Comparison between Epsilon Normalized Least means Square (-NLMS) and Recursive Least Squares (RLS) Adaptive Algorithms

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April 2, 2018

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Abstract—**There is an evidence that channel estimation in communication systems plays a crucial issue in recovering the transmitted data. In recent years, there has been an increasing interest to solve problems due to channel estimation and equalization especially when the channel impulse response is fast time varying Rician fading distribution that means channel impulse response change rapidly. Therefore, there must be an optimal channel estimation and equalization to recover transmitted data. However. this paper attempt to compare epsilon normalized least mean square (ϵ −*NLMS*) and recursive least squares (*RLS*) algorithms by computing their performance ability to track multiple fast time varying Rician fading channel with different values of Doppler frequency, as well as mean square deviation (MSD) has simulated to measure the difference between original channel and what is estimated. The simulation results of this study showed that (ϵ −*NLMS*) tend to perform fast time varying Rician fading channel better than (RLS) adaptive filter .**

Keywords: ϵ −NLMS, RLS, MSD, Rician Channel.

I. INTRODUCTION

One of the most important issues in all communication systems is that the received signal is different from the transmitted signal due to various transmission weaknesses. These weaknesses lead to random modifications that reduce the quality of analog signal, producing a fluctuation in binary bit polarity where the bit-error rate (BER) is increased. The most significant weakness is fading [1]. Fading is a random process refers to the variations in received signal strength at the receiver and it is classified into two kinds as slow fading and fast fading [2,3].

In the slow fading, channel impulse response changes much slower than transmitted signal. In the fast fading, channel impulse response changes rapidly within symbol duration because of the different propagation mechanisms described as scattering, diffraction, and reflection, which causes multipath propagation of the transmitted signal. At the receiver, the multipath signal, sometimes add constructively or sometimes destructively, which leads to a variation in the received signal power. The received signal envelope of a fast fading signal is said to follow a Rician distribution if one path line-of-sight is available between the transmitter and the receiver [4]. In order to overcome the channel effects, an efficient channel estimation must be adopted to recover transmitted data. However, this paper describes the simulation for tracking performance of two adaptive algorithms: epsilon normalized least mean square (ϵ −*NLMS*) and recursive least squares (*RLS*) to compare between their ability of tracking fast time varying Rician fading channel. Recently, tracking performance has been compared between extend recursive least square (ERLS) and recursive least square (RLS), ERLS was superior on RLS [5]. Also the applicability of RLS adaptive filter for satellite gravity gradients has investigated both in the time and spectral domain to assess the algorithm performance, the RLS filter was a very effective in the term of convergence speed but its computationally intensive [6], whereas, different variable step size strategies proposed to enhance the performance of the LMS algorithm for estimation system identification at the cost of complexity computations [7]. As well to two new improved RLS adaptive filter variable forgetting factor and variable convergence factor are proposed to minimize the mean error square problem of the noise free a post error signal and simulation results for channel equalization application demonstrated that the variable forgetting factor VFF-RLS outperforms to variable convergence factor VCF-RLS and NLMS algorithms in the terms of steady- state misalignment [8].

II. CHANNEL MODEL

The communication link involve in addition to direct signal, a reflected signals which can be modeled as a Rician, which is widely used in wireless communication channels.
The Rician PDF for a distributed envelope $r(t)$ can be written as:

$$p(r) = \frac{r}{\sigma^2} \exp\left(\frac{-(r^2+v^2)}{2\sigma^2}\right) I_o\left(\frac{rv}{\sigma^2}\right)$$

$$for\ v \geq 0, r \geq 0 \qquad (1)$$

$$p(r) = 0 \quad for \quad r < 0 \qquad (2)$$

Where σ^2 is the time average power at the envelope detector, while v is the peak amplitude of the direct Line of Sight (LOS) signal and $I_o(z)$ is the modified Bessel function of the first kind with order of zero. The distribution of Rician fading can be represented through of the Shape Parameter $K = \frac{v^2}{2\sigma^2}$, defined as the ratio of the power contributions by LOS path to the remaining multipath, and the *Scale parameter*$= v^2 + 2\sigma^2$, which can be defined as the received power from direct and indirect paths [9].

