De Rham Cohomology for Compact Kahler Manifolds

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De Rham Cohomology is shown for compact Kahler manifolds considering the Hodge theory, Kahler potential and ddbar lemma.

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Let there be an external mapping of sections for the exterior derivative Δ for the mapping of the projections that in general turned out to be the Dolbeault operators that in essence be the,

$$\Delta\colon\ \Omega^r \longrightarrow \Omega^{r+1}$$

Where for the De Rham Cohomology one can describe the 4 forms as ϵ, ϵ_1, ϵ_2, ϵ_3 for a equivalence class of closed forms $[\epsilon]$ having the representation,

$$\epsilon_1 = \epsilon_2 + \epsilon_3$$

Where for $\epsilon_1 \cong [\epsilon]$, one can get the harmonic form $\Delta\epsilon_3 = 0$ for the exact form ϵ_2. Thus, if we take a constant sheaf on $\overline{\mathcal{R}}$ for a smooth manifold S where the De Rham Cohomology showed that for any map parameterized by J one can easily make the mapping,

$$J : H^p_{dR}(S) \longrightarrow H^p\left(S, \overline{\mathcal{R}}\right) \ni \begin{cases} [\epsilon] \in H^p_{dR}(S) \\ \forall J_{\epsilon_1} \in Hom\ H_p\left(S, \overline{\mathcal{R}}\right) \equiv H^p\left(S, \overline{\mathcal{R}}\right) \end{cases}$$

This proves two identities;

1. One can have a natural isomorphism for the sheaf cohomology of $\overline{\mathcal{R}}$ in a way as to show,

$$H^*_{dR}(S) \cong H^*\left(S, \overline{\mathcal{R}}\right)$$

$\ni \mathcal{R}$ represents a Abelian group.

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2. One gets the isomorphism between the singular cohomology and de Rham cohomology such that for ant set $\mathcal{G}$ and a trivial parameter classification or class $[G]$ there exists the relation with ϵ_1 in the way,

$$T \approx [G] \longrightarrow \sum\nolimits_{G} \epsilon_1$$

$$\forall \begin{cases} T \in [G] \\ T \in H_p(S) \end{cases}$$

Thus, one can take the exact form ϵ_2 and differentiating it with i, j, k for a grouping of $\langle |i|, |j| \rangle$ the resultant factor provides the relation to $\Delta\colon\ \Omega^r \longrightarrow \Omega^{r+1}$ such that in the case of ϵ_2 one of the most important aspects of Hodge theory can be found giving the wedge form,

$$\Omega^{p,q} \ni \epsilon_2 \cong \sum_{p,q} f_{ij} \Delta z^i \wedge \Delta \bar{z}^j$$

$\exists in\ \langle |i|, |j| \rangle;\ |i| = p, |j| = q$ for $\Omega^{p,q}$

Thus, one can find $\Delta = \partial + \bar{\partial}$ in differentiating the exact form ϵ_2 such that,

1. $\partial \epsilon_2$
2. $\bar{\partial} \epsilon_2$
3. $\partial \bar{\partial} + \bar{\partial} \partial =$ $0 \begin{cases} \quad where\ Poincaré\ Lemma\ holds\ for\ \partial\ and\ \bar{\partial} \\ for\ \epsilon_2\ in\ \Delta\epsilon_2\ complex\ differential\ it\ is\ \partial\bar{\partial}\ lemma \end{cases}$

   Thus, for the $\partial\bar{\partial}\ lemma$, one can satisfy compact manifolds as K$\ddot{a}$hler provided in the consequence of Hodge Theory, if one corresponds, the $\Delta\epsilon_2$ norm then, for the compact K$\ddot{a}$hler, a global form of this lemma holds.

   Let L be a compact Riemannian manifold, then for the relation: $\epsilon_1 = \epsilon_2 + \epsilon_3$. When $\Delta\epsilon_3 = 0$ then there exists exactly one $\epsilon_2 - form$ for the De Rham Cohomology class in $H_{dR}^{K}(L)$. Then for the space of the harmonic (ϵ_3) $k - forms\ L$ is isomorphic to $H^k\left(L, \overline{\mathcal{R}}\right)$ taking the sheaf Cohomology for $K^{th} -$ Betti numbers in each of such finite spaces. Thus in this case it can be assumed that the manifold $L \cong S$ for the Abelian group of $\mathcal{R}$.

Therefore, taking the complex manifold (K$\ddot{a}$hler) K having a exact form $\Omega^{p,q} \ni \epsilon_2$ the $\partial\bar{\partial}$ $lemma$ takes the bidegree $(1,1)$ form of $\Omega^{p,q}$ $\forall p, q \geq 1$ for a relation with the De Rham Cohomology such that in the $k - forms$ one can get the exact form of the K$\ddot{a}$hler $\Omega^k(K)$ whose class is zero is De Rham Cohomology for $H_{dR}^{p+q}(K, \mathbb{C})$ has also the $\partial\bar{\partial} = 0$. This bidegree form is essential for the K$\ddot{a}$hler potential for $[\epsilon]$ such that in the case of the relation $\epsilon_1 \cong [\epsilon]$, the potential is defined,

$$\epsilon_1 = i^{-2}\partial\bar{\partial}\rho$$

Where for the K$\ddot{a}$hler manifold (K, ϵ_1) for the potential of K$\ddot{a}$hler to be defined as ρ for $[\epsilon]$, there exists the neighborhood μ of $\bar{\rho}$ where $\bar{\rho}$ is a local K$\ddot{a}$hler potential for the exact differential of $(K, \mathbb{C})$ such that, in compact forms of K$\ddot{a}$hler one can get the ϵ_2 form of the potential in the local potential $\bar{\rho}$ for the form in $\mu \subset K$,

$$i\partial\bar{\partial}(K, \mathbb{C}) \equiv \Omega^{1,1}(K)$$

For the $\partial\bar{\partial}$ −manifolds, when it has been assumed the compact space to be L previously, where the compact K$\ddot{a}$hler denoted as K then its not difficult to say that,

$$L \approx K$$

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