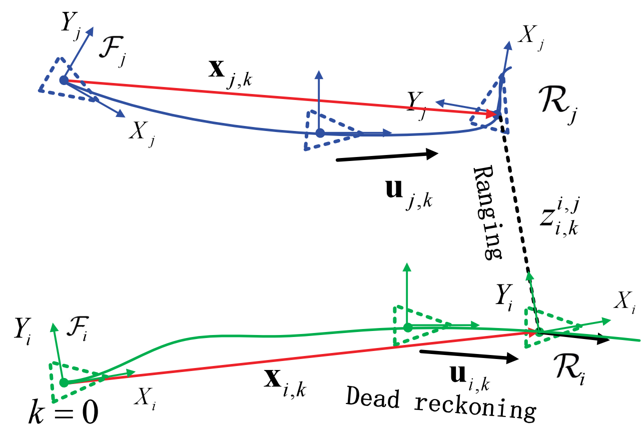

Fig. 1. Illustration of the considered localization setup. Two of the multiple robots are localizing cooperatively in two dimensions by means of DR and ranging to other robots.

To achieve this, efficient algorithm should be developed to make full use the information of DR and the measurements of UWB. We first define

$$\mathbf{u}_{ij,k} = \mathbf{g}(\mathbf{u}_{j,k}) - \mathbf{u}_{i,k}, \tag{4}$$

then write $\mathcal{M} = \{\mathcal{M}_1, ..., \mathcal{M}_M\}$, $\mathcal{M}_j = \{\mathbf{u}_{ij,1}, ..., \mathbf{u}_{ij,k}\}$, $\mathcal{Z} = \{\mathcal{Z}_1, ..., \mathcal{Z}_M\}$ and $\mathcal{Z}_j = \{z_{i,1}^{i,j}, ..., z_{i,k}^{i,j}\}$. From a probabilistic-based framework, our goal is to obtain

$$p(\mathbf{x}_{i,k}^{i,1}, ..., \mathbf{x}_{i,k}^{i,M} | \mathcal{M}, \mathcal{Z}).$$

We assume target motion and ranging measurements are independent. Based on the Bayesian theory and the assumption that target state follows a first order Markov process, $p(\mathbf{x}_{i,k}^{i,1}, ..., \mathbf{x}_{i,k}^{i,M} | \mathcal{M}, \mathcal{Z})$ can be factorized into [12]

$$\begin{aligned} & p(\mathbf{x}_{i,k}^{i,1:M} | \mathcal{M}, \mathcal{Z}) \\ & \approx \prod_{j=1}^{M} p(\mathbf{x}_{i,k}^{i,j} | \mathcal{M}_j, \mathcal{Z}_j) \\ & \propto \prod_{j=1}^{M} p(\mathbf{x}_{i,k}^{i,j} | \mathcal{M}_j) p(\mathcal{Z}_j | \mathbf{x}_{i,k}^{i,j}) \\ & \propto \prod_{j=1}^{M} \Big\{ \underbrace{p(\mathbf{x}_{i,k}^{i,j} | \mathbf{x}_{i,k-1}^{i,j}, \mathbf{u}_{ij,k})}_{\text{Dead Reckoning}} \underbrace{\prod_{j=1}^{M} p(z_{i,k}^{i,j} | \mathbf{x}_{i,k}^{i,j})}_{\text{UWB Ranging}} \Big\}, \end{aligned} \tag{5}$$

where $p(\mathbf{x}_{i,k}^{i,j} | \mathbf{x}_{i,k-1}^{i,j}, \mathbf{u}_{ij,k})$ represents the prediction process in Bayesian filtering, which is given by

$$\begin{aligned} \mathbf{x}_{i,k}^{i,j} &= \mathbf{g}(\mathbf{x}_{j,k}) - \mathbf{x}_{i,k} \\ &= \mathbf{g}(\mathbf{x}_{j,k-1}) + \mathbf{g}(\mathbf{u}_{j,k}) - \mathbf{x}_{i,k-1} - \mathbf{u}_{i,k} \\ &= \mathbf{x}_{i,k-1}^{i,j} + \mathbf{u}_{ij,k}, \end{aligned} \tag{6}$$

and $p(\mathbf{z}_{i,k}^{j,i} | \mathbf{x}_{i,k}^{i,j})$ denotes the updating process, which can be obtained from (3), i.e., the ranging model of UWB measurement, which represents the likelihood of receiving a ranging measurement $\mathbf{z}_{i,k}^{j,i}$ given the states of two robots $\mathbf{x}_{i,k}$ and $\mathbf{x}_{j,k}$.