III. MATHEMATICAL MODEL

A case study approach is to clarify the mathematical model for Tracking Fast Time Varying of Rician Fading Channel of two algorithms:

A. Recursive Least Squares RLS algorithm.

Recursive least squares (RLS) filter is a blind algorithm which recursively finds coefficients of the filter that minimize a weighted linear least square cost function relating to the input signal. Equations (3 and 4) illustrate the mathematical model of RLS [5].

$$w_i = w_{i-1} + \frac{\lambda^{-1}\alpha P_{i-1} u_i^*}{1+\lambda^{-1} u_i P_{i-1} u_i^*}[d(i) - u_i w_{i-1}] \qquad (3)$$

Where covariance matrix P_i :

$$P_i = \lambda^{-1}\left[P_{i-1} - \frac{\lambda^{-1} P_{i-1} u_i^* u_i P_{i-1}}{1+\lambda^{-1} u_i P_{i-1} u_i^*}\right] \qquad (4)$$

B. Epsilon Normalized Least Mean Square ϵ −NLMS Algorithm.

The pure least mean square LMS adaptive algorithm suffer from difficulty to choose a learning rate (step-size) μ that responsible of stability, convergence speed and steady state error of the adaptive filter because it sensitive to the scaling of it's input. The epsilon least mean square ϵ −NLMS algorithm came to overcome this problem by normalizing the input power and added to epsilon value, when epsilon value is some small positive value that responsible of algorithm regularization to reduce error as described equations (5-7) [10].

$$w_i = w_{i-1} + \mu \frac{u_i^*}{g[u_i]} e \qquad (5)$$

$$g[u_i] = \epsilon + ||u_i||^2 \qquad (6)$$

$$e(i) = d(i) - u_i w_{i-1} \qquad (7)$$

Where w_i is the weight vector that represent to estimate the channel, while lambda λ denotes the forgetting factor and its is some positive scalar value should be less than one to operate in time varying environment, when alpha α is some scalar value $|\alpha| \leq 1$ that represent the filter coefficient used to investigate tractability and adaptability of filter. The u_i refer to input power vector, while the $g[u_i]$ are some positive value function of u_i to normalizing the input power.

and the measurements $d(i)$ satisfy

$$d(i) = u_i w_i^o + v(i) \qquad (8)$$

$$w_{i+1}^o = \alpha w_i^o + n_i \qquad (9)$$

Where w_i^o is the unknown weight vector that we intend to estimate, when $v(i)$ is the noise measurement is white with unit variance, the n_i represent noise disturbance and $e(i)$ denoted the estimation error.

IV. SIMULATION RESULT

A case study characteristics of Racian channel was used to be fast time varying with two values of Doppler frequencyf_D, low value 10Hz and high value 200Hz as shown in the figures (1-7) to maintain tracking performance of algorithms when sampling frequency 1KHz, number of channel samples equal to 500 samples and number of simulated channel are three for each transmission. The table below illustrates the main parameters of simulated algorithms.

TABLE I. SIMULATION PARAMETERS

Parameters	**Specifications**
All initial values	Zero
Scalar value λ	0.995
epsilon $\boldsymbol{\epsilon}$	1e-6
Scalar value α	1
Step-size μ	0.25

Figures (1,2,3) present three color lines; pink, blue and green. These lines represent original channel, estimated channel by $\epsilon - NLMS$ algorithm and estimated channel by RLS algorithm respectively when Doppler frequency $f_D = 10Hz$.

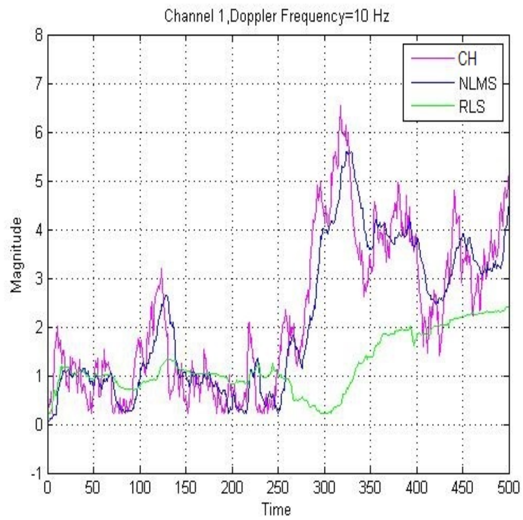

Fig. 1. Performance of ϵ −NLMS and RLS algorithms for tracking Ricain channel 1 when$\mathrm{f_D} = 10\mathrm{Hz}$.

