A Robust Control Strategy for Air Conditioner Group to Participate in Power System Frequency Regulation

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Abstract: With the increasing penetration of renewable energy, the stability of power system face the challenge. Controlling the air conditioning loads based on the demand response strategy would improve the stability of power system and the renewable energy absorption ability. Firstly, the transfer function model of aggregated air conditioner group is developed in this paper. Then the \( H_\infty \) robust controller is designed based on the theory of linear matrix inequalities to improve the robustness of the power system. Finally, a comprehensive case study of frequency response model considering DR is performed to verify the effectiveness of the \( H_\infty \) robust controller.

Index Terms—Air conditioning loads; demand response; frequency regulation; \( H_\infty \) robust control

I. INTRODUCTION

With the continuous consumption of fossil energy, the energy crisis and environmental pollution would become increasingly serious. The development of renewable energy has attracted more attention. However, due to the characteristics of intermittence, the renewable energy power generation system can’t output stable active power, which poses a threat to the stability of the power system. With the development of smart grid, the importance of user-side resources in the frequency regulation process has gradually gained attention.

The demand response (DR) provides a feasible solution for the renewable energy connected to the grid. The electrical equipment that participates in the demand response mainly includes water heaters, refrigerators, air conditioners, and these devices have low requirement for power supply continuity. Closing the equipment in a short time would not affect the users. When the frequency of the power system fluctuates, the load aggregator would change the working state of the participating demand response load according to the dispatching instruction, and the frequency fluctuation of the power system would be suppressed.

There are some research reports at home and abroad on using demand response resources for frequency-assisted adjustment. Reference 0 proposed a load frequency modulation model between the group load temperature setting value and the system frequency linear response. Reference 0 studied the power system load dispatching using AC load, and proposed a two-layer optimal scheduling model of AC loads based on direct load control.

The above literatures mainly focus on how to control the air conditioners, and do not study the equivalent thermal parameter model of the air conditioner group. Reference 0 proposed a collaborative control strategy of generator-air conditioner group load participating in power system frequency regulation based on the function of frequency regulation of thermostatically controlled loads. Reference 0 studied the transfer function model of an AC group, and proposed a closed-loop PID controller for AC load to stabilize power system frequency. However, the PID controller has poor anti-interference ability when random disturbance occurs.

Robust \( H_\infty \) control theory provides an effective solution for random disturbance, and this method can effectively suppress power system frequency fluctuations caused by random disturbance. This paper comprehensively studies the situation that the AC group participates in the power system frequency regulation, and establishes the equivalent thermal parameter model of the air conditioner group. And in this paper, a robust \( H_\infty \) controller is designed based on the linear matrix inequality (LMI) theory, and it can improve the robustness of the system. Section 2 establishes the second-order transfer function model of AC group. Section 3 proposes the state space equations of frequency response model. Section 4 studies how to design \( H_\infty \) robust controllers based on LMI. Section 5 verifies the effectiveness of \( H_\infty \) robust controllers. Section 6 is the summary of this paper.

II. TRANSFER FUNCTION MODEL OF AIR CONDITIONERS

A. An Equivalent thermal parameters model of AC

The involved air conditioning loads in the demand response would be periodically turned on and off. This paper mainly investigates the air conditioner in the cooling state, and the working state \( S_i(t) \) and \( dT_i(t)/dt \) can be written as following 0:

\[
S_i(t) = \begin{cases} 1 & \text{if the AC is on} \\ 0 & \text{otherwise} \end{cases}
\]

\[
dT_i(t)/dt = \begin{cases} -K_i & \text{if the AC is on} \\ 0 & \text{otherwise} \end{cases}
\]
\[ S_i(t^*) = \begin{cases} 0 & T(t) \leq T_{\text{min}} + u(t) \\ 1 & T(t) \geq T_{\text{max}} + u(t) \\ S_i(t) & \text{otherwise} \end{cases} \]  

(1)

\[
\frac{dT(t)}{dt} = -\frac{1}{C_i R_i} \left[ T_i(t) - T_e + S_i(t) R_i Q_i \right]
\]

(2)

When \( S_i(t) = 1 \), the air conditioner is on; when \( S_i(t) = 0 \), the air conditioner is off, \( u(t) \) represents the deviation of the temperature set point.

