Simulation of Two-Dimensional Distribution Laws of Random Correlated Quantities of Natural-Climatic Factors in Context of Probabilistic Assessment of Reliability of Hydraulic Structures of Cascades of Hydroschemes

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Simulation of two-dimensional distribution laws of random correlated quantities of natural-climatic factors in context of probabilistic assessment of reliability of hydraulic structures of cascades of hydroschemes

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Abstract. When performing calculations to assess reliability of hydraulic structures of cascades of hydroschemes on the basis of probabilistic methods, the necessity to simulate random natural-climatic phenomena producing loads and effects on hydraulic structures arises. In particular, statistical series of random quantities of such important natural-climatic phenomena are considered: annual lowest average monthly temperatures, annual maximal amplitudes of average monthly temperatures. Each of the enumerated natural-climatic phenomena is characterized by presence of close correlation connections between random quantities when passing from one hydroscheme of the cascade to another. The necessity to consider correlation connections requires construction (simulation) of joint distribution law of random quantities system. The purpose of the work is simulation of joint distribution law of system of random variables that do not satisfy the normal distributions, taking into account correlation connections between random variables when passing from one hydroscheme of the cascade to another. Methods of the theory of correlation and methods of mathematical statistics with the use of software package MathCad were used in the course of the investigation. Simulation of joint law of distribution of system of random variables that do not satisfy normal distributions, taking into account correlation connections between random variables when passing from one hydroscheme of the cascade to another, and also assessment of accuracy of results, that were performed, have shown advantages of this approach from the viewpoint of accuracy of results obtained by different procedures. The results can be used in probabilistic calculations of reliability of hydraulic structures and cascades of hydroschemes.

1 Introduction

Assessment of safety and reliability of hydraulic structures on the basis of probabilistic methods is regulated by normative documents [1–9]. Taking into account the extremely high potential danger of hydraulic structures, improvement of methods of assessment of their reliability is an important and relevant problem. During performing calculations on assessment of reliability of hydraulic structures of hydroscheme cascades, necessity to simulate distribution laws of random natural-climatic phenomena that create loads and effects on hydraulic structures arises. In this investigation the approaches that allow simulating a joint law of distribution of system of random quantities that do not satisfy the normal distributions in the closed form, and also obtaining the conditional distribution laws of random quantities of natural-climatic phenomena taking into account correlation connections, are realized.

2 Analysis of recent researches

Statistic series of random quantities of such important natural-climatic phenomena, obtained by direct measurements in dam sites of hydroschemes of the Dnieper cascade of hydroelectric stations: annual maximal flood discharges $Q_{max,i}$, annual maximal ice thickness $h_{max,i}$, annual lowest average monthly temperatures $t_{min,i}$, annual maximal amplitudes of average monthly temperatures $\Delta t_{min,i}$ were investigated by methods of probability theory and mathematical statistics with substantiation of the proposed distribution laws in investigations [10–12]. Each of the enumerated natural-climatic phenomena is characterized by presence of close correlation connections between the random quantities when passing from one hydroscheme of the cascade to another. Investigations [11, 13] deal with revealing correlation connections between random

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quantities of natural-climatic phenomena in hydroschemes of the Dnieper cascade of hydroelectric stations. The necessity to take into account correlation connections between natural-climatic phenomena requires construction (simulation) of joint distribution law of random quantities system, which is realized in investigation [11]. In the mentioned sources, distribution laws of random variables of natural-climatic phenomena, that enter into the system, do not satisfy the normal distributions, therefore approaches to transform them into the normal laws by way of the use of the corresponding transformations were used. Principles of construction of the joint distribution law of system of random variables that satisfy the normal distributions are widely presented in present-day investigations [14–16]. Investigations of two-dimensional and multidimensional joint distribution laws of systems of discrete and continuous random variables that do not satisfy the normal distributions are proposed in investigations [15–31]. Two-dimensional and multidimensional distributions with multiple correlation connections are presented in investigations [21, 23–26]. Application of non-linear regression models is presented in investigation [22]. Multidimensional distribution laws of random variables simulated with the use of the copula theory are presented in investigations [29, 32–39], in particular, hydrologic mode of hydrosystem in flood period is simulated in investigation [32].

