



## Planning Optimal Path Networks Using Dynamic Behavioral Modeling

---

Sergei Kudinov, Egor Smirnov, Gavriil Malyshev and  
Ivan Khodnenko

EasyChair preprints are intended for rapid dissemination of research results and are integrated with the rest of EasyChair.

April 24, 2018

# Planning Optimal Path Networks Using Dynamic Behavioral Modeling

Sergei Kudinov<sup>1</sup>, Egor Smirnov<sup>1</sup>, Gavriil Malyshev<sup>1</sup> and Ivan Khodnenko<sup>2</sup>

<sup>1</sup> Institute for Design and Urban Studies, ITMO University, Birzhevaya Liniya 14, 199034 Saint Petersburg, Russia

<sup>2</sup> High-Performance Computing Department, ITMO University, Birzhevaya Liniya 4, 199034 Saint Petersburg, Russia

sergei.kudinov@corp.ifmo.ru, smirnov.egor.v@gmail.com,  
g.malyshev@corp.ifmo.ru, ivan.khodnenko@corp.ifmo.ru

**Abstract.** Mistakes in pedestrian infrastructure design in modern cities decrease transfer comfort for people, impact greenery due to appearance of desire paths, and thus increase the amount of dust in the air because of open ground. These mistakes can be avoided if optimal path networks are created considering behavioral aspects of pedestrian traffic, which is a challenge. In this article, we introduce Ant Road Planner, a new method of computer simulation for estimation and creation of optimal path networks which not only considers pedestrians' behavior but also helps minimize the total length of the paths so that the area is used more efficiently. The method, which includes a modeling algorithm and its software implementation with a user-friendly web interface, makes it possible to predict pedestrian networks for new territories with high precision and detect problematic areas in existing networks. The algorithm was successfully tested on real territories and proved its potential as a decision making support system for urban planners.

**Keywords:** Path Formation, Agent-Based Modeling, Human Trail System, Group Behavior, Pedestrian Flows Simulation, Stigmergy.

## 1 Introduction

Pedestrian infrastructure is a crucial part of urban environment, forming the basis of city territory accessibility because the last part of a trip is normally walked [1]. Thus, planning and organizing a comfortable pedestrian infrastructure is vitally important for urban development. Path network optimality is among key factors determining the comfort value of the way [2], as pedestrians tend to consider the optimal route to be the most comfortable [3].

From the pedestrian's point of view, the decisive factor when choosing the route is the highest connectivity that enables the pedestrian to get from the departure point to the destination point with minimum effort and in the minimum time possible, i.e. using the shortest way [4]. However, in terms of city planning, economics and environmental protection, minimizing the costs of path network creation is equally im-

portant, as well as minimizing the paved area in order to increase the green area and for other purposes. A compromise is possible which would provide a comfortable pedestrian infrastructure without linking all possible attraction points to each other using paved paths, although finding this kind of solution might be challenging.

In this article, a computer simulation method is discussed which makes it possible to design optimal path networks. The method considers both pedestrians' behavioral demands and the need to minimize the total length of the paths. The method was tested on real urban territories, showed high accuracy in predicting problematic areas of existing pedestrian networks, and demonstrated a good calculation speed.

## 2 Related Work

Usage of behavioral modeling methods for designing pedestrian infrastructure is currently underrepresented in research literature. Today, many simulation methods and software tools allow for modeling pedestrian flow motion in a predefined route network, which makes it possible to predict interaction between agents and prevent jams during public events and in emergency situations [5]. These are based on the social force model [6] and the cellular automata model [7], and their main application area is capacity estimation, but using these methods for calculating optimal path networks seems to be impossible.

Nevertheless, simulation methods aimed at building an optimal path network do exist, although they are not widespread due to their restricted application or their unsuitability for practical implementation.

