Modeling and Analytical Analysis of the Effect of Atmospheric Temperature to the Planktonic Ecosystem in Oceans

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Modeling and Analytical Analysis of the Effect of Atmospheric Temperature to the Planktonic Ecosystem in Oceans

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Abstract. In marine ecosystem, plankton is considered as the primary food producer. The growth of plankton depends on the efficiency of saturation carbon dioxide, saturation oxygen, nutrition, temperature of the water, sunlight, saturated or unsaturated toxic chemical, plastic etc. But the growth of phytoplankton mostly depends on the photosynthetic activity of plankton. On the other hand, the photosynthetic activity varies with the different atmospheric temperature. In this study, we discuss the effect of atmospheric temperature to the plankton in marine ecosystem including the concentration of dissolved oxygen. To investigate the effect of atmospheric temperature, we formulate a mathematical model consists of non-linear ordinary differential equations considering four dynamical variables as the amount of atmospheric temperature, the density of phytoplankton, the density of zooplankton and the concentration of dissolved oxygen. After testing the positivity, stability analysis has been performed at different critical points of the proposed model. From numerical simulation, approximate solution of every dynamical species has been found.

Keywords: Atmospheric temperature, Photosynthetic activity, Plankton.

1 Introduction

Total energy of the marine ecosystem in the ocean is supplied by the plankton population and the density of plankton greatly depends on the photosynthesis. Photosynthetic activity of plankton is a chemical reaction which mostly interact with temperature [1]. Generally, it starts to increase with the increase of temperature (up to 77°F or 25°C) and starts to decrease with high temperature (from 25°C up to 40°C). The photosynthetic activity is performed with very small rate above 40°C temperature and at a time it will be stopped. Generally, the photosynthesis runs well from 22°C to 28°C. Therefore, this temperature is considered as perfect temperature and 25°C is considered as the optimum temperature for photosynthetic activity of phytoplankton [2].

Schabhuttl et al. [3] discussed the statistical analysis of the combined effect of temperature and diversity on phytoplankton's growth considering 15 species of freshwater phytoplankton. Striebel et al. [4] discussed the statistical analysis of abiotic and biotic differences in shaping the response to two aspects of temperature change: permanent increase of mean temperature versus pulse disturbance in form of a heat wave. Destania et al. [5] and Khare et al. [6] described the effect of nutrients on plankton through mathematical modelling. Sekerci and Petrovskii [7] described the effect after climate change on plankton-oxygen dynamics. Promrak and Rattanakul [8] described the effect of increasing global temperature on the green lacewings and the life cycles of mealybugs. Besides some papers [9, 10] described the analytical analysis of the effect of temperature on phytoplankton.

In this study, we proposed a nonlinear mathematical modeling to describe the effect of atmospheric temperature on plankton in marine ecosystem including concentration of dissolved oxygen. To formulate the model, four dynamical species are considered and other interactions are neglected in this model.

We can easily find out the acuity of photosynthesis of phytoplankton in the ocean with respect to the depth of water from the proposed model. It also helps to acquire knowledge about the relationship between atmospheric temperature and depth from the water surface level, and the corresponding results to the growth of plankton.

2 Model Formulation

To formulate the model of the effect of atmospheric temperature to the planktonic of the marine ecosystem, a system of nonlinear differential equations consists of four dynamical species considering the atmospheric temperature (T), the density of Phytoplankton (P), the density of Zooplankton (Z), the concentration of dissolved Oxygen (D). The interrelationship among them can be represented through a diagram as

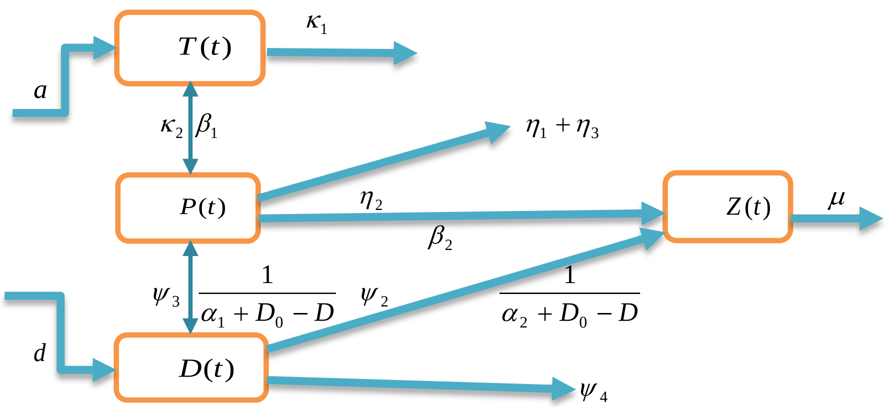

Fig. 1: The schematic diagram of the system representing the interaction among the considered species in marine ecosystem.

