Multilevel Fast Multipole Acceleration for Fast ISAR Imaging based on Compressive Sensing

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***Abstract*—In this paper, compressive sensing (CS) is introduced into an efficient numerical method multilevel fast multipole acceleration (MLFMA) for the electromagnetic scattering problem over a wide incident angle. The resulted data from CS based MLFMA are processed to inverse synthetic aperture radar (ISAR) imaging. Simulation results in maritime surveillance for the classification of ships and aircrafts show the received data for ISAR imaging from MLFMA with CS can reach the outperforms of MLFMA and achieves a quality similar to that of classical ISAR imaging. Furthermore, the computational complexity is improved by CS obviously through the reduced matrix computation for the fewer incident waves. An illustration with Aircraft Model Imaging confirms the usefulness of CS-MLFMA.**

***Index Terms*—multilevel fast multipole acceleration (MLFMA), compressive sensing (CS), electromagnetic (EM), inversed synthetic aperture radar (ISAR), Fast Fourier Transform (FFT)**

I. Introduction

Inverse synthetic aperture radar (ISAR) technique can supply high resolution images of targets for long distance. In ISAR imaging, the observation durations must belong enough so that a considerably high cross-range resolution can be obtained through traditional fourier transform (FFT) based range doppler (RD) method. To predict the radar signal of the target accurately, it is necessary to study full wave electromagnetic (EM) scattering mechanism of the complex target.

In real life applications, due to high economic cost and complex field measurement of multiple scattering effects, the EM computation is an effective and economical way to simulate the scattering echoes of targets for ISAR imaging research. However, due to system and maneuver limitations, it is often impossible to achieve enough echo samples incross-range, making the imaging result with poor resolution and high sidelobes, which cannot be achieved in the real world due to the expensive equipment or cost. The numerical methods to solve fast simulation of EM scattering problems over a wide angle can be classified into two groups: integral equation and differential equation. The traditional integral equation, such as method of moment (MoM) and finite element method can also be used to get the received data for ISAR imaging, which need to repeat the solution of the system matrix equation using the iteration method in every incident angle. A variety of fast algorithms, such as multilevel fast multipole acceleration and adaptive integral method are developed to greatly reduce the computational cost for the analysis of electrically large objects. Fortunately, the concept of compressed sensing (CS) is an interesting way for a potentially large reduction in the sampling and computation costs for sensing signals that have a sparse or compressible representation. The basic concept of compressive sensing [1], [2] is that one can reconstruct an unknown signal vector with a few samples with high precision by solving the optimization problem with l_1 norm, many methods have been proposed (e.g.,least absolute shrinkage and selection operator and gradient projection for sparse reconstruction).

In this paper, compressive sensing is combined with multilevel fast multipole acceleration (CS-MLFMA), which reduced the number of the incidents angles. The compressed data can be obtained to extract the independently distributed function of the scattering centers for ISAR imaging.

The remainder of the paper is organized as follows. Section II recalls CS theory. Section III introduces first the MLFMA for a wide-angle EM scattering problem and then the CS approach implemented in MLMFA, it is also proposed how to get the received data, which are then exploited to recover the images in ISAR imaging by FFT. Section IV gives out the experimental results of CS-MLFMA compared to MLFMA, and also the imaging results. Moreover, computational complexity and computing time comparison are detailed in this section. Finally, in Section V, conclusions and suggestions for future work are discussed, more particularly some prospects for using the proposed method under severe conditions are detailed.

II. Compressive Sensing

The aim of compressive sensing is to acquire an unknown signal, assumed to have a sparse representation, using the minimal number of linear nonadaptive measurements and then recovering it by means of efficient optimization procedures.