According to the working conditions of every single air conditioner, the percentage of working AC in the group can be written as following:

\[
D_{w}(t) = \sum_{i=1}^{n} S_i(t) \frac{Q_i}{\eta_i} / \sum_{i=1}^{n} \eta_i \]

(3)

wherein, the numerator of \( D_{w}(t) \) represents the total power of the working air conditioners, and the denominator represents the total power of the air conditioners; \( n \) represents the number of ACs that participate in demand response; \( \eta_i \) represents the cooling energy efficiency ratio; \( P_i \) represents the electric power consumed per unit time.

The above analysis is mainly for the thermodynamic model of air conditioner. Each air-conditioned room can be regarded as a dynamic, non-linear, independent system. In the actual control process, on the one hand, it is necessary to face the dimensional disaster problem that may be caused by a large number of air conditioners; on the other hand, the model includes both the continuous signal (temperature signal) and the discrete signal (the switching signal of the air conditioner). Therefore, the next section would study the air conditioner aggregation model and obtain a simplified air conditioner group transfer function model.

B. Transfer function model of air conditioner group

In the actual control process, each air-conditioned room is a nonlinear independent system, so it is difficult to get an accurate mathematical model. Therefore, it is necessary to simplify the air conditioner group.

In order to obtain a simplified mathematical model, we first make the following assumptions:\(^5\): 1) At the initial moment, the indoor temperature of the air conditioner group is evenly distributed between the upper and lower temperature set point; 2) the air conditioners participating in the demand response have the same temperature set point; 3) the equivalent heat capacity \( C_e \) of the air conditioners obeys a lognormal distribution with a standard deviation \( \sigma_c \) and an average value \( \mu_c \).

From the above assumptions, the relationship between the equivalent thermal resistance \( R_e \) of the air conditioner and the rated cold power \( Q_i \) can be derived as following:

\[
R_e Q_i = \frac{T_e - T_{\text{ref}}}{d}
\]

(4)

where \( T_{\text{ref}} = (T_{\text{max}} + T_{\text{min}}) / 2 \), \( d \) is the duty cycle of air conditioner in steady state. In this paper, the duty cycle is 0.5. Substituting the formula (4) into the formula (2) can get the expression of the indoor temperature change rate (5):

\[
\frac{dT_i}{dt} = \frac{T_e - T_{\text{ref}}}{C_i R_i} = v_i
\]

(5)

\( v_i \) indicates the rate of the indoor temperature change. In order to get the working state of the air conditioner at any time, the intermediate variable \( x_i(t) \) is introduced, and its expression is as following:

\[
x_i(t) = x_i^0 + v_i t
\]

(6)

where

\[
x_i^0 = \begin{cases} 1 + T_i(0) - T_{\text{min}} & \frac{dT_i(0)}{dt} > 0 \\ T_{\text{max}} - T_i(0) & \frac{dT_i(0)}{dt} < 0 \end{cases}
\]

(7)

It can be obtained from (6) and (7) that when \( x_i(t) \in [2k-1, 2k] \), the air conditioner is in the off state; when \( x_i(t) \in [2k, 2k+1] \), the air conditioner is turned on. For a population of \( n \) ACs, the ratio \( D_i(t) \) of air conditioners being worked at any time can be written by (8):\(^5\):

\[
D_i(t) = \frac{Pr[x_i(t) < 1]}{3} + \sum_{k=1}^{n} [Pr[x_i(t) < 2k+1] - Pr[x_i(t) < 2k]]
\]

(8)

\( Pr \) is the probability operator.

Since every AC has the same power, so when the power of a single air conditioner and the number of air conditioners in the AC group are known, the total power provided by the air conditioner group can be calculated.