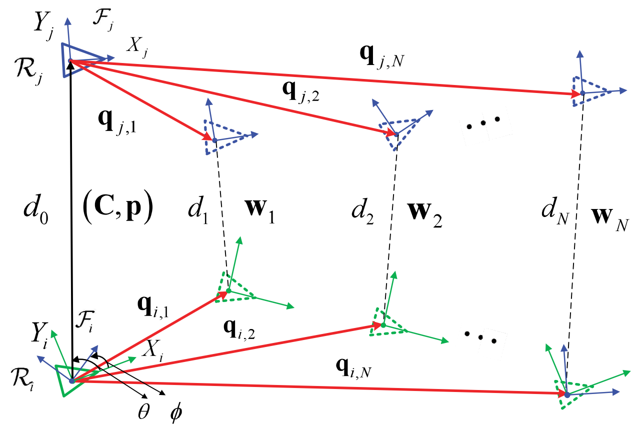

Fig. 2. Illustration of the considered localization setup. Multiple robots are localizing cooperatively in two dimensions by means of dead reckoning and ranging to other robots.

However, to implement (5) in a practical application, two main problems have to be addressed.

Problem 1: As we see in (5), it is easy to infer that the prior information $p(\mathbf{x}_{i,0}^{i,j})$ should be given at $k=0$. Additionally, each robot $\mathcal{R}_i$ only knows its position $\mathbf{x}_{i,k}$ and displacements $\mathbf{u}_{i,k}$ in its global frame $\mathcal{F}_i$. According to this, the transition and rotation of two arbitrary frame should be given, i.e., the function $\mathbf{g}(\cdot)$ should be obtained at the initialization stage.

Problem 2: We intend to implement (5) with a consensus-based DPF. Therefore, further derivation of (5) should be given, and the details of DPF implementation should be presented.

Remark 1: In (5), it is worthy to point out that $\mathbf{x}_{i,k}^{i,i}=\mathbf{x}_{i,k}$. Since robot i will not produce any measurement when $j=i$, the term $p(\mathbf{x}_{i,k}^{i,j}|\mathcal{M}_j,\mathcal{Z}_j)$ in (5) can be simplified to

$$p(\mathbf{x}_{i,k}^{i,j}|\mathcal{M}_j,\mathcal{Z}_j)=p(\mathbf{x}_{i,k}|\mathbf{x}_{i,k-1},\mathbf{u}_{i,k}),$$

which means that only prediction step will be executed in $\mathcal{R}_i$.

III. Relative Localization Scheme

In this section, to address the two problems mentioned in Section II, a two-stage RL scheme is proposed. In the first stage, we address the initialization problem by utilizing the measurements of wheel encoder and UWB as they travel though a sequence of poses. In the second stage, a consensus-based DPF is developed to implement the Bayesian filter, i.e., equation (5), where a least square strategy is used to approximate likelihood function to reduce the communication cost between robots.

A. Initialization

Consider two arbitrary robots $\mathcal{R}_i$ and $\mathcal{R}_j$ whose initial poses are indicated by the frames of reference $\mathcal{F}_i$ and $\mathcal{F}_j$ (see Fig. 2), respectively. The two robots acquire N robot-to-robot distance measurements $d_l, l=1,...,N$ while moving in a 2D plane, as shown in Fig. 2. According to the geometric relations, we define that

$$\mathbf{g}(\mathbf{q}_{j,l})=\mathbf{p}+\mathbf{C}\mathbf{q}_{j,l}, \tag{7}$$

where $\mathbf{q}_{i,l}$ and $\mathbf{q}_{j,l}, l=1,...,N$ denote their positions of $\mathcal{R}_i$ and $\mathcal{R}_j$ in their respective global frames at initialization stage, N denotes the moving step, $\mathbf{p}$ is the transition vector, and $\mathbf{C}$ is the rotation matrix, and we define $\mathbf{C}$ as

$$\mathbf{p}:=d_0\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}, \tag{8}$$

$$\mathbf{C}:=\begin{bmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{bmatrix}, \tag{9}$$

where ϕ is the relative orientation of robot i and j at $l=0$. Then, the robot-to-robot distance d_i can be expressed as the length of vector $\mathbf{p}_l$, connecting the two robots at the time of measurement:

$$d_l=\|\mathbf{p}_l\|_2=\sqrt{\mathbf{w}_l^T\mathbf{w}_l}, \tag{10}$$

where

$$\mathbf{w}_l:=\mathbf{g}(\mathbf{q}_{j,l})-\mathbf{q}_{i,l}=\mathbf{p}+\mathbf{C}\mathbf{q}_{j,l}-\mathbf{q}_{i,l}.$$