The three points of compressive sensing theory are the sparsity of the received signals, incoherent measurement matrix and robust algorithm of reconstruction . Let $x \in \mathbb{R}^N$ be a signal having a sparse representation and its sparsity is $K, (K \ll N)$, so that x can be expressed as:

$$x = \Psi a \tag{1}$$

where $\Psi = (\psi_1, \psi_2, .., \psi_N)$ is the basis matrix, $a \in \mathbb{R}^{N\times 1}$ is the column vector of weighting coefficients with

$$a_i =< x_i, \psi_i >= \Psi^T x$$

using $\Phi \in \mathbb{R}^{M\times N}$ sensing matrix the dimension measure reducing into $K \sim M$, where the measuring times

$$M = O(K \log_2(\frac{N}{K})).$$

The signal obtained as $y = \Phi x = Aa$, $A = \Phi\Psi$ and A is the $(M \times N)$ matrix witch satisfy the restricted isometry property (RIP) with $0 < \delta < 1$ and for all K-sparse, $x \in \mathbb{R}^N$ the condition is true $1 - \delta \leq \frac{\|Ax\|_{l_p}}{\|x\|_{l_p}} \leq 1 + \delta$ with $\| \, . \, \|_{l_p}$ being the l_p-norm.
The reconstruction of x from y can be obtained via the l_1 norm minimization as

$$\hat{a} = min\| \; a \; \|_{l_1} \quad \text{subject to} \quad y = Aa \tag{2}$$

Finally, the original signal can be estimated as $\hat{x} = \Psi\hat{a}$.
Among the recovery algorithms, greedy algorithms are in some sense a good compromise between computational complexity and the required number of measurements. Greedy algorithms iteratively approximate the coefficients and the support of the original signal. They have the advantage of being very fast and easy to implement. Often the theoretical performance guarantees are very similar to, for instance, l_1 minimization results.
The most well-known greedy approach is Orthogonal Matching Pursuit and other prominent examples of greedy algorithms are stage wise OMP (StOMP), regularized OMP (ROMP), and compressive sampling MP (CoSamp) [3].

III. Implementation of compressive sensing in MLFMA

A. MLFMA

MLFMA [4], [5] has been widely used to solve the electromagnetic scattering of complex object with the surface integral equation approach. When it is applied to analyze the scattering from the multi-scale objects where dense discretization is necessary to capture geometric features accurately, the memory usage of MLFMA is very large. It is because of that the boxs size of tree structure cannot be set to less than 0.2λ (where λ is the wavelength), the number of unknowns in the near-field boxes is very large. The near-field matrices of MLFMA is filled by direct method.
Originally, the multilevel fast multipole algorithm (MLFMA) was proposed to reduce complexity and memory requirements. Song and Chew [6], implemented the MLFMA with $O(N \log_2 N)$ complexity and memory requirement using translation, interpolation, anterpolation (adjoint interpolation), and a grid-tree data structure.
Applied on smooth and conducting surface, based of the three principal boundary integral equations of electromagnetic scattering theory: the electric field integral equation (EFIE), the magnetic field integral equation (MFIE) and the combined field integral equation (CFIE) respectively defined as:

$$\hat{t} \times \overrightarrow{E^{inc}(\overrightarrow{r})} = \hat{t} \times (j\omega \overrightarrow{J_s(\overrightarrow{r})} + \nabla\Phi_s(\overrightarrow{r})) \tag{3}$$

$$\hat{n}\overrightarrow{H^{inc}(\overrightarrow{r})} = \overrightarrow{J_s(\overrightarrow{r})} - \hat{n}\nabla\Phi_s(\overrightarrow{r}) \tag{4}$$

$$CFIE = \alpha EFIE + (1 - \alpha)Z_0 \times MFIE \tag{5}$$

with $\alpha \in [0, 1]$, $Z_0 = \sqrt{\frac{\varepsilon_0}{\mu_0}}$ is the intrinsic impedance of space, $\overrightarrow{E^{inc}(\overrightarrow{r})}$ and $\overrightarrow{H^{inc}(\overrightarrow{r})}$ are the electric and magnetic field intensity of incident wave respectively.
Φ_s is an electric scalar potentiel, $\hat{n}$ and $\hat{t}$ are the unit vectors perpendicular and tangential to the scatterer surface, respectively.
$\overrightarrow{r}$ is a field point, $j = \sqrt{-1}$, ω is the angular frequency and $\overrightarrow{J}_s$ is the magnetic potential vector.
One can write a dense matrix equation

$$ZX = V \tag{6}$$

with Z is MoM impedance matrix, X represent unknown coefficient vector, V an excitation vector. The required matrix-vector multiplications (MVM) are performed efficiently with MLFMA, which reduces both storage complexity and computational complexity into $O(N \log_2 N)$.