Fig. 1 shows that temperature helps phytoplankton to produce food and phytoplankton serves the energy and oxygen to zooplankton. Thus they make a balancing marine ecosystem among themselves.

From the above discussion and according to Fig. 1, the proposed four species ecosystem can be represented by a system of non-linear ordinary differential equations as:

$$\frac{dT}{dt} = a - \kappa_1 T - \kappa_2 PT \tag{1}$$

$$\frac{dP}{dt} = \frac{\beta_1 TP}{\alpha_1 + D_0 - D} - \eta_1 P - \eta_2 PZ - \eta_3 P \tag{2}$$

$$\frac{dZ}{dt} = \frac{\beta_2 PZ}{\alpha_2 + D_0 - D} - \mu Z \tag{3}$$

$$\frac{dD}{dt} = d + \psi_1 PT - \psi_2 DZ - \psi_3 DP - \psi_4 D \tag{4}$$

with initial conditions $T(0) > 0,\ P(0) \geq 0,\ Z(0) \geq 0,\ D(0) \geq 0$

The brief description of the parameters used in the model are shown in the following table.

Table 1. The brief description of the parameters in the model are as follows:

Symbol	Meaning	Values
a	Atmospheric temperature of the earth	$25\ ^0C$
κ_1	Rate of system loss	$0.78\ K\,l^{-1}$
κ_2	Absorbing rate of temperature for photosynthesis	$0.300\ K\,l^{-1}$
β_1	Proportional constant	$0.50\ day^{-1}$
α_1	Saturation constant	$0.51\ mg\ l^{-1}$
η_1	Natural death rate of phytoplankton	$0.009\ day^{-1}$
η_2	Predation rate of zooplankton	$0.41\ l\ mg^{-1}\ day^{-1}$
η_3	Density of water (muddy and dirty)	$0.01\ mg^{-1}\ l^{-1}$
β_2	Proportional constant	$0.33\ day^{-1}$
α_2	Saturation constant	$0.41\ mg\ l^{-1}$
μ	Natural death rate of zooplankton	$0.01\ day^{-1}$
d	Concentration of dissolved oxygen enters into the system	$24\ mg\ l^{-1}\ day^{-1}$
ψ_1	Producing rate of O_2 by photosynthetic activity	$0.652\ mg\ l^{-1}\ day^{-1}$
ψ_2	Absorbing rate of O_2 by zooplankton for breathing	$0.02\ mg\ l^{-1}\ day^{-1}$
ψ_3	Absorbing rate of O_2 by phytoplankton for respiration	$0.025\ day^{-1}$
ψ_4	Natural depleting rate	$3\ day^{-1}$
D_0	Saturation value of dissolved oxygen	$30\ mg\ l^{-1}$

3 Analytical Analysis

In the analytical section, we perform the positivity test of the dynamical variables, stability analysis at equilibrium points and numerical simulation [11, 12].

3.1 Boundedness of the System

Now we establish that the system is bounded by using the following lemma.

Lemma 1: The set $\Phi = \left\{ (T, P, Z) \in \Re_4^+ : 0 \le T + P + Z \le \frac{a}{\delta_n}, D \le \frac{d}{\psi_4} \right\}$ is a region of attraction for each solution and initially all the variables are positive, and where δ_n = Min $\{\kappa_1, (\eta_1 + \eta_3), \mu\}$.