Next, by squaring two sides of the equation (10), we get

$$(\mathbf{p}+\mathbf{C}\mathbf{q}_{j,l}-\mathbf{q}_{i,l})^T(\mathbf{p}+\mathbf{C}\mathbf{q}_{j,l}-\mathbf{q}_{i,l})=d_l^2. \tag{11}$$

Rearranging the terms in (11) after squaring both sides yields

$$(\mathbf{q}_{i,l}-\mathbf{p})^T\mathbf{C}\mathbf{q}_{j,l}+\mathbf{q}_{i,l}^T\mathbf{p}=\varepsilon_l, \quad l=1,...,N, \tag{12}$$

where

$$\varepsilon_l:=\frac{1}{2}(d_0^2+\mathbf{q}_{i,l}^T\mathbf{q}_{i,l}+\mathbf{q}_{j,l}^T\mathbf{q}_{j,l}-d_l^2).$$

Note that ε_l on the left-hand side of (12) are known (from the measurements of DR information and UWB ranging), while the unknown variable θ and ϕ, embedded in $\mathbf{p}$ and $\mathbf{C}$, appear only on the right-hand side expressions. To solve (12) with the constraints

$$\begin{aligned}\sin(\theta)^2+\cos(\theta)^2&=1\\ \sin(\phi)^2+\cos(\phi)^2&=1,\end{aligned} \tag{13}$$

we use the homotopy continuation (PHCpack) method in [19] to obtain a coarse result. However, in practice, the robot-to-robot distances d_l will have to be replaced by noisy measurements z_l, which is produced according to (3), i.e.,

$$z_l=d_l+v_k=\sqrt{\mathbf{w}_l^T\mathbf{w}_l}+v_k. \tag{14}$$

Therefore, to correctly account for the measurement noise, the results obtained from the algebraic method requires further processing. Typically, a WLS problem is formulated for further refinement. Specifically, solving the weighted normal equations [18]

$$\min_{\theta,\phi}\quad \frac{1}{2}\mathbf{G}^T\mathbf{W}^{-1}(\mathbf{z}-\hat{\mathbf{z}}), \tag{15}$$

[1]Note that $\mathbf{q}_{i,l}$ has the same definition as $\mathbf{x}_{i,k}$, we use a new variable $\mathbf{q}$ here just for clarity. That is, $\mathbf{q}_{i,l}$ is specially used for the initialization step.

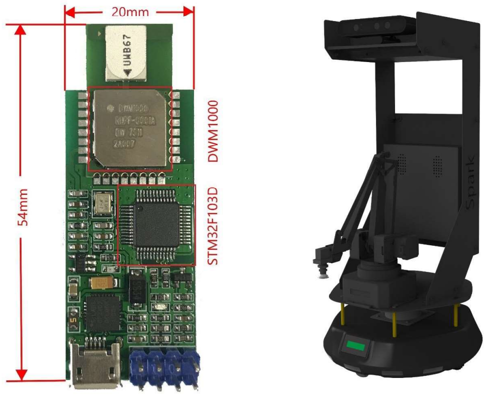

Fig. 3. The UWB module is consist of a DWM1000 module and the STM32F103 Discovery board, which is used to get the range information, with up to 30 meters ranging capability and programmed to send the ranging measurements every second. STM32F103 discovery board serves as host controllers to the DWM1000 to record and process the ranging information; We placed IMU sensor and wheel encoder on the bottom of each robot for dead reckoning.