### 2.1 Active Walkers

The Active Walkers method based on a greedy pathfinding algorithm was developed by Dirk Helbing and was aimed at modeling the forming of animal and human paths [8]. It makes it possible to model the forming of desire paths across lawns on territories with non-optimal path networks. The territory for the algorithm is defined by a grid with outlined borders and preset attraction points between which the agents simulating the pedestrians are distributed. The agent motion equation considers, among other things, the direction to the destination point and presence of existing paths nearby. This way the forming of desire paths is modeled as the agents move across the grid cells. At the end of the simulation, the modeled path network is formed by the grid cells through which the highest number of agents moved.

The drawback of this method is that the greedy pathfinding algorithm is not predictive, so an agent within the simulation makes its way to the destination based only on the comfort of each next step and the direction to the destination. The agent has no information on the complexity of the landscape or the location of obstacles, so it cannot start bypassing an obstacle until coming close to it [9]. This limits the applicability of Active Walkers to particular cases where territories have no complex shaped obstacles or dead ends, which makes the algorithm inefficient for creating an optimal path network on real urban territories with a complex configuration.

## 2.2 The Method by the Central Research and Project Institute for Urban Planning

This method was developed by the USSR Institute for Urban Planning that worked on planning developing urban territories and public accommodation. The method is stated in a set of instructions for mathematical and geometrical calculation of an optimal pedestrian network [10]. These design guidelines are based on a method of designing optimal networks for pedestrian communications [11]. Location of all destination points and obstacles, as well as a set of significant links between the given points needs to be considered as input data. The optimality criteria for the network created is the observance of the network feasibility condition which means that the angle between the pedestrian's motion direction and the direction to the destination point does not exceed  $30^\circ$ . This condition is of geometric nature and is closely related to the psychological mechanism regulating pedestrians' behavior as they move towards the destination. A subconscious visual on-site estimation of the angle between the motion direction in each point of the route and the direction to the destination plays the main role in this mechanism.

The algorithm allows for mathematical calculation and design of optimal path networks on urban territories, as it considers pedestrians' behavioral demands as well as economic and environmental factors, which makes it possible to create comfortable path networks with a minimum total length. The main drawback of the algorithm is lack of software implementation, which makes its wide use impossible. Moreover, the algorithm can only be used for pedestrian infrastructure planning for new territories and cannot be applied to optimize existing pedestrian networks where it is unreasonable to reconstruct the territory completely.

## 3 Proposed Methodology

The optimal path network creation method proposed in this article is called Ant Road Planner and is based on agent modeling performed by A\* algorithm, a modification of Dijkstra's pathfinding algorithm. An important feature of this algorithm is its ability to consider changes to the area map introduced by agents as optimal paths are formed by them.

This method is somewhat similar to algorithms of the so called ant colony optimization family. In these algorithms, ant-like agents choose their ways randomly based on "pheromone" traces left by other ants [12]. Trampledness of the lawn in the task in question can be compared to the pheromone traces in ant colony optimization algorithms. However, there are differences as well. The suggested method uses determined pathfinding based on full information on the navigation graph, unlike ant colony optimization algorithms in which the next step is chosen randomly. This helps to avoid problems typical of all greedy and randomized algorithms which find non-optimal paths in case there are complex-shaped obstacles.

The method is implemented in a software solution written in Java with a web interface, which makes it possible to use it as a practical support tool for decision making

in pedestrian infrastructure design [13]. This enables testing the algorithm on a large number of real territories with the help of urban planning experts.

### 3.1 Input Data

As input data, the algorithm requires detailed information on the configuration of the territory for which an optimal path network is being created. This information includes the location of obstacles, attraction points (shops, building entrances, playgrounds etc.), existing elements of pedestrian infrastructure, and different types of landscape surface. For this purpose, the algorithm uses a vector map of the territory. The web interface supports GeoJSON maps imported from GIS systems as well as DXF files from CAD systems.

The attraction points within the algorithm are divided into several types:

- Generators which agents go out from but which cannot be their destinations
- Attractors which can be agents' destinations but cannot generate agents
- Universal points performing both functions

A combination of different types of attraction points can handle situations when pedestrians do not move between certain attraction points. For example, pedestrians do not normally walk between different entrances to the same house, so these entrances can be marked as generators.