Proof: Let's consider a function $\dot{x}(t) = f(x,t)$, where $x(t) = (T(t), P(t), Z(t))$. If δ_n = Min $\{\kappa_1, (\eta_1 + \eta_3), \mu\}$, then we obtain the following inequality:

$$\frac{dx(t)}{dt} + \delta_n\, x(t) \le a$$

Applying the differential inequalities, we have $0 \le x(t) \le \frac{a}{\delta_n}$. Similarly from equation (4), we get $0 \le D(t) \le \frac{B}{c_1}$, where $c_1 = \psi_2 Z + \psi_3 P + \psi_4$ and $B = d + \psi_1 PT$.

Hence the solution of the system is bounded in Φ .

3.2 Equilibrium Points

We obtain three equilibrium points of the system (1-4) by setting $\frac{dT}{dt} = 0$, $\frac{dP}{dt} = 0$, $\frac{dZ}{dt} = 0$ and $\frac{dD}{dt} = 0$. The equilibrium points are

(i) $E_1(\overline{T}, 0, 0, \overline{D})$, (ii) $E_2(\overline{T}, \overline{P}, 0, \overline{D})$ and (iii) $E_3(\overline{T}, \overline{P}, \overline{Z}, \overline{D})$

3.3 Stability Analysis

The system of equations (1)-(4) can be represented into Jacobian matrix as

$$J_i = \begin{bmatrix} -\kappa_1 - \kappa_2 P & -\kappa_2 T & 0 & 0 \\ \frac{\beta_1 P}{\alpha_1 + D_0 - D} & \frac{\beta_1 T}{\alpha_1 + D_0 - D} - \eta_1 - \eta_2 Z - \eta_3 & -\eta_2 P & \frac{\beta_1 TP}{(\alpha_1 + D_0 - D)^2} \\ 0 & \frac{\beta_2 Z}{\alpha_2 + D_0 - D} & \frac{\beta_2 P}{\alpha_2 + D_0 - D} - \mu & \frac{\beta_2 PZ}{(\alpha_2 + D_0 - D)^2} \\ \psi_1 P & \psi_1 T - \psi_3 D & -\psi_2 D & -\psi_2 D - \psi_3 P - \psi_4 \end{bmatrix} \tag{5}$$

where $i = 1, 2, 3$

Stability Analysis at E_1. After solving the characteristic equation of (5) at E_1, we get four eigen values as

$$\lambda_1 = -\kappa_1,\ \ \lambda_2 = \frac{a\beta_1 / \kappa_1}{\alpha_1 + D_0 - \dfrac{d}{\psi_4}} - \eta_1 - \eta_3,\ \ \lambda_3 = -\mu,\ \ \lambda_4 = -\psi_4$$

Among the four eigen values, three of them are negative and one of them may be negative or positive. Then the equilibrium point E_1 will be stable if $\lambda_2 < 0$.

Stability Analysis at E_2. After solving the characteristic equation of (5) at E_2, we get four eigen values as

$$\lambda_1 = -\kappa_1 - \kappa_2 \overline{P},\ \ \lambda_2 = \frac{\beta_1 \overline{T}}{\alpha_1 + D_0 - \overline{D}} - \eta_1 - \eta_3,\ \ \lambda_3 = -\psi_3 \overline{P} - \psi_4,$$

$$\lambda_4 = \frac{\beta_2 \overline{P}}{\alpha_2 + D_0 - \overline{D}} - \mu$$

Where two of them are negative and two of them may be negative or positive. If they are negative, E_2 will be stable, else E_2 will be unstable saddle point.

Stability Analysis at E_3. After solving the characteristic equation of (5) at E_3, we get four eigen values as

$$\lambda_1 = -\kappa_1 - \kappa_2 \overline{P},\ \ \lambda_2 = \frac{\beta_1 \kappa_1 \overline{T}}{(\alpha_1 + D_0 - \overline{D})(\kappa_1 + \kappa_2 \overline{P})},\ \ \lambda_3 = \frac{\beta_2 \overline{P}}{\alpha_2 + D_0 - \overline{D}} - \mu,$$

$$\lambda_4 = -\psi_2 \overline{Z} - \psi_3 \overline{P} - \psi_4$$

Where two of them are negative and two of them may be negative or positive. If they are negative, E_3 will be stable and if they are positive, E_3 will be unstable saddle point.