where $\mathbf{G} = [\mathbf{G}_{\mathbf{p}_{i,k}}, \mathbf{G}_{\mathbf{p}_{j,k}}]$, and $\mathbf{W}$ is given by

$$\mathbf{W} = \mathbf{G}_{\mathbf{p}_{i,k}} \mathbf{P}_{i,k} \mathbf{G}_{\mathbf{p}_{i,k}}^T + \mathbf{G}_{\mathbf{p}_{j,k}} \mathbf{P}_{j,k} \mathbf{G}_{\mathbf{p}_{j,k}}^T + \mathbf{R}_l, \tag{16}$$

and the parameters can be obtained by using the chain rule, we compute the Jacobians as

$$\mathbf{G}_{\mathbf{p}} = \frac{\partial z_i}{\partial \mathbf{p}}, \qquad \mathbf{G}_{\phi} = \frac{\partial z_i}{\partial \phi}, \tag{17}$$

$$\mathbf{G}_{\mathbf{q}_{i,l}} = \frac{\partial z_i}{\partial \mathbf{q}_{i,l}}, \qquad \mathbf{G}_{\mathbf{q}_{j,l}} = \frac{\partial z_i}{\partial \mathbf{q}_{j,l}}. \tag{18}$$

Now, an iterative WLS can be employed to address (15), and the final covariance of the estimate is given by

$$\mathbf{P}_{\text{ini}} = \mathrm{E}\left[\begin{bmatrix} \delta\phi \\ \tilde{\mathbf{p}} \end{bmatrix} \begin{bmatrix} \delta\phi \\ \tilde{\mathbf{p}} \end{bmatrix}^T\right], \tag{19}$$

where

$$\tilde{\mathbf{p}} = \mathbf{p}^{\zeta+1} - \mathbf{p}^{\zeta}, \delta\phi = \phi^{\zeta+1} - \phi^{\zeta}, \tag{20}$$

and $\mathbf{p}^{\zeta}$ and ϕ^{ζ} denote the estimate transition and orientation at the ζth iteration of the WLS algorithm.

B. Consensus-Based Distributed Particle Filter

Particle filter is a useful tool for implementing a recursive Bayesian filter. Considering it has an excellent performance in nonlinear systems, an implementation of the exact Bayesian solution, i.e., equation (5), using particle filter is developed here. Then, motivated by the numerous advantages of distributed architecture, especially computational sharing, a consensus-based DPF approach is proposed.

As we see in (5), the posterior pdf of each robot $\mathcal{R}_i$ actually can be calculated independently, thus we write

$$p(\mathbf{x}_{i,k}^{i,j} | \mathcal{M}_j, \mathcal{Z}_j) \propto p(\mathbf{x}_{i,k}^{i,j} | \mathbf{x}_{i,k-1}^{i,j}, \mathbf{u}_{i,k}) \prod_{j=1}^{M} p(\mathbf{z}_{i,k}^{i,j} | \mathbf{x}_{i,k}^{i,j}). \tag{21}$$

We assume a sample-based approximation of the posterior pdf $p(\mathbf{x}_{i,k}^{i,j} | \mathcal{M}_j, \mathcal{Z}_j)$ is expressed as

$$p(\mathbf{x}_{i,k}^{i,j} | \mathcal{M}_j, \mathcal{Z}_j) \approx \sum_{q=1}^{Q} \omega_{k|k}^{(q)} \delta(\mathbf{x}_{i,k}^{i,j} - \mathbf{x}_{i,k}^{i,j,(q)}), \tag{22}$$

where $q = 1, 2..., Q$ is the index of the particle $\mathbf{x}_k^{(q)}$ at time k, Q is the particle number size, $\omega_{k|k}^{(q)}$ is the particle weight of the qth particle. Substituting (22) into (21), yields

$$\omega_{k|k}^{(q)} \propto \omega_{k|k-1}^{(q)} \prod_{j=1}^{M} p(\mathbf{z}_{i,k}^{i,j} | \mathbf{x}_{i,k}^{i,j,(q)}). \tag{23}$$

Take the logarithm form, we can obtain

$$\log(\omega_{k|k}^{(q)}) \propto \log(\omega_{k|k}^{(q)}) + \sum_{j=1}^{M} \log(p(\mathbf{z}_{i,k}^{i,j} | \mathbf{x}_{i,k}^{i,j,(q)})). \tag{24}$$

In the distributed implementation, robot i has restricted access limited to its local measurement $\mathbf{z}_{i,k}^{i,j}$ and can, therefore, only evaluate its local likelihood $p(\mathbf{z}_{i,k}^{i,j} | \mathbf{x}_{i,k}^{i,j,(q)})$ based on its vector particle $\mathbf{x}_{i,k}^{i,j,(q)}$. The likelihoods of other robots are not available at robot i and need to be communicated for updating the weights. Obviously, equation (25) for updating the weights can be iteratively implemented using an average consensus algorithm. Therefore, we define

$$\xi := \frac{1}{M} \sum_{j=1}^{M} \log(p(\mathbf{z}_{i,k}^{i,j} | \mathbf{x}_{i,k}^{i,j,(q)})) \tag{25}$$