B. MLFMA with CS

Using the moment method (MoM), after discretization and testing, (6) can be rewritten as:

$$\sum_{i=1}^{N} Z_{ji} x_i = v_j, j = 1...N \tag{7}$$

Where Z_{ji} means the impendence matrix, v is the incident wave and x is the current coefficient. Equation (7) expressed as:

$$Z(f)X(\Theta, f) = V(\Theta, f) \tag{8}$$

Θ is the incident angle, f is the incident frequency, V the excitation vector related to the incident angle Θ and X is the current coefficient vector to be solved.
Fixed f to f_0 and $\Theta = (\Theta_1, ..., \Theta_n)$, (8) rewritten by taking the transpose of Z, X and V as:

$$\begin{pmatrix} X^T(\Theta_1, f_0) \\ X^T(\Theta_2, f_0) \\ \vdots \\ X^T(\Theta_n, f_0) \end{pmatrix} Z^T(f_0) = \begin{pmatrix} V^T(\Theta_1, f_0) \\ V^T(\Theta_2, f_0) \\ \vdots \\ V^T(\Theta_n, f_0) \end{pmatrix} \tag{9}$$

For the monostatic radar cross section (RCS), EM scattering over a wide angle, the computation of solving the matrix equation at each incident angle is very time-consuming than we introduce theory of compressive sensing then (9) can be described as by using (1)

$$\Psi_{n\times n} \begin{pmatrix} a(\Theta_1, f_0) \\ a(\Theta_2, f_0) \\ \vdots \\ a(\Theta_n, f_0) \end{pmatrix} Z^T(f_0) = \begin{pmatrix} V^T(\Theta_1, f_0) \\ V^T(\Theta_2, f_0) \\ \vdots \\ V^T(\Theta_n, f_0) \end{pmatrix} \tag{10}$$

The incident waves can be defined with adding the incident waves from different angles together randomly as below:

$$V^{CS} = \sum_{i=1}^{N} o_i V^T(\Theta_i, f_0) \tag{11}$$

where o_i is the coefficient of each incident wave. The invariable Z is related to the incident angle, the induced current invoked by the new kind incident wave is equivalent to the sum of the separately induced current by each incident wave from each angle as:

$$X^{CS} = \sum_{i=1}^{N} o_i X^T(\Theta_i, f_0) \tag{12}$$

Compressive sensing theory can be implemented to reduce the number of the incidence from the original n times to m times by linear combination.

$$\begin{pmatrix} X^{CS}(\Theta_1, f_0) \\ X^{CS}(\Theta_2, f_0) \\ \vdots \\ X^{CS}(\Theta_m, f_0) \end{pmatrix} = \Phi_{m\times n} \begin{pmatrix} X^{T}(\Theta_1, f_0) \\ X^{T}(\Theta_2, f_0) \\ \vdots \\ X^{T}(\Theta_n, f_0) \end{pmatrix} \tag{13}$$

The reconstructed signal can be obtained by

$$\begin{pmatrix} X^{T}(\Theta_1, f_0) \\ X^{T}(\Theta_2, f_0) \\ \vdots \\ X^{T}(\Theta_n, f_0) \end{pmatrix} = \Psi_{n\times n} \begin{pmatrix} \hat{a}(\Theta_1, f_0) \\ \hat{a}(\Theta_2, f_0) \\ \vdots \\ \hat{a}(\Theta_n, f_0) \end{pmatrix} \tag{14}$$

where $(\hat{a}(\Theta_i, f_0); i = 1, .., n)$ can be estimated from (2). The new incident waves can be expressed as:

$$\begin{pmatrix} X^{CS}(\Theta_1, f_0) \\ X^{CS}(\Theta_2, f_0) \\ \vdots \\ X^{CS}(\Theta_m, f_0) \end{pmatrix} = \Phi_{m\times n}\Psi_{n\times n} \begin{pmatrix} \hat{a}(\Theta_1, f_0) \\ \hat{a}(\Theta_2, f_0) \\ \vdots \\ \hat{a}(\Theta_n, f_0) \end{pmatrix} \tag{15}$$