3.4 Numerical Simulations

Graphical representation through numerical simulation is the most useful task to represent the interactions among the dynamical variables. Here to check the feasibility of our analysis concerning stability axioms, we use Maple coding. Some numerical computations have been driven by using these coding choosing a set of parameters shown in the Table 1. The conditions for the existence of interior equilibrium E_3 are satisfied under these parametric values and the numerical solutions for each dynamical species are obtained at temperature 25 ^{0}C (shown in Fig. 3) given as

$$\overline{T} = 11.53,\ \ \overline{P} = 0.851,\ \ \overline{Z} = 1.379,\ \ \overline{D} = 9.547$$

Fig.2 represents the effect of lower temperature (18^{0}C) to the photosynthesis and corresponding effect to zooplankton and oxygen. At temperature 18^{0}C, the rate of photosynthetic activity of phytoplankton is not optimum and so grows on. On the other hand, the growth rate of zooplankton and oxygen is increasing because of the increasing rate of phytoplankton. Fig. 2(a) shows that the atmospheric temperature decreases proportionally with the depth of ocean measured from water surface layer.

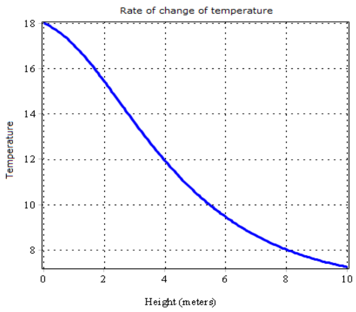

Fig. 2(a)

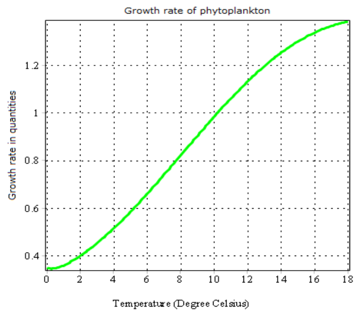

Fig. 2(b)

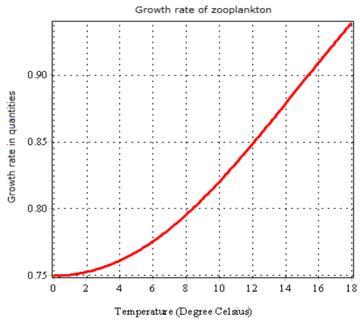

Fig. 2(c)

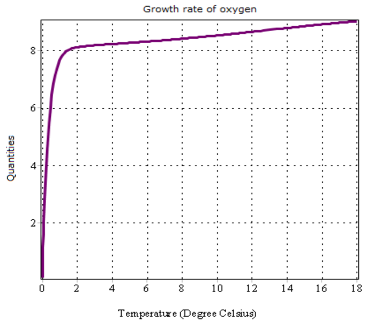

Fig. 2(d)

Fig. 2: Effect of temperature on planktonic ecosystem under $a = 18\ ^0C$, $\kappa_1 = 0.72\ K\ l^{-1}$ and $\kappa_2 = 0.250\ K\,l^{-1}$

When the temperature reaches to the optimum temperature ($25\ ^0C$), the rate of photosynthetic activity is maximized shown in Fig. 3(b). So at that temperature, the rate of photosynthesis remains constant with time. As a result, the growth of zooplankton and oxygen become maximized with highest growth rate shown in Fig. 3(c) and Fig. 3(d) respectively. We notice that the absorbing rate of temperature by phytoplankton and system loss of temperature are proportionally changed with the atmospheric temperature.

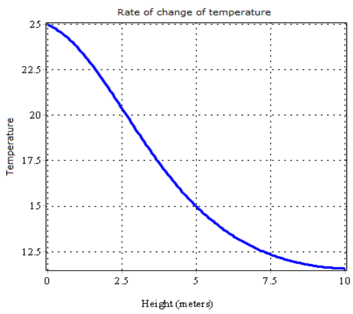

Fig. 3(a)

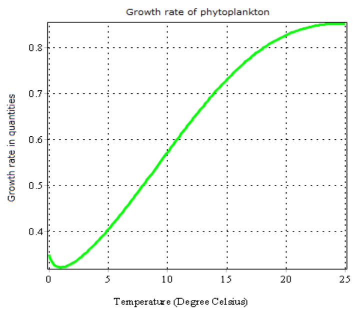

Fig. 3(b)