Locations of agent generators are shown on the map, as well as walkability of the territory parts ranging from zero for obstacles to maximum for official paths (Fig. 1). In order to obtain high-quality results, it is important to set relative popularity of agent attraction points correctly. The attraction points within the model are divided into two types: "popular" and "less popular", which correspond to the relative number of people choosing them.

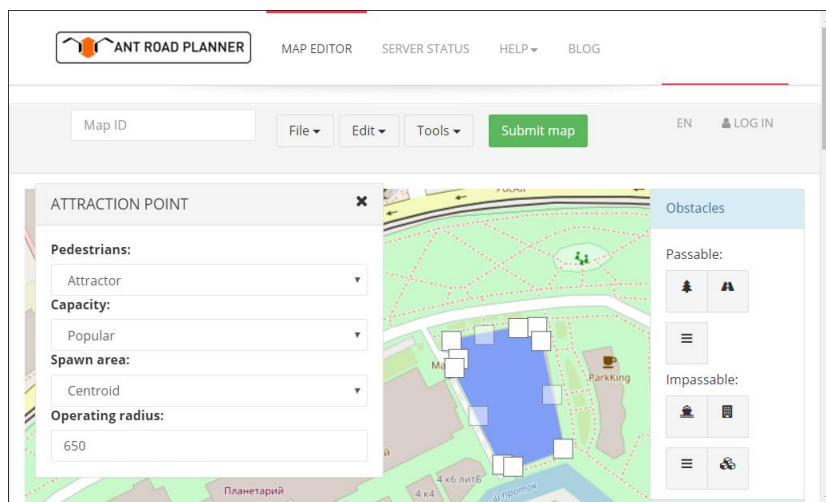
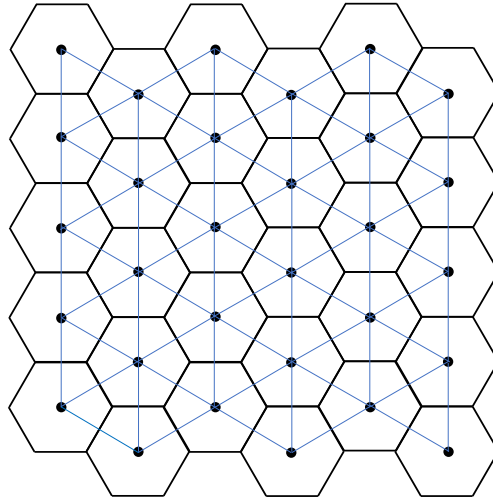


Fig. 1. Preparing territory map in Ant Road Planner web interface.

### 3.2 Building the Navigation Graph

At the initialization step, the input data is processed by the algorithm for future simulation. A navigation graph  $G(V,E)$  is built based on the map. In order to do this, a hexagonal grid is applied over the map, the centers of the hexangular blocks forming the vertex set of the graph  $V$ . If there is no impassable obstacle between the centers of the two adjacent blocks, i.e. an agent can walk between them, these nodes are linked with edges constituting set  $E$ . In **Fig. 2**, the points represent the vertex set, the vertices corresponding to the hexangular cells of the grid, and the thin lines between the points represent the edge set. Hexagonal grid was chosen instead of more common orthogonal one in order to increase the precision of route forming [14].



**Fig. 2.** Hexagonal grid and the navigation graph.

The weight  $W$  of each edge  $e$  is represented by the difference of two components: constant  $W_{const}(e)$  determined by the type of surface, and variable  $W_{var}(e)$  representing the trampledness:

$$W(e) = W_{const}(e) - W_{var}(e) \quad (1)$$

Initial trampledness equals 0.  $W_{const}(e)$  equals 1 for official paths with hard pavement; these have no variable component. For lawns,  $W_{const}(e)$  is suggested to be 2.7. This value was calculated empirically in a series of algorithm tests on reference territory maps. In order to do this, such values were selected for the variables that the pedestrian network resulting from the simulation for each territory was as close as possible to the official and desire path network existing on the real territory.