Assuming that at the initial moment, the temperature set value is increased by 0.5°C, the response of the air conditioner group can be written as following:

\[
D_i(t) = D_{w} (T_{\text{ref}}) + \mathcal{L}^{-1}(G_{i} (s) * 0.5 / s)
\]

(9)

In formula (9), the first term indicates the power demand response of the AC group at the temperature set point; the second term indicates the aggregated demand response of a 0.5°C rise in temperature set point. The general form of the power demand response of AC group is as following, and \( T_r = T_{\text{max}} - T_{\text{min}} \):

\[
D_{w}(T) = (1 + \frac{\log(1 + \frac{T_r - T}{T_{\text{ref}} - T_e / 2})}{\log(1 + \frac{PR - T_{\text{ref}} + T_e / 2}{T_{\text{ref}}})})^{-1}
\]

(10)

By analyzing the relationship between \( D_i(t) \) and temperature, the second-order transfer function of the AC group can be derived as following: \(^5\):
\[ G_p(s) = \frac{b_2 s^2 + b_1 s + b_0}{d_1 s^3 + 2 \xi \omega_n s + \omega_n^2} \]  

(11)

III. FREQUENCY RESPONSE STATE SPACE MODEL WITH AIR CONDITIONING LOAD

In the power system, the system model with AC group participating in frequency modulation is as following:

\[
\begin{align*}
\dot{x} &= Ax + B_i w + B_u u \\
y &= C x
\end{align*}
\]

where \( u \) is the control input, \( w \) is the interference signal(\( \Delta P_d \)), and \( A, B_1, B_2, \) and \( C \) are the coefficient matrices, and the expressions are as following:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Delta f + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Delta P_d \\
y &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T x
\end{align*}
\]

\[
A = \begin{bmatrix} 0 & 0 & 0 & -K_1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -w_1^2 & d_1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \end{bmatrix} \]

\[
B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix} \]

\[
C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}
\]

IV. ROBUST CONTROLLER DESIGN BASED ON LMI

When the output power of renewable energy power generation system is fluctuating, the frequency stability of the power system would be affected. So, the corresponding controller must be designed to ensure the stability of the power system frequency. In this paper, a robust \( H_\infty \) controller is designed based on LMI theory, which was first proposed by Canadian professor Zames of McGill University in 1981. This theory is based on rigorous mathematical derivation. And the designed controller based on LMI theory has achieved good control effect in practical applications.

According to the feedback signal, the \( H_\infty \) control can be divided into \( H_\infty \) state feedback control and \( H_\infty \) output feedback control. \( H_\infty \) state feedback control needs to feed back all the state variables in the system to the controller. And \( H_\infty \) output feedback control only needs to feed back the output signal of the system to the controller, but the design principle of \( H_\infty \) output feedback control is more complicated. In this paper, we will design these two \( H_\infty \) controller and compare the control effects of the two.

A. \( H_\infty \) output feedback controller

\( H_\infty \) output feedback controller is designed in this section and its state space equation can be expressed as following:

\[
\begin{align*}
\dot{\xi} &= A_{K} \xi + B_{K} y \\
u &= C_{K} \xi + D_{K} y
\end{align*}
\]

In the state space equation, \( \xi \) is the state variable of the \( H_\infty \) output feedback controller, and \( A_{K}, B_{K}, C_{K}, D_{K} \) are the parameter matrices of the \( H_\infty \) controller to be sought. Substituting the controller into the state equation of the frequency response model, the whole state equation of the closed-loop system can be derived as following:

\[
\begin{align*}
\dot{\zeta} &= A_{C1} \zeta + B_{C1} w \\
u &= C_{C1} \zeta + D_{C1} w
\end{align*}
\]

\( \zeta \) contains both the state variables in the frequency response model and the state variables in the controller. \( A_{C1}, B_{C1}, C_{C1}, \) and \( D_{C1} \) are determined by the coefficient matrices of the controller and the coefficient matrices of the system. For the
specific case in this paper, the expressions of state variable and coefficient matrices are as following:

\[
\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} = \begin{bmatrix}
A_2 + B_1 B_2 C_2 \\
B_2 C_2
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
B_3 \\
C_3
\end{bmatrix} c_3 = [C \ 0]; \ D_3 = [0]
\]

The task of solving the \( H_\infty \) output feedback controller is to solve the four coefficient matrices in (14). According to the correlation law of \( H_\infty \) norm, the design theorem of \( H_\infty \) controller can be obtained. 0.