The performed critical analysis of the present-day investigations and publications made it possible to formulate the purpose and determine the objective of the investigation. The objective of the investigation is development of the algorithm of construction of joint distribution law of random variables system taking into account correlation dependences between the natural factors: between annual maximal flood discharges of the watercourse (r. Dnieper); between annual lowest average monthly temperature at hydroschemes of the Dnieper cascade; between annual maximal amplitude of variations of temperature of outdoor air at the hydroschemes of the Dnieper cascade; between annual maximal ice thickness at the hydroschemes of the Dnieper cascade.

The purpose of the work is simulation of joint distribution law of system of random variables that do not satisfy the normal distributions, taking into account correlation connections between random variables when passing from one hydroscheme of the cascade to another.

Methods of the theory of correlation and methods of mathematical statistics with the use of software package MathCad were used in the course of the investigation.

3 Results and discussion

Joint density of distribution of continuous system of random variables (X_1, X_2), that satisfy the lognormal distributions [15, 19, 27, 28, 31] is presented by expression (1):

$$f(\gamma_1,\gamma_2)=\frac{\xi^2}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}\,\gamma_1\gamma_2}\times$$
$$\times exp\left\{-\frac{1}{2(1-r^2)}\left[\begin{array}{l}\left(\dfrac{10\,log_{10}\gamma_1-m_1}{\sigma_1}\right)^2+\\ +\left(\dfrac{10\,log_{10}\gamma_2-m_2}{\sigma_2}\right)^2-\\ -2r\left(\dfrac{10\,log_{10}\gamma_1-m_1}{\sigma_1}\right)\times\\ \times\left(\dfrac{10\,log_{10}\gamma_2-m_2}{\sigma_2}\right)\end{array}\right]\right\}, \quad (1)$$

where $\xi=\dfrac{10}{ln(10)}$, $\gamma_1=10^{\frac{X_1}{10}}$, $\gamma_2=10^{\frac{X_2}{10}}$;

σ_1, σ_2 – root-mean-square deviations of random variables X_1, X_2;

m_1, m_2 – mathematical expectations of random variables X_1, X_2;

r – correlation coefficient of random variables X_1, X_2.

Distribution (1) presents lognormal distribution on the plane. In this case each of random variables X_1 or X_2 has density of lognormal distribution:

$$f(\gamma)=\frac{\xi}{\sigma\sqrt{2\pi}\gamma}exp\left\{-\frac{(10\,log_{10}\gamma-m)^2}{2\sigma^2}\right\}. \quad (2)$$

Conditional law of distribution of random variable X_2 at a fixed value of variable X_1 has form [31]:

$$f(\gamma_2|\gamma_1)=\frac{\xi}{\sigma_2\sqrt{2\pi(1-r^2)}\gamma_2}\times$$
$$\times exp\left\{-\frac{1}{2(1-r^2)}\left[\begin{array}{l}\left(\dfrac{10\,log_{10}\gamma_2-m_2}{\sigma_2}\right)-\\ -2r\left(\dfrac{10\,log_{10}\gamma_1-m_1}{\sigma_1}\right)\end{array}\right]^2\right\}. \quad (3)$$

But in practical calculations it is more convenient to use expressions (2–3) in closed form [14]:

$$f(\gamma_1)=\int_{-\infty}^{\infty}f(\gamma_1,\gamma_2)d\gamma_2\,, \quad (4)$$

$$f(\gamma_2|\gamma_1)=\frac{f(\gamma_1,\gamma_2)}{\int_{-\infty}^{\infty}f(\gamma_1,\gamma_2)d\gamma_2}. \quad (5)$$

Conditional mathematical expectation of random variable X_2 at a fixed value of variable X_1 has form [14]:

$$m(\gamma_2|\gamma_1) = \int_{-\infty}^{\infty} \gamma_2 \frac{f(\gamma_1, \gamma_2)}{\int_{-\infty}^{\infty} f(\gamma_1, \gamma_2) d\gamma_2} d\gamma_2, \quad (6)$$

and conditional dispersion and standard deviation of random variable X_1 are determined by expressions:

$$D(\gamma_2|\gamma_1) = \int_{-\infty}^{\infty} (\gamma_2 - m_2)^2 \frac{f(\gamma_1, \gamma_2)}{\int_{-\infty}^{\infty} f(\gamma_1, \gamma_2) d\gamma_2} d\gamma_2, \quad (7)$$