### 3.3 Agents' Behavioral Model

Agents  $p(i)$  that model pedestrians within the algorithm are divided into two groups – "decent" and "indecent" – to simulate the behavior of different types of pedestrians. For agents of the first type, the key factor when choosing the direction is the condition of the surface (lawn). "Decent" agents will not leave the path and start crossing the lawn if it is not significantly trampled. Moreover, they will stick to this type of behavior even if the way along official paths is longer than along desire paths that are not trampled enough. "Indecent" agents tend to always take the shortest way regardless of the existence and trampledness of the path across the lawn. That is,  $W_{var}(e)$  for them is always taken to equal the maximum acceptable value  $W_{max}$ . Thus, the weight of the edges representing the lawn is always minimal, almost equal to that of the edges representing paved paths. As a result, these pedestrians use nearly the geometrically shortest ways directly across the lawns and serve as a starting point for forming long narrow paths which are then used by other, "decent" pedestrians forming wide stable paths. This behavior represents pedestrians' psychology and the influence of the broken windows theory: People are more prone to do things not welcomed by the society (in this case – walking across lawns) if they see someone else has already done so [15].

### 3.4 Simulation Process

The attraction points of types "generator" and "universal point" have a capacity  $C$  which represents the number of agents generated in unit time. In the current version of the algorithm, the performance of "popular" and "less popular" attraction points differs by a factor of two. Such a rough division is due to labor efficiency of measurements and prediction of precise values for pedestrian flows in all attraction points of real territories. Thus, in order to make the method easier to use for urban territory designers, we suggest dividing the attraction points into those having a high pedestrian flow (e.g. public transport stops) and those having a lower flow (e.g. one of the entrances to a residential building).

Agents of different types are distributed equally within each attraction point but "indecent" agents constitute 5-10% of the total number of simulated agents. This proportion in the algorithm is chosen empirically.

Attraction points of types "attractor" and "universal point" have an operating radius  $R$ . It determines the maximum straight line distance between attraction points creating agents for this destination point. Agents' destinations are chosen randomly from a list of attractors and universal points with suitable operating radius.

The following happens at each step of the simulation:

1. Agents  $p(i)$  walk a certain distance  $S$  proportional to the specified speed  $v$ . At the end of the simulation step, agent's position on the current edge is saved to parameter  $S_L$ :

$$S_L = (S \bmod L) / L, \text{ where } L \text{ is the length of the graph edge.} \quad (2)$$

2. Trampledness of the graph edge  $W_{var}(e)$  increases by a constant value of the trampledness increment  $\Delta W_{ped}$  after each agent who walked the whole length of the edge till the end on this step of the simulation:

$$W_{var}'(e) = W_{var}(e) + \Delta W_{ped} \quad (3)$$

Trampledness of surrounding edges increases as well. The purpose and mechanism of this process are described in detail below.

3. Agents reaching their destinations disappear. New agents appear in attraction point of types "generator" and "universal point". Each point generates a new agent after a set number of simulation steps, while popular points generate pedestrians two times more often. Agent creation frequency can be set manually (if statistics or an estimation of the number of pedestrians are available) or equals 2 pedestrians a minute by default. This value was chosen empirically and is explained below.
4. Trampledness of each graph edge  $W_{var}(e)$  decreases by a constant value  $\Delta W_{dis}$  reflecting the path "dissolution" process, for example as a result of greenery re-growth.

$$W_{var}''(e) = W_{var}'(e) - \Delta W_{dis} \quad (4)$$

Increasing the trampledness of the edges surrounding the edge walked enables the algorithm to model realistic width of desire paths and implement a path adhesion mechanism. This mechanism is necessary to replace multiple parallel paths with a single one which is equally preferable for pedestrians using the neighboring paths.