The average consensus algorithm considered in this manuscript is represented by

$$\xi^{(s+1)} = U_{i,i}^{(s)} \xi_k^{(s)} + \sum_{j \in \mathcal{N}_i} U_{i,j}^{(s)} \xi_k^{(s)} \tag{26}$$

where $\xi_k^{(s)}$ is the consensus state variables at robot i, s is the consensus iteration index, and $\mathcal{N}_i$ represents the set of neighboring robots for robot i, choices of weights $U_{i,j}^{(s)}$ can be found in [20], and we use the Metropolis weights in this paper.

IV. Experimental Results

In this section, real world RL experiments are designed to demonstrate the effectiveness of the initialization strategy, and the influence of the number of UWB ranging measurements is analysed. Besides, the advantage of the proposed consensus-based DPF is highlighted since we can obtain a good RL result with significantly reduced computation cost.

A. Experiment setup

The proposed RL algorithm has been tested on a team of robots moving in $9m \times 2m$ wide arena, each robot traveled with an average velocity of around 0.6m/s. As shown in Fig. 3, UWB module (DWM1000) is used to get the range information, with up to 30 meters' ranging capability and programmed to send the ranging measurements with 1Hz frequency. The STM32F103 discovery board serves as host controllers to the DWM1000. Meantime, we placed an IMU

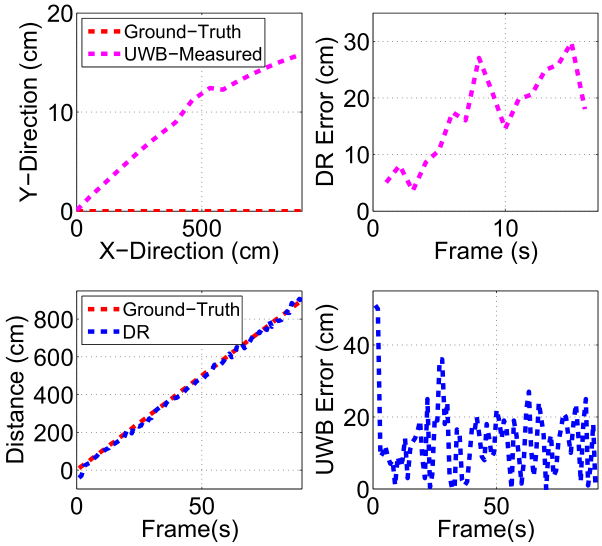

Fig. 4. Two robots move according to the predefined trajectories.

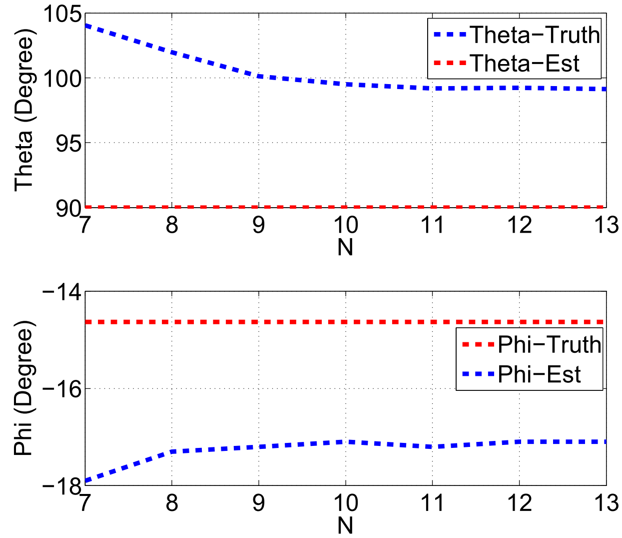

Fig. 6. Different N versus estimation result of θ and ϕ.