$$\sigma(\gamma_2|\gamma_1) = \sqrt{D(\gamma_2|\gamma_1)}. \quad (8)$$

Conditional mathematical expectation $m(\gamma_1|\gamma_2)$, dispersion $D(\gamma_1|\gamma_2)$ and standard deviation $\sigma(\gamma_1|\gamma_2)$ of random variable X_1 at a fixed value of variable X_2 are calculated analogously By this means five parameters are determined: $m(\gamma_1|\gamma_2)$, $m(\gamma_2|\gamma_1)$, $\sigma(\gamma_1|\gamma_2)$, $\sigma(\gamma_2|\gamma_1)$, r of density of distribution of continuous system of random variables (X_1, X_2), that satisfy lognormal distributions.

The value of random variable X_2 is determined by conditional law of distribution with parameters $m(\gamma_2|\gamma_1)$, $\sigma(\gamma_2|\gamma_1)$:

$$f_{simulated}(\gamma_2) = \frac{\xi}{\sigma(\gamma_2|\gamma_1)\sqrt{2\pi}\gamma_2} \times \times exp\left\{ -\frac{(10 log_{10} \gamma_2 - m \cdot (\gamma_2|\gamma_1))^2}{2\sigma^2(\gamma_2|\gamma_1)} \right\}. \quad (9)$$

Let us illustrate the presented approach by an example. We simulate the joint law of distribution of two-dimensional system of random variables (X_1, X_2), that satisfy lognormal distributions. Analysis of statistical data on annual maximal amplitudes of average monthly temperatures at hydroschemes of the Dnieper cascade, and also determination of parameters of their distribution functions is performed in investigation [12]. It is presented in Tabl. 1. Selection of function of distribution has been performed by comparison of deviations of probabilities σp and maximal amplitude of monthly average temperatures $\sigma \Delta t$ of actual values from analytical distribution. It is presented in Tabl. 2.

It is found that probabilities of annual maximal amplitudes of average monthly temperatures in t. Kaniv, t. Kremenchuk can be presented by lognormal distribution. Parameters of the distributions for t. Kaniv: mathematical expectation $m_{\Delta t1}$ = 24.59 °C, standard deviation $\sigma_{\Delta t1}$ = 1.16 °C; for t. Kremenchuk: mathematical expectation $m_{\Delta t2}$ = 25.77 °C, standard deviation $\sigma_{\Delta t2}$ = 1.17 °C.

Investigation of correlation connections between annual maximal amplitudes of average monthly temperatures by statistical data of observations at t. Kaniv and t. Kremenchuk, carried out in investigation [13], indicates close correlation dependence between annual maximal amplitudes of average monthly temperatures in dam sites of hydroschemes of the Dnieper cascade.

Table 1. Parameters of distribution functions of probability of annual maximal amplitude of monthly average temperatures at geographical places of location of hydroschemes Dnieper cascade.

Item observation	Logarithmic-normal distribution	
	mathematical expectation	standard deviation
t. Vyshhorod	25.16	1.16
t. Kaniv	24.59	1.16
t. Kremenchuk	25.77	1.17
t. Kamyanske	27.43	1.16
t. Zaporizhzhia	27.00	1.16
t. Nova Kakhovka	26.72	1.16

Table 2. Results of assessment of accuracy of calculations of probability of annual maximal amplitude of monthly average temperatures at geographical places of location of hydroschemes of Dnieper cascade.

Item observation	Logarithmic-normal distribution	
	deviations of probabilities σp, %	deviations of maximal amplitude of monthly average temperatures σΔt, %
t. Vyshhorod	2.8	0.4
t. Kaniv	3.7	0.6
t. Kremenchuk	4.2	0.8
t. Kamyanske	2.8	0.5
t. Zaporizhzhia	6.9	0.9
t. Nova Kakhovka	2.9	0.5

By results of correlation analysis of statistical samples of maximal amplitude of average monthly temperatures of outdoor air, °C, correlation coefficient of two samples at t. Kaniv and t. Kremenchuk is r = 0.871. It is presented in Fig. 1.