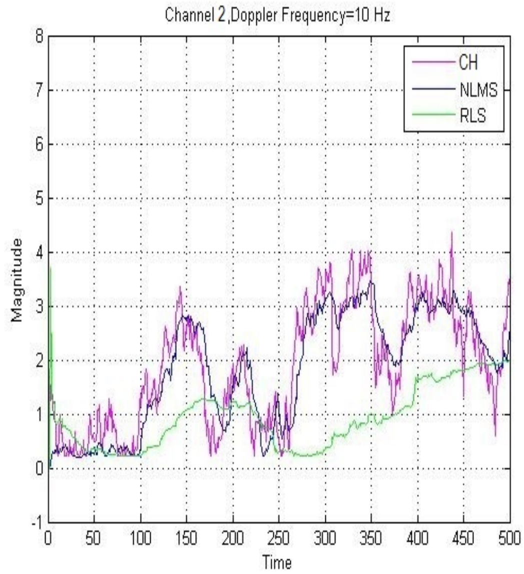

Fig. 2. Performance of ϵ −NLMS and RLS algorithms for tracking Ricain channel 2 when$\mathrm{f_D} = 10\mathrm{Hz}$.

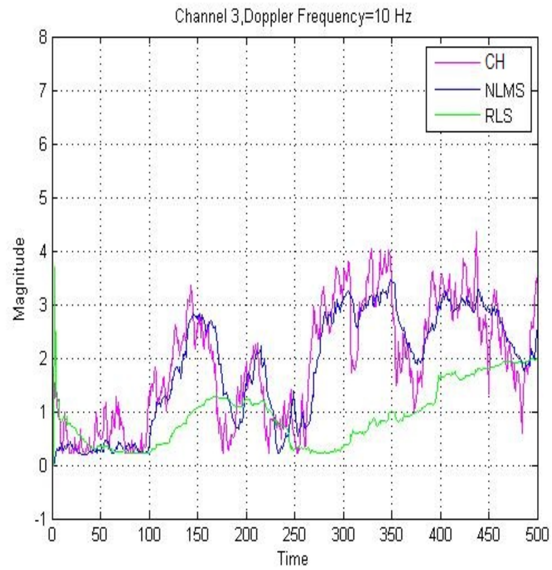

Fig. 3. Performance of ϵ −NLMS and RLS algorithms for tracking Ricain channel 3 when$\mathrm{f_D} = 10\mathrm{Hz}$.

Further analysis in figures (4, 5, 6) show difference performance between $\epsilon - NLMS$ and RLS algorithms when $f_D = 200Hz$.

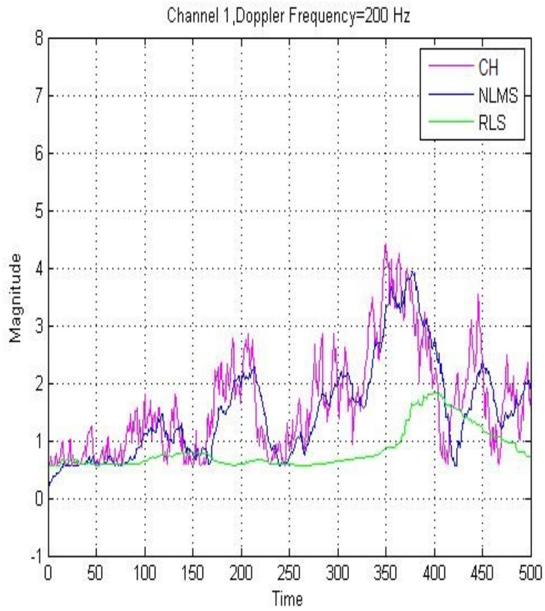

Fig. 4. Performance of ϵ −NLMS and RLS algorithms for tracking Ricain channel 1 when $f_D = 200$Hz..