It is known that the new constructed induced current by compressive sensing which is equivalent to implement the compressing and measuring processes at the same time can be obtained by reconstructing the m $\;(m \approx K\log(\frac{n}{K}) \ll n)$ measurements, where K is the sparsity of the induced current signal. The measurement matrix $\Phi_{m\times n}$ is constructed as (16), which should not only satisfy RIP but also need to void the recycling computation for each incidence by the linear combination of incident waves without repeated information but with completive information.

$$\begin{pmatrix} o_{1,1} & \dots & 0 & o_{1,m+1} & \dots & 0 & o_{1,j} & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & o_{m,m} & 0 & \dots & o_{2m,m} & 0 & \dots & o_{m,n} \end{pmatrix} \tag{16}$$

C. ISAR Imaging by MLFMA with CS

ISAR is a method for obtaining high resolution images of a targets, the images are obtained as a result of the coherent processing of signals received from a sequence of locations and the information about the target position in the range/cross-range domain will be contained in two-dimensional sinusoids with corresponding frequencies. Hence, range/cross-range position can be easily obtained by using $2D$-FT.

ISAR can be realize the image of a moving object from a fixed radar such that the trajectory of the object is known [8]. Let $R(\theta)$ the distance from the observed point on the target to the radar, is expressed as

$$R(\theta) = \sqrt{x_1^2 + (y_1 + r_0)^2} \tag{17}$$

The target on the turntable turns a very small angle θ during the twice observation duration is shown in Fig. 1.

r_0 is a distance between horizontal axis-X and axis-X', $R(\theta)$ is distance from the observed point on the target to the radar

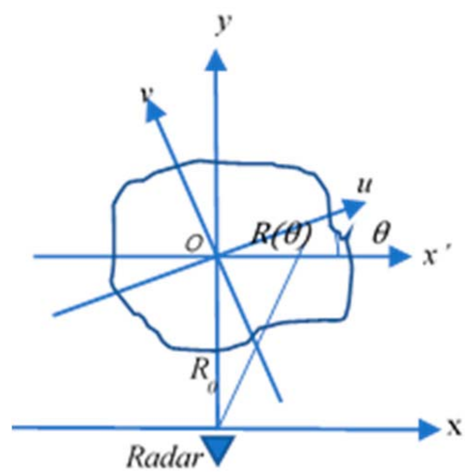

Fig. 1. ISAR imaging model.

$$x_1 = ucos\theta + vsin\theta \quad , \quad y_1 = vcos\theta - usin\theta \tag{18}$$

using (18) and equation (17) can be expressed as follow:

$$R(\theta) = \sqrt{r_0^2 + u^2 + v^2 + 2r_0vcos\theta - 2r_0usin\theta} \tag{19}$$

$$R(\theta) \approx r_0 + vcos\theta - usin\theta \tag{20}$$

The received data can be obtained as:

$$X(\theta, t) = \int\int g(u, v)e^{j2\pi f[t-\frac{2R(\theta)}{c}]}dudv \tag{21}$$

such that the transmitted signal of the radar is $s_t = e^{j2\pi f}$, f is the frequency and $g(u, v)$ is $2D$ distribution function of the target, c is the light velocity. In the frequency domain, the received signal can be transformed by FFT as in (22)

$$X(\theta, f) = \int\int g(u, v)e^{-j\frac{4\pi f}{c}(r_0+vcos\theta-usin\theta)}dudv \tag{22}$$

Therefore, the distribution function of the scattering center $g(u,v)$ obtained by FFT for ISAR imaging as

$$g(u,v) = \int\int X(\theta,f)e^{j\frac{4\pi f}{c}(r_0+vcos\theta-usin\theta)}d\theta df \quad (23)$$

IV. Numerical Examples

To prove effectiveness of the compressive sensing theory used in EM, some models are experimented by utilizing the proposed method CS-MLFMA. The RCS data are then used to ISAR imaging which proves that can recovery the received data successfully.

A. MLFMA with CS

The implementation of CS reduces the computation time while keeping the recovery validity. For a more complex ship model in Fig. 2, the simulation environment is 90 and the measuring time m is set to 90. The relative errors between the original and reconstructed current is shown in Fig. 3, which can be acceptable expect some special points. The RCS by both methods in Fig. 4 show the feasibility of the CS implementation in MLFMA.

