Intuitionistic fuzzy soft linear spaces

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Abstract:
We define Intuitionistic fuzzy soft linear spaces (IFSLS) and its properties, characteristics are studied with examples. We introduce the Cartesian product of Intuitionistic fuzzy soft linear spaces and are illustrated by some examples. Besides that, we define Intuitionistic soft subspaces and given examples.

Keywords: Intuitionistic fuzzy soft linear space (IFSLS), Intuitionistic fuzzy soft subspace (IFSSS), intuitionistic fuzzy soft vector.

1. Introduction:
The soft set was defined by Molodtsov [2]. K.Attansov [3][4] introduces the intuitionistic fuzzy set. The concept of intuitionistic fuzzy soft sets was introduced by Maji et al [6]. Moumita Chiney and S. K. Samanta describe the concept of Intuitionistic fuzzy vector spaces [5]. A.Sezgin Sezer, A.O.Atagin introduced the concept of soft vector spaces [1].

In this paper, we introduce intuitionistic fuzzy soft linear spaces (IFSLS) and some of its properties and characteristics are studied with examples. We introduce the Cartesian product of Intuitionistic fuzzy soft linear spaces and are illustrated by some examples. Besides that, we define Intuitionistic soft subspaces and given examples.

2. Preliminaries:
Definition 2.1: A continuous t-norm is defined by a binary operation \(*: [0,1] \times [0,1] \to [0,1]\) if \(\ast\) satisfies the below mentioned properties:
(i) \(\alpha \ast \beta = \beta \ast \alpha\)
(ii) \((\alpha \ast \beta) \ast \gamma = \alpha \ast (\beta \ast \gamma)\)
(iii) \(\ast: [0,1] \times [0,1] \to [0,1]\) is continuous
(iv) \(\alpha \ast 1 = 1 \ast \alpha\)
(v) \(\alpha \ast \beta \leq \gamma \ast \delta\) if \(\alpha \leq \gamma, \beta \leq \delta\) for all \(\alpha, \beta, \gamma, \delta \in [0,1]\)

Some examples are \(\alpha \ast \beta = \alpha \beta, \alpha \ast \beta = \min\{\alpha, \beta\}, \alpha \ast \beta = \max\{\alpha + \beta, 1\}\).

Definition 2.2: A continuous t-co norm (s-norm) is defined by a binary operation \(\Delta: [0,1] \times [0,1] \to [0,1]\) if \(\Delta\) satisfies the below mentioned properties:
(i) \(a\Delta \beta = \beta \Delta a\)
(ii) \((a\Delta \beta) \Delta \gamma = a\Delta (\beta \Delta \gamma)\)
(iii) \(\Delta: [0,1] \times [0,1] \to [0,1]\) is continuous
(iv) \(\alpha \Delta 0 = 0 \Delta \alpha\)
(v) \(\alpha \Delta \beta \leq \gamma \Delta \delta\) if \(\alpha \leq \gamma, \beta \leq \delta\) for all \(\alpha, \beta, \gamma, \delta \in [0,1]\)
Some examples are \(\alpha \Delta \beta = \alpha + \beta - \alpha \beta, \ \alpha \Delta \beta = \max\{\alpha, \beta\}, \alpha \Delta \beta = \min\{\alpha + \beta, 1\}\).

If \(\alpha \ast \alpha = \alpha\) then \(\ast\) is called an idempotent t-norm and if \(\alpha \Delta \alpha = \alpha\) then \(\Delta\) is called an idempotent s-norm \(\forall \alpha \in [0,1]\).

**Definition 2.3:** Let \(x\) be an element in space of points (objects) \(X\). An intuitionistic fuzzy set in \(X\) is defined by a truth membership function \(T_A: X \rightarrow [0,1]\) and falsity membership function \(F_A: X \rightarrow [0,1]\) and \(0 \leq T_A(x) + F_A(x) \leq 1\).

**Definition 2.4:** If \(S\) is a mapping defined by \(S: B \rightarrow P(U)\) then a pair \((S, B)\) is called a soft set over \(U\) for \(B \subseteq E\). Here \(P(U)\) denotes the power set of \(U\), \(U\) denotes an initial universal set and \(E\) denotes a set of parameters.