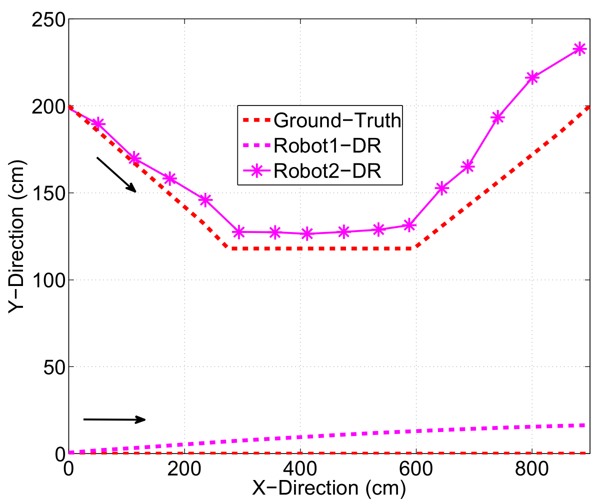

Fig. 5. Illustration of the setup of a preliminary test: the accuracy of UWB ranging and and DR, ground truth is provided as a benchmark

sensor and a wheel encoder on the bottom of each robot for DR (see 3). Additionally, to obtain a ground truth against which to evaluate the performance of the proposed RL algorithm, a manual calibration of our experiment was made in advance, where the distance angle and errors are limited less than $0.05m$ and $5°$, respectively. Finally, the recorded data (DR and UWB ranging information) is further processed in Matlab2013a on a Intel core i7 with a 3.6-GHz processer.

Considering that our following experiments are largely dependent on the information of UWB ranging and DR, we provid a test result of their accuracy in Fig. 4 for preliminary evaluation, where one robot moves in a straight line, and displaces 9m in 15 frame (seconds). As we see, while it is true that DR can determines each robot's location, it may not be accurate due to accumulative error, and the error approaches to 30cm at the last frame. Besides, we see that UWB module has a measurement error around $20\mathrm{cm}$ Within the $9\mathrm{m}$ range.

B. Effectiveness of Initialization

To show the effectiveness of our initialization strategy, we designed an experiment that two robots move according to the predefined trajectories (see Fig. 5, the ground true are provided as a benchmark.), and the trajectories and distance measurements were generated as follows: i) the two robots start at initial positions 2m apart from each other, robot 2 is located at the vertical direction of robot 1, i.e., $\theta = 90^o$. Besides, the two robots has a relative orientation $\phi = -14.6^o$; ii) each robot moves with the velocity of $0.6\mathrm{m/s}$; and iii) the robots record DR and UWB ranging information at their new positions. Steps ii) and iii) are repeated until 15 distance measurements are collected.

In our algorithm, seven distance measurements are capable to get a unique solution of initial pose. To evaluate the influence of N, $N = 7, ..., 13$ measurements are all tested in our algorithm. As we see in Fig. 7, we compared different $\mathbf{N}$ versus estimation result of θ and ϕ. When $N = 7$, it has the capability to provide an accurate estimation of initial relative pose. Besides, we can see that there is a small improvement of the estimation result with the increase of the number of measurements before $N = 10$.

C. Low Computation Cost

In this subsection, the main advantage of our consensus-based DPF, i.e., computation sharing, is highlighted by a three robot RL experiment. As shown in Fig. 8, we add another robot on the basis of the experiment in the last subsection, where the third robot moves from a position of $\mathbf{p} = [9, 2]$ position with respect to robot 1 and $\phi = 180^o$. Note that the third robot also moves with the velocity of $0.6\mathrm{m/s}$.

With respect to the initialization step, we choose $N = 9$ to ensure a good estimation of initial relative pose. For the consensus-based DPF, we choose the number of particles

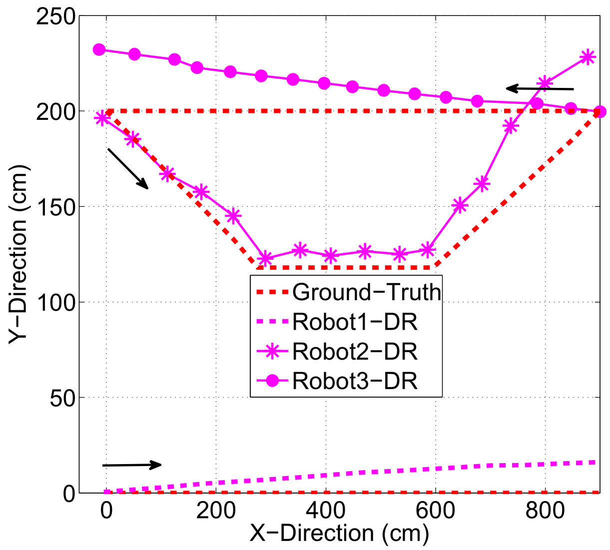

Fig. 7. Three mobile robots RL scenario: another robot is added on the basis of the experiment in the last subsection, where the third robot moves from a position of $\mathbf{p} = [9, 2]$ position with respect to robot 1 and $\phi = 180^o$.