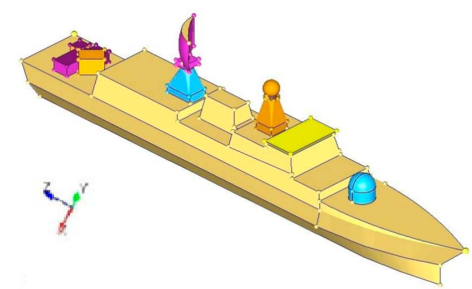

Fig. 2. Ship Mode.

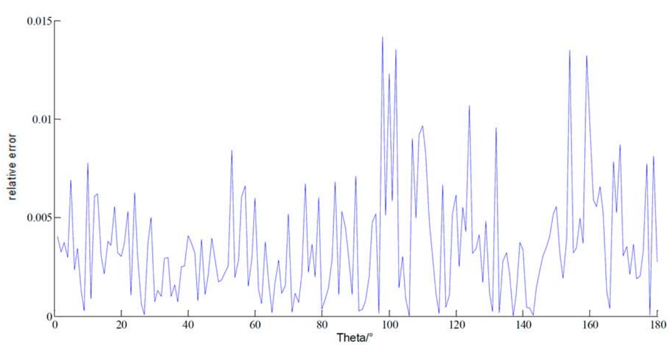

Fig. 3. The relative error of the current of ship model by MLFMA and CS-MLFMA based on some basis.

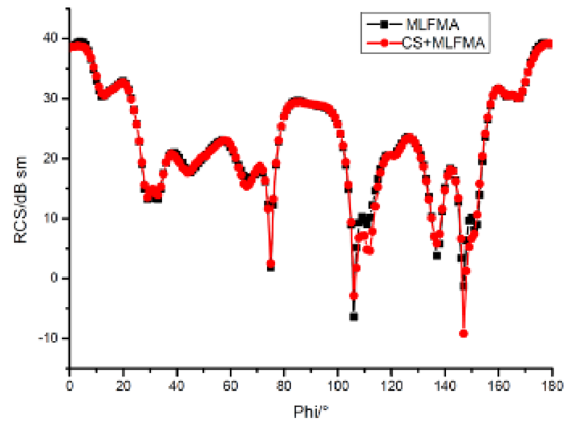

Fig. 4. RCS for ship model by MLFMA and CS-MLMFA.

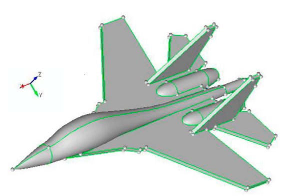

Fig. 5. Aircraft Model.

B. ISAR Imaging by using CS-MLMFA

The received data for the wide- band and multi-angle signal can be obtained by MLFMA with CS, which then can be utilized for ISAR Imaging. An aircraft model in Fig. 5 is taken as a numerical example and the parameters are detailed in Tab. I. The ISAR imaging of the aircraft is feasible, and the computation time can be reduced by CS (see Tab. II).

In the following paragraph, we compare analytically the computational complexity in the case of MLMFA vs CS-MLMFA case.

The number of operations for MLMFA is $O(pnN\log_2 N)$ and the computational complexity for CS-MLMFA is combined by computing the M measurement values $O(pM\log_2 N)$ on the one hand, and on the other the computational complexity of signal reconstruction with length N is $O(nKMN)$.

Combining these two computational complexities, we obtain

$$\frac{pM\log_2 N + nKMN}{pnN\log_2 N} = \frac{M}{nN} + \frac{KM}{p\log_2 N} \ll 1 \quad (24)$$

such that $M \ll N, M \ll n, K \ll p$ and p is the number of iterations for matrix computation.

Consequently, the ISAR imaging is feasible and the computational complexity can be reduced by CS.