**Definition 2.5:** If \(F\) is a mapping given by \(F: S \rightarrow IF(U)\) then a pair \((F, S)\) is called an intuitionistic fuzzy soft set over \(U\) for \(S \subseteq E\). Here \(U\) denotes an initial universal set and \(E\) denotes a set of parameters.

If \(F(s)\) is an Intuitionistic fuzzy set of \(U\) for every \(s \in S\) then it is called an Intuitionistic fuzzy value set of parameter \(s\). Then \(F(s)\) is an Intuitionistic fuzzy value set such that \(F(s) = \{<x, \mu_{F(s)}(x), \lambda_{F(s)}(x)> | x \in U\}\) where \(\mu_{F(s)}\) is a membership function and \(\lambda_{F(s)}\) is a non membership function. Then intuitionistic fuzzy soft class is defined as set of all intuitionistic fuzzy soft sets over \(U\) with parameters from \(E\) and is denoted by IFF(U,E).

**Example 2.6:**
Let \(U = \{v_1, v_2, v_3\}\) be a set of vehicles and \(E = \{e_1(\text{mileage}), e_2(\text{cheap}), e_3(\text{costly})\}\) be a set of parameters w.r.t which the nature of vehicles are described. Let
\[
\begin{align*}
  f(e_1) &= \{<v_1(0.5,0.3), v_2(0.4,0.6), v_3(0.6,0.3)>\} \\
  f(e_2) &= \{<v_1(0.6,0.3), v_2(0.7,0.3), v_3(0.8,0.2)>\} \\
  f(e_3) &= \{<v_1(0.7,0.3), v_2(0.6,0.2), v_3(0.5,0.2)>\}
\end{align*}
\]
Then \(I = \{<e, f_i(e_1)>, <e, f_i(e_2)>, <e, f_i(e_3)>\}\) is an intuitionistic fuzzy soft set over \((U, E)\).

**Definition 2.7:** The complement of IFSS \(I\) is denoted by \(I^c\) and is defined as \(I^c = \{e, <x, F_1(e), T_1(e)> | x \in U, e \in E\}\).

**Definition 2.8:** Let \(I_1\) and \(I_2\) be two IFS sets over \((U, E)\). Then \(I_1\) is said to be an intuitionistic fuzzy soft subset of \(I_2\) if \(T_{f_i(e_1)}(x) \leq T_{f_i(e_2)}(x), F_{f_i(e_1)}(x) \geq F_{f_i(e_2)}(x) \forall e \in E, x \in U\).

**Definition 2.9:** Let \(I_1\) and \(I_2\) be two IFS sets over common universe \((U, E)\). The union is denoted by \(I_1 \cup I_2 = I_3\) and it is defined as:
\[
I_3 = \{s, <x, T_{f_1(s)}(x), F_{f_1(s)}(x)> | x \in U, s \in E\}
\]
Where
\[
\begin{align*}
  T_{f_1(s)}(x) &= T_{f_1(s)}(x) \Delta T_{f_2(s)}(x) \\
  F_{f_1(s)}(x) &= F_{f_1(s)}(x) * F_{f_2(s)}(x)
\end{align*}
\]
Their intersection is defined by \(I_1 \cap I_2 = I_4\) and it is defined as
Case IV: If \( x = (0, 5) \) and \( y = (5, 1) \) then \( x + y = (0, 5) \).

**Definition 2.10:** Let \( I_1 \) and \( I_2 \) be two IFSS over \((U, E)\). Then ‘and’ operation is denoted by \( I_1 \wedge I_2 = I_5 \) and it is defined as

\[
I_5 = \{ (s, t) | <x, T_{f_1(s)}(x), F_{f_1(s)}(x)> / x \in U, (s, t) \in EXE \}
\]

Where

\[
T_{f_1(s)}(x) = T_{f_1(s)}(x) \ast T_{f_2(s)}(x)
\]

\[
F_{f_1(s)}(x) = F_{f_1(s)}(x) \Delta F_{f_2(s)}(x)
\]

**Definition 2.11:** An IFS set \( I \) over \((U, E)\) is said to be null IFS set denoted by \( \emptyset_I \) if

\[
T_{f_1(e)}(x) = 0, F_{f_1(e)}(x) = 1 \ \forall e \in E, x \in U.
\]