Let  $W_{var}(ej)$  be the trampledness of edge  $j$  that neighbors edge  $i$  which the agent walks. After the agent walks the edge  $i$ , trampledness  $W_{var}(ej)$  of the surrounding edges increases by the induced trampledness  $\Delta W_{ind}$ :

$$W_{var}'(ej) = W_{var}(ej) + \Delta W_{ind}, \quad (5)$$

Induced trampledness is calculated as the product of the trampledness increment of the edge walked  $\Delta W_{ped}$  and a variable remoteness factor  $D(x)$  representing the distance between the node located at the far end of the calculated edge  $j$  and the node located at the far end of the walked edge  $i$ , where remoteness  $x$  is the distance between the nodes:

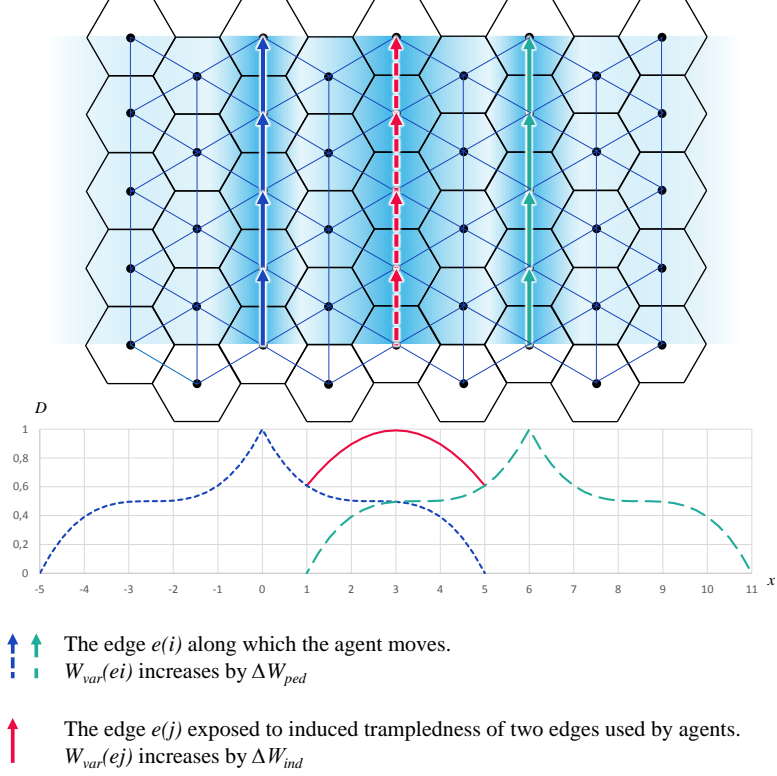
$$\Delta W_{ind} = \sum_i \Delta W_{ped i} * D(x)_i, \{D \in \mathbb{R}: 0 \leq D \leq 1\} \quad (6)$$

Range  $r$  of induced trampledness depends on the stage of the simulation on which the calculation takes place. As part of path adhesion mechanism development, experimental estimation of maximum range and possible curves illustrating the dependence of the factor  $D$  on the distance  $x$  was carried out. The task was to find such a curve that adding induced trampledness caused by neighboring edges used by agents would change the trampledness of the unused edge located between them by a value comparable to  $\Delta W_{ped}$ . It was found out that a suitable dependence is described by an equation of a cubic parabola:



$$D(x) = -4 \left| \frac{x}{r} \right|^3 + 6 \left| \frac{x}{r} \right|^2 - 3 \left| \frac{x}{r} \right| + 1 \quad (7)$$

**Fig. 3** shows how induced trampledness emerges when simulating path adhesion.



**Fig. 3.** Path adhesion process at the first stage of the simulation.

Range  $r$  is chosen to equal 5 meters for the first half of all the simulation steps. This range of induced trampledness is enough to start the adhesion process for paths located close to each other, which was determined by experiments. However, wide areas of high trampledness appear as a result of this process. For the path resulting from adhesion to have a realistic width, at the second stage of the simulation the trampledness of the surrounding edges is spread over a distance of  $r \approx 1.5$  meters from the edge walked.