**Theorem 1** For the system (12), the controller (13) is one of its \( H_\infty \) output feedback controllers. The controller should make the closed-loop system (14) progressively stable, and the necessary and sufficient condition for the \( H_\infty \) norm from the interference signal \( w \) to the measured output \( y \) to be less than 1 is that there is a symmetric positive definite matrix \( X \) that makes the following matrix inequality established.

\[
\begin{bmatrix}
A_1^T X_c + X_c A_1 & X_c B_{c1} \\
B_{c1}^T X_c & -I
\end{bmatrix} +
\begin{bmatrix}
C_{c1}^T \\
D_{c1}^T
\end{bmatrix} < 0
\]

By solving the matrix inequality, the parameter matrixes of the \( H_\infty \) controller and \( X \) can be obtained.

**B. \( H_\infty \) state feedback controller**

Assuming that the state variables in the system are measurable, designing a \( H_\infty \) state feedback controller makes the closed-loop system (14) progressively stable, and its expression is as following:

\[
u(t) = K x(t)
\]

where \( K \) is the required controller. State space equation of the closed-loop system after adding the controller is as following:

\[
\begin{align*}
\dot{x} &= (A + B_1 K)x + B_1 w \\
y &= C x
\end{align*}
\]

**Theorem 2** For the system (12) in which the state variables can be measured, there is a \( H_\infty \) state feedback controller and its necessary and sufficient condition is that there exists a matrix \( W \) and a symmetric positive definite matrix \( X \) that make the following matrix inequality established.

\[
\begin{bmatrix}
AX + B_1 W + (AX + B_2 W)^T & B_1 X C^T \\
B_1^T & -I & 0 \\
CX & 0 & -I
\end{bmatrix} < 0
\]

Once established, there is a \( H_\infty \) state feedback controller \( K = WX^{-1} \) that makes the closed loop system progressively stable.

**V. SIMULATION AND ANALYSIS**

Based on the linear inequality described by (15) and (18), the corresponding controller can be obtained by solving the linear matrix inequality. The results of \( H_\infty \) output feedback controller \( K_1 \) and \( H_\infty \) state feedback controller \( K_2 \) are as following:

![Fig.2 Simulation results under sudden generation loss](image)

It can be seen from Fig. 2 that when \( \Delta P_s = 0.17 \) p.u., the maximum frequency deviation reaches -0.68 Hz without the participation of the controller. Comparing the three controllers, it can be found that frequency response model with \( H_\infty \) output feedback controller and \( H_\infty \) output feedback controller perform better with minimum generator output power, maximum output power of AC group, and highest utilization rate of AC load. When the generation loss happens, the utilization rate of AC load of the system without controller is very small, so its frequency deviation is pretty large.

Fig. 3 shows the relationship between sudden generation loss and the maximum value of the output frequency deviation. It can be seen from the figure that when the sudden generation
loss is equal, the \( H_\infty \) output feedback controller and \( H_\infty \) state feedback controller have the best control effect; the relationship between the maximum frequency drop and \( \Delta P_d \) is linear.

![Fig. 3 Maximum frequency drop under different magnitude of \( \Delta P_d \)](image)

Due to the variable nature of wind speed and sunshine intensity, the renewable energy generation system can’t output stable power. So each control method under variable generation loss should be tested. And the simulation results are shown in Fig.4.

![Fig.4 Simulation results under variable generation loss](image)

From Fig.4, it can be seen that when the variable generation loss are the same, \( H_\infty \) output feedback control method and \( H_\infty \) state feedback controller can well suppress the fluctuation of the frequency. Among the three methods, \( H_\infty \) output feedback control method results in minimum frequency error, minimum output power of generator, maximum output power of AC group.

VI. CONCLUSION

Aiming at the problem of frequency regulation after the fluctuating renewable energy is connected to the grid, this paper studies the method of using the energy storage characteristics of the AC load to participate in the system frequency regulation, and designs the corresponding controller. Through simulation analysis, the conclusions can be drawn as following:

1) A large number of ACs can participate in the system frequency control after being connected to the power system, and the dynamic frequency response of the system can be improved by designing corresponding controllers.

2) Comparing with the linear quadratic regulator, the \( H_\infty \) robust controller has better control effect, and it can greatly reduce the frequency deviation value.

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