An IFS set \( I \) over \((U, E)\) is said to be absolute IFS set denoted by \( I_A \) if

\[
T_{f_1(e)}(x) = 1, F_{f_1(e)}(x) = 0 \ \forall e \in E, x \in U.
\]

3. Intuitionistic fuzzy soft linear spaces

**Definition 3.1:** An intuitionistic fuzzy set \( B = \{ <x, T_B(x), F_B(x) > / x \in V \} \) on a vector space \( V \ (K) \) is called a intuitionistic fuzzy sub vector space \( V \ (K) \) if

(i) \( T_B(x + y) \geq T_B(x) \ast T_B(y) \)
(ii) \( F_B(x + y) \leq F_B(x) \Delta F_B(y) \ \forall x, y \in V \)
(iii) \( T_B(\lambda x) \geq T_B(x) \)
(iv) \( F_B(\lambda x) \leq F_B(x) \ \forall x, y \in V, \lambda \in K \).  

An intuitionistic fuzzy soft set \( I \) on \( V \ (K) \) is called an intuitionistic fuzzy soft vector space/linear space (IFSLS) if \( f_1(e) \) is a intuitionistic fuzzy sub vector space on \( V \ (K) \) for all \( e \in E \).

**Example 3.2:** If \( E = \{ e_1, e_2, \ldots, e_n \} \) denotes the parametric set and \( R^n(R) \) be the \( n \)-dimensional Euclidean space. Let us define a mapping \( f_1 : E \rightarrow IFS(R^n) \) for any \( t \in R^n \) as following:

\[
T_{f_1(t)}(e) = \begin{cases} 
  \frac{1}{2}, & \text{if } i \text{ th coordinate of } t \text{ is zero } \\
  0, & \text{otherwise }
\end{cases}
\]

\[
F_{f_1(t)}(e) = \begin{cases} 
  0, & \text{if } i \text{ th coordinate of } t \text{ is zero } \\
  1/10, & \text{otherwise }
\end{cases}
\]

If \( a \ast b = \min\{a, b\}, a \Delta b = \max\{a, b\} \).

Then \( I \) forms an IFSS as well as IFSLS over \( R^n(R) \) w.r.t parametric set \( E \).

For convenience, we take an attempt for the parameter \( e \) and Euclidean space \( R^2(R) \).

Then the following four cases arise to choose \( x, y \in R^2 \).

Case I: If \( x = (0, 4) \) and \( y = (0, 2) \) then \( x + y = (0, 6) \).
Case II: If \( x = (0, 3) \) and \( y = (3, 2) \) then \( x + y = (3, 5) \).
Case III: If \( x = (1, 2) \) and \( y = (5, 1) \) then \( x + y = (6, 3) \).
Case IV: If \( x = (5, 1) \) and \( y = (-5, 4) \) then \( x + y = (0, 5) \).
From these four cases, the first and second set of conditions can be verified.

**Example 3.3:** Consider a real vector space $C=\{a+ib/a,b\in R,i=\sqrt{-1}\}$ and the parametric set $E=\{a,\beta,\gamma\}$. We divide the elements of $C$ into four cases e.g:

(C1) $\{ib/b\in R-\{0\}\}$ when real part is zero

(C2) $\{a/a\in R-\{0\}\}$ when imaginary part is zero

(C3) $\{a+ib/a,b\in R-\{0\}\}$ when both parts are non-zero

(C4) $\{0+i0\}$ the null vector

If $x\in C1$ and $y\in C2$ then $x+y\in C3$. We write $C1+C2=C3$.

We define IFSS $I$ over $(C,E)$ is given by $a*b=\max\{a+b-1,0\}$, $a\Delta b=\{a+b,1\}$. Then $I$ forms an IFSL over $(C(R),E)$.

From these four cases, the first and second set of conditions can be verified.

**Corollary 3.4:** Let $I$ be an IFSL over $(V(K),E)$. Then for $x\in V$ and $\lambda(\neq0)\in K$,

$T_f(x)\lambda x=F_f(x)\lambda x=F_f(x)$ hold.

**Proof:** $T_f(x)\lambda x=F_f(x)$

If $a*b=\min\{a,b\}$, $a\Delta b=\max\{a,b\}$ and $\theta$ is null vector of $V$.