The weight  $W(e)$  of the same edge  $e$  for different agents  $p$  within the model can differ. The weight determines the attractiveness of the territory part for the given agent, which is inversely related to the weight. Agents walking the territory choose the direction for the next step based on the edge weight. As the agents walk along the edge, its weight may decrease as the trampledness  $W_{var}(e)$  increases, which reflects the increase of attractiveness as the path becomes more trampled.  $W_{var}(e)$  is limited from

below by zero for intact lawn (which has not been walked by agents yet) and from above by  $W_{max}$  which equals 1.6. This value is chosen in such a way that the weight of the edge across the lawn area always exceeds that of the edge following a paved path. As a result, even a lawn area with maximum trampledness will have a slightly lower attractiveness (up to 10%) than a similar official path, all other factors held equal.

The following formula is used for the weight  $W(e)$  of the edge  $e$  for the agent:

$$W(e) = (W_{const}(e) - W_{var}(e)) * L \quad (8)$$

Based on the parameter limits described above, untouched lawn is 2.7 times less comfortable than a paved path for a "decent" agent, and a well-trampled lawn is only 1.1 times less comfortable. An "indecent" agent pays no attention to the trampledness of the lawn, so for it the weight of the edge across the lawn always equals 1.1.

Trampledness  $W_{var}(e)$  for the edge  $e$  after simulation step  $i$  can be expressed as follows:

$$W_{var}(e) = \Delta W_{ped} * P_{count}(e,i) + \Delta W_{ind} - \Delta W_{dis}, \text{ where} \\ P_{count}(e,i) \text{ is the number of agents who walked the edge } e \text{ at step } i. \quad (9)$$

In the model, agents plot their routes according to the A\* algorithm. The simulation continues until the preset number of steps is reached. Intermediate results can be estimated at each step.

After the simulation finishes, Ant Road Planner software environment forms a graphical layout representing the distribution of trampledness over the territory and showing the areas with the most intensive flow, where agents typically leave official paths and form desire paths.

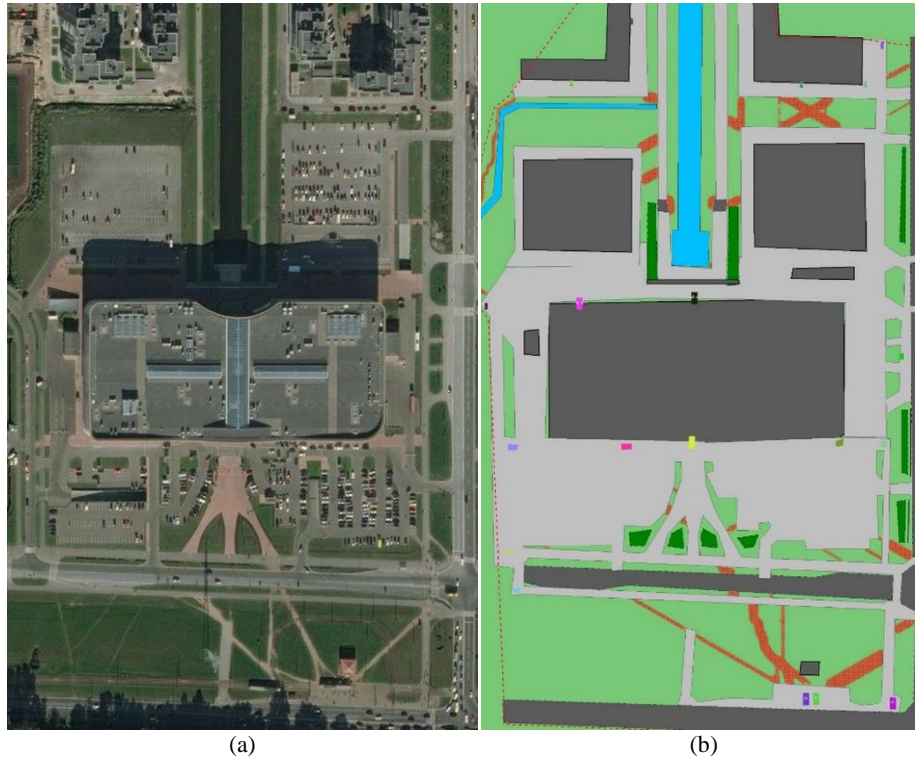
## 4 Experiments and Results

The main parameters of the algorithm, such as the proportion of "indecent" agents or  $W_{const}(e)$  for different types of surface, were chosen empirically based on experiment results. Three examples of existing urban territories were used: a small 50×50 meter backyard, a large 150×150 meter yard and a 500×300 meter park section. A comprehensive examination of possible parameter values and their combinations was carried out with a simulation run for each set of values. Then the prediction suggested by the algorithm was visually compared to on-site data on the path layout. A parameter set was selected that produced a simulation result as close to the real path layout as possible.