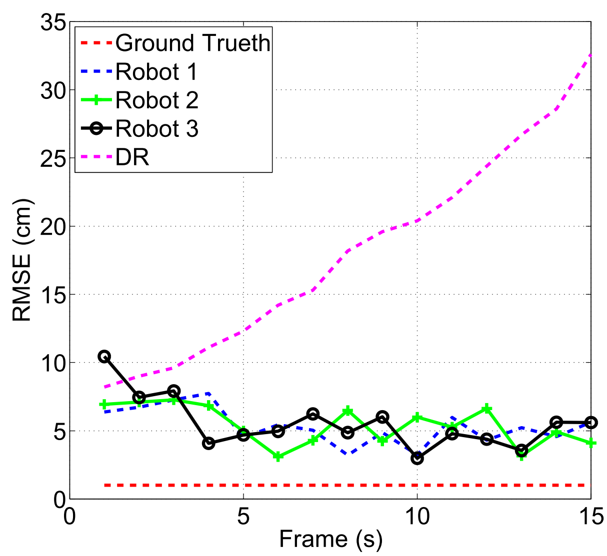

Fig. 8. The RL result of robot 1.

(PNS) $Q = 200$, and we execute consensus process only one time in this experiment, i.e., $S = 1$. The estimation accuracy is evaluated by

$$\text{Error} = \sqrt{(x_{i,k} - \hat{x}_{i,k})^2 + (y_{i,k} - \hat{y}_{i,k})^2},$$

where $[\hat{x}_{i,k}, \hat{y}_{i,k}]$ denotes the estimation result of relative position.

In Fig. 8, the RL result of robot 1 is provided. As we see, the RL error curve of each robot is much lower than the DR line, which means that there is an improvement of RL accuracy by using our method. Besiedes, the RL error line of each robot also approaches to the benchmark line

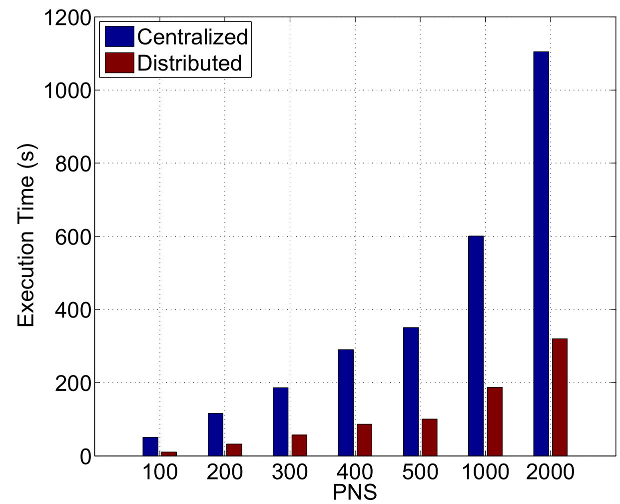

Fig. 9. The comparison of execution time of the posed consensus-based DPF and centralized approach with different PNS.

(Ground truth), which further demonstrates the effectiveness of our method. Note that, due to the estimation accuracy of the other two robots i and j are almost the same as robot 1, the RL error graph of robot 2 and 3 are not provided. Finally, we compared the execution time of each robot, which indicates the computation cost of each robot used to obtain the RL result of the other two robots. As shown in Fig. 9. It can be seen that our consensus-based DPF has much lower computational cost than the centralized approach with different PNS. Specifically, with regard to our 3 robots RL scenario, the execution time is exactly reduced about 3 times.

V. Conclusion

In this paper, combining DR and UWB ranging information to achieve RL is considered. Generally, a novel two-stage RL method was proposed to address two main problems, i.e., pose initialization and distributed implementation, in a practical multi-robot system. At the first stage, we use robot-to-robot measurements acquired at different vantage points during robot motion to determine initial pose, where we built a linear system to compute a coarse estimate for relative pose firstly, and then we employed an iterative WLS method for further refinement. Secondly, we developed a consensus-based DPF to fuse the data from local robot and neighbors to realize RL. Experiments results on a team of differentially driven mobile robots show initialization strategy is able to obtain an accurate estimation of relative pose and highlighted that the proposed approach has much lower computation cost than the centralized approach at each local robot.

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