**Proof:** (i) For $\lambda=-1$, the result directly follows from above corollary.

(ii) For the null vector $\theta\in V$

$T_f(x)\theta=F_f(x)\theta=F_f(x)$

$F_f(x)\theta=F_f(x)$

Hence proved.

**Proposition 3.5:**

Let $I$ be an IFSL over $(V(K),E)$. Then for each $v\in V$, following hold.

(i) $T_f(v)^-\lambda v=F_f(v)$

(ii) $T_f(v)\lambda v=F_f(v)$

if $a*b=\min\{a,b\}$, $a\Delta b=\max\{a,b\}$ and $\theta$ is null vector of $V$.

**Proof:** (i) For $\lambda=-1$, the result directly follows from above corollary.

(ii) For the null vector $\theta\in V$

$T_f(\theta)=F_f(\theta)$

$F_f(\theta)=F_f(\theta)$

Hence proved.

**Proposition 3.6:**

An IFSS $I$ on $(V,K)$ is called an IFSL with respect to the set $E$ if and only if the following equations hold

$T_f(\lambda u+\mu v)\geq T_f(u)*T_f(v)$

$F_f(\lambda u+\mu v)\leq F_f(u)\Delta F_f(v)$ $\forall u,v\in V,\lambda,\mu\in F, e\in E$

If $a*b=\min\{a,b\}$, $a\Delta b=\max\{a,b\}$

**Proof:** First suppose $I$ be an IFSL on $V(K)$ w.r.t.$E$.

Then $T_f(\lambda u+\mu v)\geq T_f(\lambda u)*T_f(\mu v)\geq T_f(u)*T_f(v)$

$F_f(\lambda u+\mu v)\leq F_f(\lambda u)\Delta F_f(\mu v)\leq F_f(u)\Delta F_f(v)$

Conversely by proposition 3.5,

$T_f(\lambda u)=T_f(\lambda u)\geq T_f(\lambda u)*T_f(\lambda u)\geq T_f(u)*T_f(v)$

$F_f(\lambda u)=F_f(\lambda u)\leq F_f(\lambda u)\Delta F_f(\lambda u)\leq F_f(u)\Delta F_f(v)$
\[ F_{f_{j(e)}}(u + v) = F_{f_{j(e)}}(u + (-1)(-v)) \leq F_{f_{j(e)}}(u)\Delta F_{f_{j(e)}}(-v) \leq F_{f_{j(e)}}(u)\Delta F_{f_{j(e)}}(v) \]

Hence the proof.

**Theorem 3.7:** Let \( I_1 \) and \( I_2 \) be two IFSLSs over \((V(K), E)\). Then \( I_1 \cap I_2 \) is also an IFSLS over \((V(K), E)\).

**Proof:** Let \( I_1 \cap I_2 = P \). Now for \( u, v \in V \)

\[ T_{f_{p(e)}}(u + v) = T_{f_{i_1(e)}}(u + v) * T_{f_{i_2(e)}}(u + v) \geq [T_{f_{i_1(e)}}(u) * T_{f_{i_1(e)}}(v)] * [T_{f_{i_2(e)}}(u) * T_{f_{i_2(e)}}(v)] \]

\[ = T_{f_{i_1(e)}}(u) * [T_{f_{i_1(e)}}(v)] * T_{f_{i_2(e)}}(u) \]

\[ = T_{f_{i_1(e)}}(u) * [T_{f_{i_2(e)}}(v)] * T_{f_{i_2(e)}}(u) \]

\[ = T_{f_{i_1(e)}}(u) * T_{f_{i_2(e)}}(u) * T_{f_{p(e)}}(v) \]

\[ = T_{f_{p(e)}}(u) \]

Hence \( T_{f_{p(e)}}(u + v) \geq T_{f_{p(e)}}(u) * T_{f_{p(e)}}(v) \)

Also, \( T_{f_{p(e)}}(\lambda u) = T_{f_{i_1(e)}}(\lambda u) * T_{f_{i_2(e)}}(\lambda u) \geq T_{f_{i_1(e)}}(u) * T_{f_{i_2(e)}}(u) = T_{f_{p(e)}}(u) \)

Thus, \( T_{f_{p(e)}}(\lambda u) \geq T_{f_{p(e)}}(u) \) for \( \lambda \in K \).