After that, several simulations of new territories (not used for parameter selection) were carried out in order to test the quality of the model obtained.

As an example, we analyzed a pedestrian network on a territory of a housing estate in St. Petersburg, Russia. This territory has a complex configuration with numerous obstacles and attraction points and has an existing path network but many of its parts are non-optimal and do not correspond to pedestrians' demands. As a result, there are a lot of desire paths on the territory.

For the purpose of the experiment, the attraction points of the territory were analyzed. The territory map and the data gathered was uploaded to the simulation using the Ant Road Planner web interface, after which a simulation was performed using the suggested algorithm. The calculations were performed with Intel Core i5-760 CPU (8MB Cache, 2.80GHz) and 16GB DDR3 667 MHz RAM. The following parameters were set for the simulation: territory area – 192,500 square meters, grid density – 0.451 sq. m per 1 hexagonal block, simulation step duration – 5 seconds, simulation duration – 5,760 steps. The calculation time for the chosen territory was 3 hours 56 minutes. The simulation result is a sketch map of the territory with highlighted areas recommended for inclusion into the official path network. Here is the resulting map together with a satellite shot of the territory for side-by-side comparison. Satellite shots from Yandex.Maps (**Fig. 4**) are used in this article.



**Fig. 4.** Simulation result visualization for pedestrian motion across the territory. (a) Satellite shot of the territory, (b) A sketch map by Ant Road Planner.

Areas suggested by the algorithm to be included in the official path network are marked in red. Colored rectangles denote the locations of agent attraction points.

In order to estimate the precision of predictive simulation, the sections of path layout suggested by the algorithm were compared to the gathered on-site data on the location of desire paths on the territory. Typical examples of non-optimal network

areas for which the algorithm suggested creating additional official paths are listed below.

**Fig. 5a** shows a satellite shot and the simulation result for the area between a tram stop and a housing estate (location coordinates: 59.847732, 30.144792). Existing sideways only go along the carriageway and bypass the lawn, which encourages pedestrians to make desire paths. The paths suggested by the algorithm mainly coincide with the existing desire paths. **Fig. 5b** shows a photo of the area between a sideway and a car parking which are separated by a lawn (location coordinates: 59.850742, 30.143564). The algorithm predicted the necessity of creating a path in this place, which is confirmed by on-site research. **Fig. 5c** shows the area near the crossroads (location coordinates: 59.848019, 30.146786). Pedestrians walking from the crossroads towards the housing estate and back also take a shortcut across the lawn because official paths suggest a longer way. In this case the algorithm also correctly predicted the need to improve the connectivity of the attraction points. Finally, **Fig. 5d** shows an interesting example of a paved path that was not included in the initial design but was created by residents on their own (location coordinates: 59.851035, 30.143597). However, a typical mistake was made by locating the two paths perpendicularly, which resulted in trampling the surrounding area. For this case, the algorithm also predicted the necessity of paving a diagonal path.



**Fig. 5.** Comparison of areas suggested by the algorithm for improvement of the territory with desire paths existing in the territory

Thus, using Ant Road Planner when this territory was designed would have helped to avoid lawn trampling in many places when creating an optimal path network, as well as ensure a comfortable pedestrian infrastructure.

In addition, Ant Road Planner was used in experiments in estimating the optimality of pedestrian networks, not only in residential areas but also in parks. The algorithm also demonstrated high prediction accuracy and was adopted for experimental operation by the city administration in order to estimate the optimality of pedestrian networks planned within green area creation and renovation projects.