The linear regression equation is taken as

$$y(x) = b_0 + b_1 \times x, \quad (10)$$

where $y(x)$ – regression of pairs of statistical series of annual maximum amplitudes of average monthly

temperatures in the alignments in the geographical locations of hydropower plants of the Dnieper cascade;
x – statistical series of the annual maximum amplitude of average monthly temperatures along the X axis;
b_0 , b_1 – empirical coefficients.

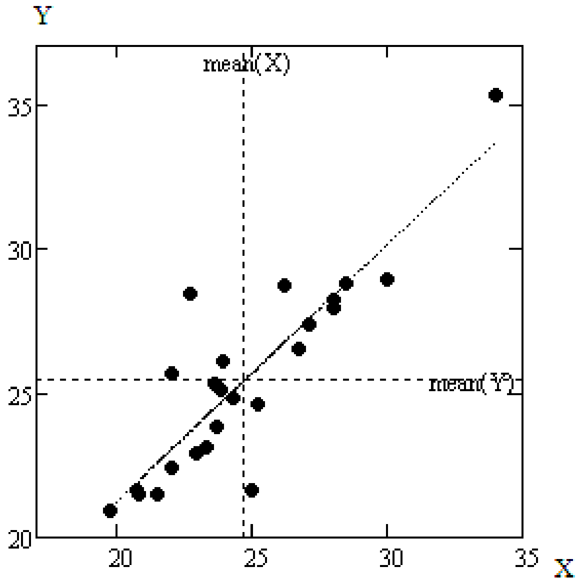

Fig. 1. Graph of the linear regression function of the statistical series of the annual maximum amplitude of the average monthly outdoor air temperatures, °C, observed in t. Kaniv (X axis), for the statistical series of the annual maximum amplitude of the average monthly outdoor air temperatures, °C, observed in t. Kremenchuk (Y axis): - - - – graph of the linear regression function; • • • – statistical series.

Sample correlation coefficients, sample covariance, standard errors are calculated. It is presented in Tabl. 3, 4, 5.

Table 3. The results of statistical processing of the annual maximum amplitude of the average monthly outdoor air temperatures, °C in the geographical locations of hydropower plants of the Dnieper cascade for the period of observations from 1966 to 1977 and from 1979 to 2008.

Item observation (reservoir)	**Selective average, °C**	**The standard deviation**	**Selective dispersion**
Kyiv Reservoir	25.7	2.7	7.3
Kaniv Reservoir	24.7	3.3	11.2
Kremenchuk Reservoir	25.5	3.4	11.5
Middle Dnieper Reservoir	26.2	3.6	12.9
Dnieper Reservoir	26.0	3.4	11.6
Kakhovka Reservoir	25.0	3.1	9.3

Using expression (1), we can construct joint density of distribution $f(\gamma_1(\Delta t_1), \gamma_2(\Delta t_2))$ of two-dimensional system of random variables ($X_1 = \Delta t_1$, $X_2 = \Delta t_2$), that satisfy lognormal distributions with parameters for t. Kaniv: $m_{\Delta t1} = 24.59$ °C, $\sigma_{\Delta t1} = 1.16$ °C; and t. Kremenchuk: $m_{\Delta t2} = 25.77$ °C, $\sigma_{\Delta t2} = 1.17$ °C. It is presented in Fig. 2.

Table 4. The empirical coefficients of linear regression equation (10) of statistical series of the annual maximum amplitude of the average monthly outdoor air temperatures, °C in the geographical locations of hydropower plants of the Dnieper cascade for the period of observations from 1966 to 1977 and from 1979 to 2008.

Item observation (reservoir)	**Coefficients**	
	b_0	b_1
Kyiv Reservoir – Kaniv Reservoir	0.412	0.946
Kaniv Reservoir – Kremenchuk Reservoir	3.583	0.884
Kremenchuk Reservoir – Middle Dnieper Reservoir	1.026	0.989
Middle Dnieper Reservoir – Dnieper Reservoir	2.034	0.914
Dnieper Reservoir – Kakhovka Reservoir	2.897	0.853

Table 5. The results of correlation analysis of statistical series of the annual maximum amplitude of the average monthly outdoor air temperatures, °C in the geographical locations of hydropower plants of the Dnieper cascade for the period of observations from 1966 to 1977 and from 1979 to 2008.