Similarly \( F_{f_{p(e)}}(u + v) \leq F_{f_{p(e)}}(u)\Delta F_{f_{p(e)}}(v) \) and \( F_{f_{p(e)}}(\lambda u) \geq F_{f_{p(e)}}(v) \) for \( \lambda \in K \).

Hence proved.

**Remark 3.8:** For two IFSLS \( I_1 \) and \( I_2 \) over \((V(K), E)\), \( I_1 \cup I_2 \) is not generally an IFSLS over \((V(K), E)\). It is possible if one is contained in another. For instance, let us consider two IFSLSs \( I_1 \) and \( I_2 \) over the real vector space \( V = \mathbb{R}^2 \) and the parametric set \( E = \{ e_i \mid i = 1, 2 \} \) as following:

\[ T_{f_{i_1(e_i)}}(x) = \begin{cases} 1/2 & \text{if } \text{ith coordinate of } x \in \mathbb{R}^2 \text{ is non zero only} \\ 0 & \text{otherwise} \end{cases} \]

\[ F_{f_{i_1(e_i)}}(x) = \begin{cases} 1/5 & \text{if } \text{ith coordinate of } x \in \mathbb{R}^3 \text{ is non zero only} \\ 1 & \text{otherwise} \end{cases} \]

\[ T_{f_{i_2(e_i)}}(x) = \begin{cases} 1/5 & \text{if } \text{ith coordinate of } x \in \mathbb{R}^2 \text{ is non zero only} \\ 1/10 & \text{otherwise} \end{cases} \]

\[ F_{f_{i_2(e_i)}}(x) = \begin{cases} 0 & \text{if } \text{ith coordinate of } x \in \mathbb{R}^2 \text{ is non zero only} \\ 1/5 & \text{otherwise} \end{cases} \]

If \( a \cdot b = \min\{a, b\} \) and \( a \Delta b = \max\{a, b\} \).

Let \( I_1 \cup I_2 = P \)

\[ T_{f_{p(e_i)}}(x + y) = T_{f_{i_1(e_i)}}(1,1) = \max\{0, \frac{1}{10}\} = \frac{1}{10} \]

\[ T_{f_{p(e_i)}}(x) * T_{f_{p(e_i)}}(y) = \{T_{f_{i_1(e_i)}}(x) \Delta T_{f_{i_1(e_i)}}(y)\} * \{T_{f_{i_1(e_i)}}(y) \Delta T_{f_{i_1(e_i)}}(y)\} \]

\[ = \min\{\max\{1/2, 1/10\}, \max\{0, 2/5\}\} = \min\{1/2, 2/5\} = 2/5 \]

Hence \( T_{f_{p(e_i)}}(x + y) < T_{f_{p(e_i)}}(x) * T_{f_{p(e_i)}}(y) \) i.e \( I_1 \cup I_2 \) is not an IFSLS here.

Now if define \( I \) over \((\mathbb{R}^3, E)\) as following
\[
T_{f_{I_1(e_1)}}(x) = \begin{cases} 
\frac{1}{6} & \text{if } i\text{ th coordinate of } x \in \mathbb{R}^2 \text{ is non zero only} \\
0 & \text{otherwise}
\end{cases}
\]
\[
F_{f_{I_1(e_1)}}(x) = \begin{cases} 
\frac{7}{10} & \text{if } i\text{ th coordinate of } x \in \mathbb{R}^2 \text{ is non zero only} \\
1 & \text{otherwise}
\end{cases}
\]

It can be easily verified that \( I_2 \subseteq I_1 \) and \( I_1 \cup I_2 \) is an IFSLS over \((\mathbb{R}^2(R), E)\).

**Theorem 3.9:** Let \( I_1 \) and \( I_2 \) be two IFSLSs over \((V(K), E)\). Then \( I_1 \cap I_2 \) is also an IFSLS over \((V(K), E)\).