## 5 Conclusions and Future Work

Computer modeling of path networks helps avoid design errors and ensure a comfortable pedestrian infrastructure. Ant Road Planner demonstrated good results and high modeling accuracy when tested on numerous real territories. Pedestrian networks designed on the basis of its results have the highest connectivity of attraction points while maintaining the lowest possible total length of the paths and taking into account pedestrians' behavior as they move across the territory. Ant Road Planner open-source web-interface can be used by urban planners even now to design pedestrian infrastructure while considering pedestrians' demands, eliminating labor-efficient manual calculations and minimizing time costs for on-site research. Current drawbacks of the algorithm, such as presence of empirically fitted coefficients and disregarding certain environment factors, will be eliminated as part of the follow-up study by conducting on-site experiments and more detailed analysis of factors affecting pedestrians' behavior as they move across urban territories. For example, the decision making mechanism when a pedestrian chooses a desire path instead of an official one, the dependency between pedestrians' behavior and the weather, the type of surface, the time of day, and illumination need to be refined, as well as study of lawn trampledness and greenery regrowth at the sites of desire paths.

The updated method which makes it possible to suggest optimal path networks for real urban territories with numerous obstacles featuring ultimate accuracy and a user-friendly interface can be widely adopted in design and engineering activities and used to develop plans for improvement and creation of urban territories that will be comfortable for the people.

## References

1. Kumar, A.: A systems approach to assess and improve the last-mile access to mass transits, p.89. Department of Industrial and Systems Engineering National University of Singapore (2015).
2. Mudron, I., Pachta, M.: Pedestrian Network Design and Optimisation Based on Pedestrian Shortcuts and Needs. GIS Ostrava 2013 – Geoinformatics for City Transformation Proceedings, pp. 175-184. Ostrava (2013).

3. Vahidi, H., Yan, W.: How is an informal transport infrastructure system formed? Towards a spatially explicit conceptual model. *Open Geospatial Data, Software and Standards* 1(8) (2016).
4. Al-Widyan, F., Al-Ani, A., Kirchner, N., Zeibots, M.: An effort-based evaluation of pedestrian route choice. *Scientific Research and Essays* 12(4), pp. 42-50 (2017).
5. Okazaki, S., Matsushita, S.: A Study of Simulation Model for Pedestrian Movement with Evacuation and Queuing. *Proceedings of the International Conference on Engineering for Crowd Safety*, pp. 271-280 (1993).
6. Helbing, D., Molnár, P.: Social force model for pedestrian dynamics. *Physical Review E* 51(5), pp. 4282-4286 (1995).
7. Weifeng, F., Lizhong, Y., Weicheng, F.: Simulation of bi-direction pedestrian movement using a cellular automata model. *Physica A: Statistical Mechanics and its Applications* 321(3), pp. 633-640 (2003).
8. Helbing, D., Keltsch, J., Molnar, P.: Modelling the Evolution of Human Trail Systems. *Nature* 388(6637), pp. 47-50 (1997).
9. Girdhar, A., Antonaglia, J.: Investigation of Trail Formation with the Active Walker Model. *Atomic-Scale Simulations* (2013).
10. The Central Research and Project Institute for Urban Planning: *Guidelines for Pedestrian Network Design*, Moscow (1989).
11. Romm, A.P.: *Pedestrian Networks*. Academia, vol. 2 (2006).
12. Dorigo, M., Stützle, T.: Ant Colony Optimization: Overview and Recent Advances. In: Gendreau, M., Potvin, J.Y. (eds.) *Handbook of Metaheuristics*. International Series in Operations Research & Management Science, vol 146, pp. 227-263. Springer, Boston, MA (2010).
13. Smirnov, E., Gurevich, M. Ant Road Planner – Pedestrian simulator webpage, <http://antroadplanner.ru/editor/editor>, last accessed 2018/04/09.
14. Nitzsche, C.: Cellular automata modeling for pedestrian dynamics, Bachelor Thesis (2013).
15. Keizer, K., Lindenberg, S., Steg, L. The Spreading of Disorder. *Science* 322(5908), pp. 1681-1685 (2008).