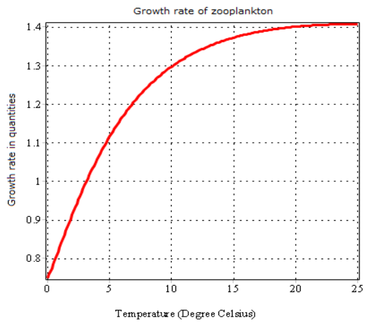

Fig. 3(c)

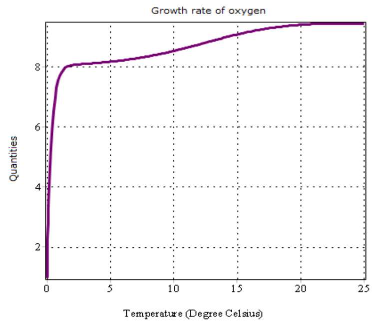

Fig. 3(d)

Fig. 3: Effect of temperature on planktonic ecosystem under $a = 25\ ^0C$, $\kappa_1 = 0.78\ K\ l^{-1}$ and $\kappa_2 = 0.300\ K\,l^{-1}$

Fig. 4 shows that the effect of over optimal temperature to the system. When the temperature crosses the optimal state, the photosynthesis starts to decrease. With the decreasing rate of phytoplankton, the growth rate of zooplankton and oxygen will be decreased proportionally shown in Fig. 4(c) and Fig. 4(d).

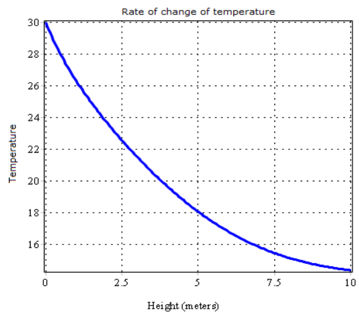

Fig. 4(a)

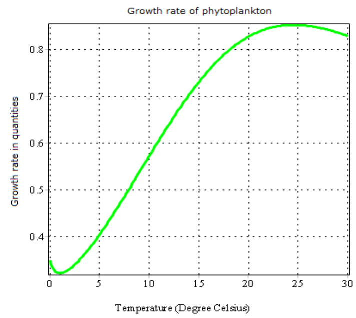

Fig. 4(b)

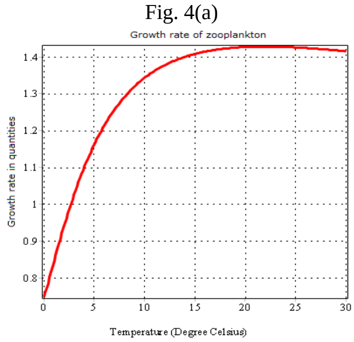

Fig. 4(c)

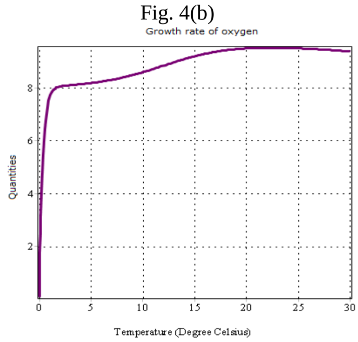

Fig. 4(d)

Fig. 4: Effect of temperature on planktonic ecosystem under $a = 30\ ^0C$, $\kappa_1 = 0.83\ K\ l^{-1}$ and $\kappa_2 = 0.320\ K\,l^{-1}$

Fig. 2, Fig. 3 and Fig. 4 represent that the ecosystem enriches gradually until the optimum temperature comes, and the system is optimum at the optimum temperature, and the system starts to decline for the high temperature (above optimal temperature).

4 Conclusions

In this study, a nonlinear mathematical model has been propounded and analyzed for the effect of temperature on marine planktonic ecosystem. The model exhibits three equilibrium points where all the critical points will be stable under some conditions. We compute numerical simulation at the optimum temperature and a comparison has been shown in this section. The growth rate of phytoplankton at 25^0C is higher than any growth rate at any temperature. When the growth rate of phytoplankton increases, the growth rate of zooplankton rises, and consequently the production rate of oxygen arises. Thus all the dynamical species reach to a stable relationship.

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