**Proof:** Let \( I_1 \cap I_2 = Q \). Now for \( x, y \in V \) and \((s, t) \in EXE\),
\[
T_{f_{Q(s,t)}}(x + y) = T_{f_{I_1(s)}}(x + y) * T_{f_{I_2(t)}}(x + y) \geq [T_{f_{I_1(s)}}(x) * T_{f_{I_1(s)}}(y)] * [T_{f_{I_2(t)}}(x) * T_{f_{I_2(t)}}(y)] = T_{f_{I_1(s)}}(x) * T_{f_{I_2(t)}}(y) \]

Hence proved

4. **Cartesian product of Intuitionistic fuzzy soft linear spaces:**

**Definition 4.1:** Let \( I_1 \) and \( I_2 \) be two IFSLSs over \((V(K), E)\) and \((W(K), E)\) respectively. Then their Cartesian product is \( I_1 \times I_2 = C \) where \( f_C(s, t) = f_{I_1(s)}X f_{I_2(t)} \) for \((s, t) \in EXE\). Analytically
\[
f_C(s, t) = \{(x, y), T_{f_C(s, t)}(x, y), F_{f_C(s, t)}(y) >/(x, y) \in VXW\}
\]

With \( T_{f_C(s, t)}(x, y) = T_{f_{I_1(s)}}(x) * T_{f_{I_2(t)}}(y) \)
\[
F_{f_C(s, t)}(x, y) = F_{f_{I_1(s)}}(x) \Delta f_{f_{I_2(t)}}(x)
\]

This can be extended for more than two IFSLSs.

**Theorem 4.2:** Let \( I_1 \) and \( I_2 \) be two IFSLSs over \((V(K), E)\) and \((W(K), E)\) respectively. Then their Cartesian product \( I_1 \times I_2 \) is an IFSLS over \((VXW)(k), EXE)\).

**Proof:** Let \( I_1 \times I_2 = C \) where \( f_C(s, t) = f_{I_1(s)}X f_{I_2(t)} \) for \((s, t) \in EXE\)

Then for \((x_1, y_1), (x_2, y_2) \in VXW\)
\[
T_{f_C(s, t)}[(x_1, y_1) + (x_2, y_2)] = T_{f_C(s, t)}[(x_1 + x_2, y_1 + y_2)] = T_{f_{I_1(s)}}(x_1 + x_2) * T_{f_{I_2(t)}}(y_1 + y_2) \geq [T_{f_{I_1(s)}}(x_1) * T_{f_{I_1(s)}}(x_2)] * [T_{f_{I_2(t)}}(y_1) * T_{f_{I_2(t)}}(y_2)]
\]
\[
= [T_{f_{I_1(s)}}(x_1) * T_{f_{I_2(t)}}(y_1)] * [T_{f_{I_1(s)}}(x_2) * T_{f_{I_2(t)}}(y_2)] = T_{f_{C(s, t)}}(x_1, y_1) * T_{f_{C(s, t)}}(x_2, y_2)
\]
Similarly $F_{f_1}(s, t)[(x_1, y_1) + (x_2, y_2)] = F_{f_2}(s, t)(x_1, y_1) + F_{f_2}(s, t)(x_2, y_2)$

Next, $T_{f_1}(s, t)[\lambda(x_1, y_1)] = T_{f_1}(s, t)[\lambda x_1, \lambda y_1] = T_{f_1}(s)(\lambda x_1) * T_{f_1}(t)(\lambda y_1) \geq T_{f_1}(s)(x_1) * T_{f_1}(t)(y_1) = T_{f_1}(s, t)(x_1, y_1)$

Similarly $F_{f_2}(s, t)[\lambda(x_1, y_1)] \leq F_{f_2}(s, t)(x_1, y_1)$

5. Intuitionistic fuzzy soft subspace:

**Definition 5.1:** Let $I_1$ and $I_2$ be two IFSLS over $(V(K), E)$. Then $I_1$ is IFSS of $I_2$ if $I_1 \subseteq I_2$ i.e. $T_{f_{I_1}}(e)(u) \leq T_{f_{I_2}}(e)(u)$

$F_{f_{I_1}}(e)(u) \geq F_{f_{I_2}}(e)(u) \forall u \in V, e \in E$

**Example 5.2:** Let us consider two IFSLS $I_1$ and $I_2$ over real vector space $V=R^3$ and parametric set $E=\{e\}$ as following:

$T_{f_{I_1}}(e)(x) = \begin{cases} \frac{1}{4} & \text{if } x \in (a, b, c) \in R^3, a + b + c = 0 \\ 0, & \text{otherwise} \end{cases}$

$F_{f_{I_1}}(e)(x) = \begin{cases} 0, & \text{if } x \in (a, b, c) \in R^3, a + b + c = 0 \\ 1/6, & \text{otherwise} \end{cases}$