Item observation (reservoir)	**Correlation coefficient of two statistical series**	**Covariance of two statistical series**	**Standard error**
Kyiv Reservoir – Kaniv Reservoir	0.761	6.6	2.2
Kaniv Reservoir – Kremenchuk Reservoir	0.871	9.5	1.7
Kremenchuk Reservoir – Middle Dnieper Reservoir	0.936	10.9	1.3
Middle Dnieper Reservoir – Dnieper Reservoir	0.964	11.3	0.9
Dnieper Reservoir – Kakhovka Reservoir	0.950	9.4	1.0

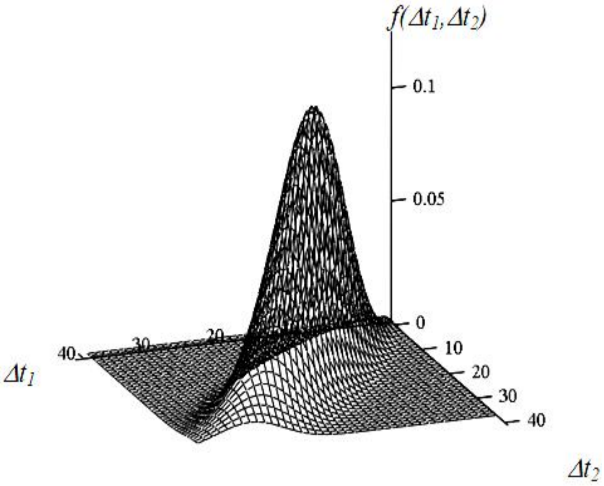

Fig. 2. Function of density of distribution $f(\gamma_1(\Delta t_1), \gamma_2(\Delta t_2))$ of system of two correlated variables Δt_1, Δt_2, that satisfy lognormal distributions.

Conditions (11–12), those function of joint density of distribution $f(\gamma_1(\Delta t_1), \gamma_2(\Delta t_2))$ of system of two correlated random variables [14] Δt_1, Δt_2 must obey, – are satisfied

$$f(\gamma_1(\Delta t_1), \gamma_1(\Delta t_1)) \geq 0\,, \qquad (11)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\gamma_1(\Delta t_1), \gamma_2(\Delta t_2))\,d\gamma_1(\Delta t_1)\,d\gamma_2(\Delta t_2) = 1\,. \qquad (12)$$

We obtain conditional law of distribution $f(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$ of random variable Δt_2 at fixed value of Δt_1 in analytical form by expressions (4–5).

Conditional mathematical expectation $m(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$ of random variable Δt_2 at fixed value of Δt_1 is obtained in analytical form by expression (6). Its numerical value is $m(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$ = 25.65299 °C. Conditional dispersion $D(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$ and standard deviation $\sigma(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$ of random variable Δt_2 at fixed value of Δt_1 is obtained in analytical form by expressions (7–8). Their numerical values equal $D(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$ = 1.36911 °C^2, $\sigma(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$ = =1.17009 °C.

Random probability of annual maximal amplitudes of average monthly temperatures $p(\Delta t_1) = p(\Delta t_2)$, distributed from 0 to 1 is specified. By known probability of amplitude of average monthly temperatures $p(\Delta t_2)$, using conditional distribution law (9) with parameters $m(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$, $\sigma(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$, we determine the quantile – the value of amplitude of average monthly temperatures $\Delta t_{2\ simulated}$.

The value of random variable Δt_2 is determined by conditional distribution law (9) with parameters $m(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$, $\sigma(\gamma_2(\Delta t_2)|\gamma_1(\Delta t_1))$. It is presented in Fig. 3.

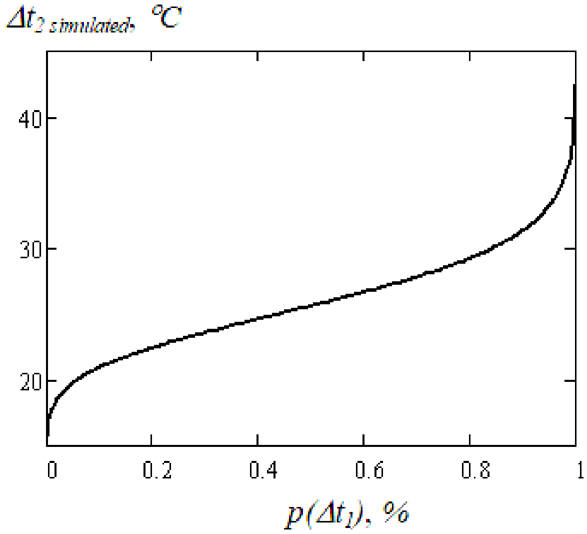

Fig. 3. Probability curve for annual maximal amplitude of average monthly temperatures at dam site of Kremenchuk hydroscheme (t. Kremenchuk) on the coordinates $\Delta t_{2\ simulated}$, °C – annual maximal amplitude of average monthly temperatures, $p(\Delta t_1)$, % – probability.