TABLE I
EXAMPLE PARAMETERS

Body Length in X-axis	9
Distance (the longest wings) in Y-axis	6
Aircraft Length in Z-axis	2.4
Initial Incident Angle	74°
Angle Interval	0.5°
Number of Incident Angel	45
Frequency Range	300-1000 MHz
Frequency Interval	15 MHz
Number of Incident Frequency	65

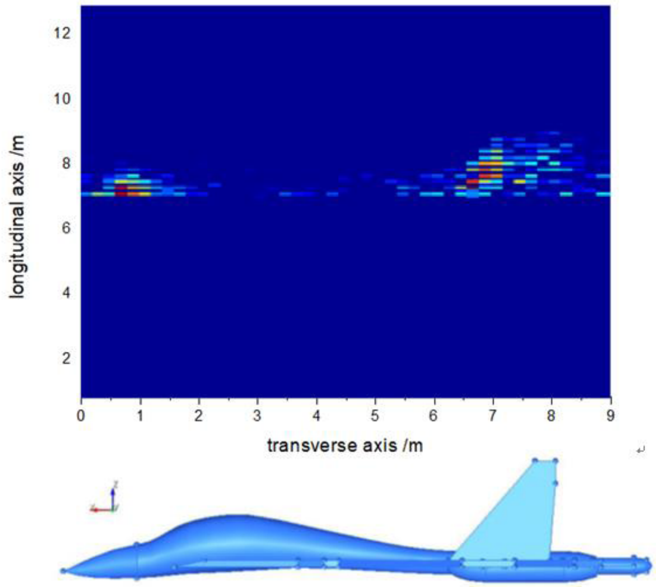

Fig. 6. Aircraft Model Imaging by CS-MLFMA.

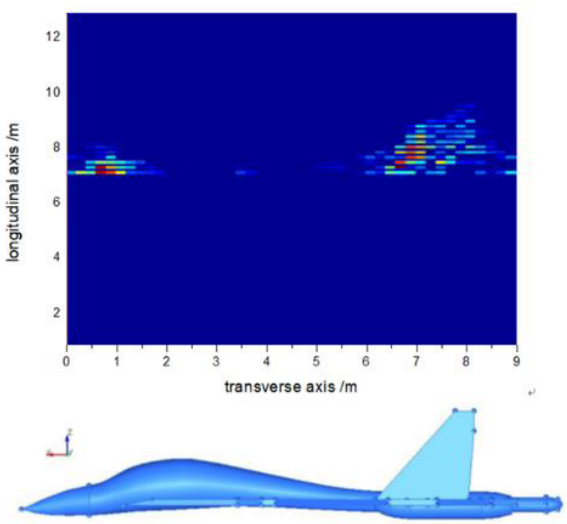

Fig. 7. Aircraft Model Imaging by MLFMA.

V. CONCLUSIONS

In this paper, compressed sensing technique is introduced in multilevel fast multipole acceleration to improve a wide angle monostatic EM scattering problem. This method makes a new kind of incident wave which reduces the computational complexity of repeated matrix computing and proved that the monostatic radar cross section obtained by CS-MLFMA is almost consistent with the data by multilevel fast multipole acceleration.

The application of CS-MLMFA makes the ISAR imaging successful with fewer incident waves and the computational complexity is reduced greatly, which makes the EM scattering

TABLE II
COMPUTATION TIME FOR AIRPLANE MODEL

Num.of Incident Angles			45	
Freq./GHz	Freq. Point	CS-MLFMA/min	MLFMA/min	Improvement
[0.3,0.4)	6	1.2	1.5	20%
[0.4,0.5)	7	1.75	2	12.50%
[0.5,0.6)	7	6.7	8	16.30%
[0.6,0.7)	6	8.3	10	17%
[0.7,0.8)	7	13	15	13.30%
[0.8,0.9)	7	17.5	20	12.50%
[0.9,1.0)	5	24.6	28	11.70%

simulation and ISAR imaging by real data meaningful. Due to the advantages of high detection rate, robust performance under all weather circumstances and ability of interference suppression, ISAR imaging is becoming an extremely important and widely used technology in military modernization and civil applications. However, if the observation times are not long enough, the resolution may pose serious problems as making the imaging result with poor resolution and high side lobes when the Fourier transform is used. This brings a challenge for acquiring a high-resolution image for real-time tracking. Multiple approaches are possible: instead of using the Fourier transform, it is better to opt for a high resolution time-frequency distribution and to consider all the disturbances as a non-stationary noise of which it is advisable to suppress [9].

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