$T_{f_{I_2}}(e)(x) = \begin{cases} \frac{1}{2} & \text{if } x \in (a, b, c) \in R^3, a + b + c = 0 \\ 2/7, & \text{otherwise} \end{cases}$

$F_{f_{I_2}}(e)(x) = \begin{cases} 0, & \text{if } x \in (a, b, c) \in R^3, a + b + c = 0 \\ 1/9, & \text{otherwise} \end{cases}$

If $a + b = \max\{a + b - 1, 0\}$ and $a \Delta b = \min\{a + b, 1\}$. Then $I_1$ is an intuitionistic fuzzy soft subspace of $I_2$ over $(R^3(R), E)$.

**Corollary 5.3:** Let $I$ be an IFSLS over $(V(K), E)$. Then for arbitrary but fixed $\lambda \in K, \lambda I = \{e, \frac{\lambda x}{\lambda} \in E\}$ is also a IFSLS over $(V(K), E)$ where $\lambda f_j(e) = \{\lambda x, T_{f_j(e)}(x), F_{f_j(e)}(x) \neq / x \in V\}$. Moreover $\lambda I$ is an intuitionistic fuzzy soft subspace of $I$.

**Proof:** Clearly $\lambda x \in V$ for $x \in V, \lambda \in K$

Since $I$ be an IFSLS over $(V(K), E)$ so by construction of $\lambda I$

$T_{f_j}(\lambda x + \lambda y) \geq T_{f_j}(\lambda x) * T_{f_j}(\lambda y)$

$F_{f_j}(\lambda x + \lambda y) \leq F_{f_j}(\lambda x) * F_{f_j}(\lambda y) \forall \lambda x, \lambda y \in V, e \in E$

$T_{f_j}(\mu(\lambda x)) \geq T_{f_j}(\lambda x)$

$F_{f_j}(\mu(\lambda x)) \leq F_{f_j}(\lambda x) \forall \lambda x \in V, e \in E, \mu \in K$

Hence $\lambda I$ is IFSLS over $(V(K), E)$

Next $T_{f_j}(\lambda x) = T_{f_j}(\lambda^{-1}(\lambda x)) \geq T_{f_j}(\lambda x)$

$F_{f_j}(\lambda x) = F_{f_j}(\lambda^{-1}(\lambda x)) \geq F_{f_j}(\lambda x)(\forall \lambda \neq 0) \in K, x \in V, e \in E$

Then $\lambda I$ is IFS subspace of $I$. 
Corollary 5.4: Let $I_1$ be an IFSLS over $(V(K), E)$. Then for arbitrary but fixed $\lambda, \mu \in K$.\(I_1 = \{ (\lambda f_{I_1} + \mu f_{I_1})(e) \mid e \in E \} \) is again an IFSLS over $(V(K), E)$ where \((\lambda f_{I_1} + \mu f_{I_1})(e) = \langle \lambda x + \mu y, T_{f_{I_1}}(\lambda x + \mu y), F_{f_{I_1}}(\lambda x + \mu y) \rangle / x, y \in V \}

Moreover $\lambda I_1 + \mu I_1$ is an IF soft subspace of $I_1$

**Proof:** Since $(V(K)$ is a vector space.
So $x+y, \lambda x + \mu y \in V$ for $x, y \in V$ and $\lambda, \mu \in F$.
Hence the proof is completed.

Corollary 5.5: Let $f_{I_1}(e), e \in E$ be a IF subspace on $(V(K), E)$. Then $\lambda f_{I_1}(e) = \{ \langle \lambda x, T_{f_{I_1}}(\lambda x), F_{f_{I_1}}(\lambda x) \rangle / x \in V \} \}

**Proof:** It is obvious
For instance, if $V=\{x, y, z \}$ and $k= \{ \lambda, \mu \}$ then $\lambda x, \lambda y, \lambda z, \mu x, \mu y, \mu z \in V \text{ and } \lambda x + \lambda x, \lambda x + \lambda y, \lambda x + \mu x, \mu y + \mu y, \ldots \in V$.
Since $f_{I_1}(e), e \in E$ is an IFS subspace on $(V(K)$ so all the inequalities hold good.

**References:**