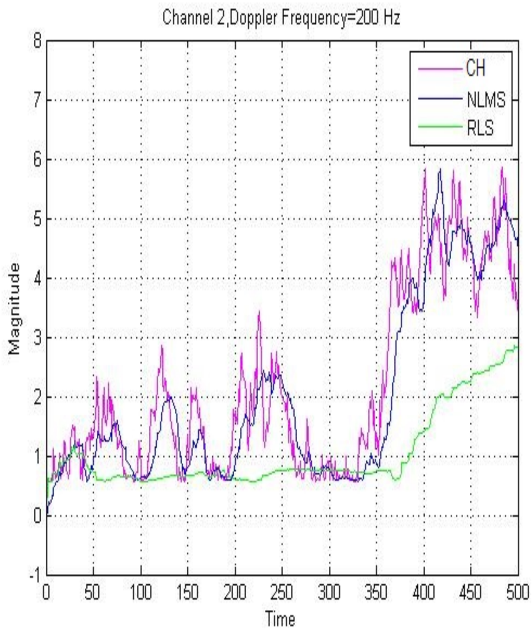

Fig. 5. Performance of ϵ −NLMS and RLS algorithms for tracking Ricain channel 2 when $f_D = 200$Hz..

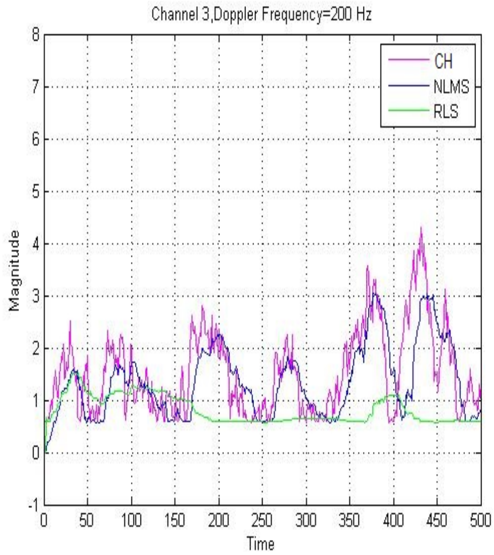

Fig. 6. Performance of ϵ −NLMS and RLS algorithms for tracking Ricain channel 3 when $f_D = 200$Hz..

From the graphs above, $\epsilon - NLMS$ algorithm reported significantly good ability for tracking Racian fading channel than RLS algorithm for the range of Doppler frequency(f_D) $10\ Hz\ and\ 200Hz$. Figure (7) describes mean square deviation (MSD) for each algorithm. The blue line represents MSE of $\epsilon - NLMS$ algorithm closes to zero level than black line (MSE of RLS). These results therefore indicate that the strength and stability of $\epsilon - NLMS$ algorithm to track fast time varying Racian fading channel than RLS estimator.

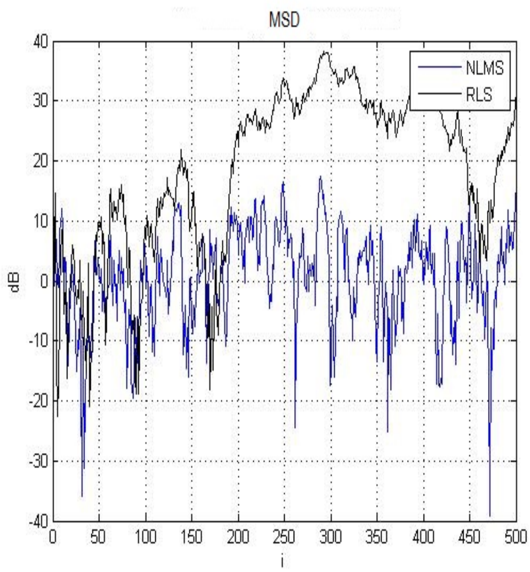

Fig. 7. Mean square Deviation (MSD) of ϵ −NLMS and RLS algorithms.

V. CONCLUTION

The contribution of this work is to compare the performance of two blind algorithms ($\epsilon -NLMS$) and (RLS) for tracking fast time varying Rician fading channel. The figures (1,2 ,3,4,5,6) showed the ($\epsilon -NLMS$) line consistent to the channel line more than RLS line for the range 10 Hz to 200 Hz of the Doppler frequency. Mean square deviation (MSD) of ($\epsilon -NLMS$) also show that when great majority of MSD values are arranged between zero and minus values more than RLS algorithm as shown in figure (7). Future work needs to be done to apply these algorithms to track other types of channel such as discrete memory less channel or Rayleigh channel for Long-term evolution (LTE) technology should concentrate on the difference between their abilities.

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