In investigations [11], substitution of

$$\Delta t_{cond,i} = a \times mean(\Delta t)\left(\frac{\Delta t_i}{mean(\Delta t)}\right)^b, \qquad (13)$$

$i = 1 \ldots n$, into output statistical series was used to transform laws of distribution of statistical data of annual maximal amplitudes of average monthly temperatures at hydroschemes of the Dnieper cascade into normal distributions, where Δt_i – corresponding members of the output statistical series, $\Delta t_{cond,i}$ – corresponding members of the transformed statistical series; $mean(\Delta t)$ – the average value of annual maximal amplitude of average monthly temperatures of the output statistical series; a, b – empirical coefficients. It is presented in Tabl. 6.

Table 6. Parameters of transformation (13) of distribution laws of annual maximal amplitude of monthly average temperatures Δt, °C at geographical sites of location of hydro schemes of Dnieper cascade.

Item observation (reservoir)	***mean*(Δt), °C**	**Coefficients**	
		a	***b***
Kaniv Reservoir (t. Kaniv)	24,733	1,05	0,15
Kremenchuk Reservoir (t. Kremenchuk)	25,458	1,01	0,25
Dnieper Reservoir (t. Zaporizhzhia)	25,975	1,04	0,24

Conditional distribution laws of lowest monthly average temperatures and of maximal amplitudes of monthly average temperatures according to [40] correspond to normal law if values of expression (14) are within the confidence interval:

$$\frac{max\left(\Delta t_{cond,i}\right)-min\left(\Delta t_{cond,i}\right)}{\sigma\left(\Delta t_{cond,i}\right)}, \qquad (14)$$

where $max(\Delta t_{cond,i})$ – maximal values of maximal amplitudes of monthly average temperatures of transformed normal distribution; $min(\Delta t_{cond,i})$ – minimal values of maximal amplitudes of monthly average temperatures of transformed normal distribution; $\sigma(\Delta t_{cond,i})$ – standard deviations of values of maximal amplitudes of monthly average temperatures of transformed normal distribution.

When the number of members of statistical series n = 24 and significance level p = 10%, the lower boundary of the interval is 3.41, the upper boundary of the interval is 4.52. It is presented in Tabl. 7.

Table 7. Confidence intervals (14) of transformation of distribution laws of annual maximal amplitude of monthly average temperatures Δt, °C at geographical sites of location of hydro schemes of Dnieper cascade.

Item observation (reservoir)	**$(max(\Delta t_{cond,i}) - min(\Delta t_{cond,i}))/\sigma(\Delta t_{cond,i})$**
Kaniv Reservoir (t. Kaniv)	3,41 < **4,20** < 4,52
Kremenchuk Reservoir (t. Kremenchuk)	3,41 < **4,40** < 4,52
Dnieper Reservoir (t. Zaporizhzhia)	3,41 < **4,21** < 4,52

For annual maximal amplitudes of average monthly temperatures $\Delta t_{1\,cond}$ and $\Delta t_{2\,cond}$ at dam sites of two hydroschemes, that are specified by normal distribution law as random correlated variables with the corresponding parameters: mathematical expectations $m_{\Delta t,1\,cond}$, $m_{\Delta t,2\,cond}$, standard deviations $\sigma_{\Delta t,1\,cond}$, $\sigma_{\Delta t,2\,cond}$, correlation coefficient $r_{\Delta t,1\,cond,\Delta t,2\,cond}$, correlation moment $K_{\Delta t,1\,cond,\Delta t,2\,cond}$, variation coefficient C_v; random probability of annual maximal amplitudes of average monthly temperatures $p(\Delta t_{1\,cond})$, distributed from 0 to 1 is specified. By normal distribution law with parameters presented above $m_{\Delta t,1\,cond}$, $\sigma_{\Delta t,1\,cond}$, quantile – the value of annual maximal amplitudes of average monthly temperatures $\Delta t_{1\,cond}$ - is determined by formulas:

$$m_{\Delta t1\,cond,\Delta t2\,cond} = m_{\Delta t2\,cond} + r_{\Delta t1\,cond,\Delta t2\,cond} \times \times \frac{\sigma_{\Delta t2\,cond}}{\sigma_{\Delta t1\,cond}} \cdot \left(\Delta t_{1\,cond} - m_{\Delta t1\,cond}\right), \qquad (15)$$

$$\sigma_{\Delta t1\,cond,\Delta t2\,cond} = \sigma_{\Delta t2\,cond}\sqrt{1-r^2_{\Delta t1\,cond,\Delta t2\,cond}}. \qquad (16)$$

Parameters of conditional distribution law $m_{\Delta t1\,cond,\Delta t2\,cond}$, $\sigma_{\Delta t1\,cond,\Delta t2\,cond}$ are being determined. By known probability of the value of annual maximal amplitudes of average monthly temperatures $p(\Delta t_{2\,cond})$, using conditional distribution law, quantile – the value of quantity of annual maximal amplitudes of average monthly temperatures $\Delta t_{2\,cond}$ - is being determined. Recalculation of the value of annual maximal amplitudes of average monthly temperatures $\Delta t_{1\,cond}$, $\Delta t_{2\,cond}$, presented by conditional distribution law with substitution of formula (13), into real annual maximal amplitudes of average monthly temperatures $\Delta t_{1\,real}$, $\Delta t_{2\,real}$ at dam sites of hydroschemes is being performed. It is presented in Fig. 4.

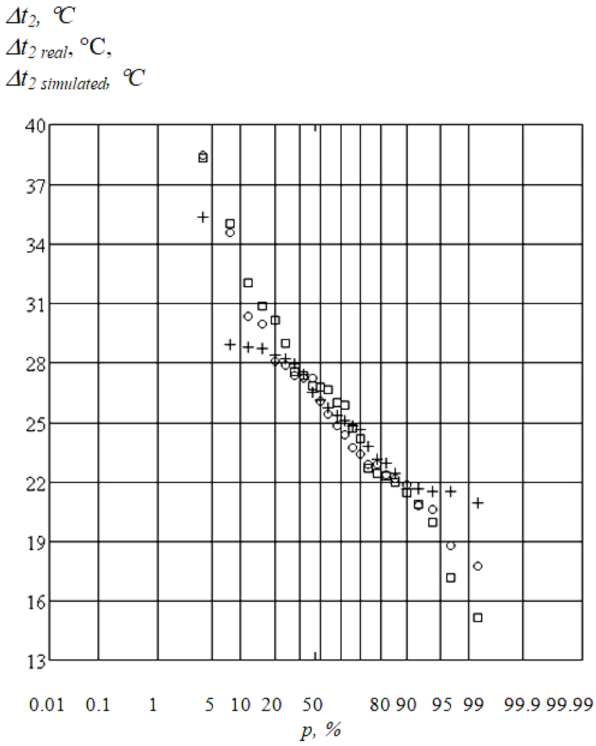

Fig. 4. Points of the probability curve of annual maximal amplitude of average monthly temperatures at dam site of the Kremenchuk hydroscheme (t. Kremenchuk) on the coordinates Δt_2, °C – annual maximal amplitude of average monthly temperatures, p, % – probability: + – observed amplitude of average monthly temperatures Δt_2, °C; □ – amplitude of average monthly temperatures $\Delta t_{2\,real}$, °C, calculated by transformation into conditional normal distribution law [11]; o – amplitude of average monthly temperatures $\Delta t_{2\,simulated}$, °C, calculated by two-dimensional lognormal distribution law.

Deviations of the values of amplitudes of average monthly temperatures, obtained by different procedures, from the observed data were assessed by comparison of their standard deviations. It was found that deviation of amplitudes of average monthly temperatures $\Delta t_{2\,real}$, °C, calculated by transformation into conditional normal distribution law [11], from the observed points of annual maximal amplitude of average monthly temperatures is $\sigma(\Delta t_{2\,real})$ = 2.227 °C. Deviation of amplitudes of average monthly temperatures $\Delta t_{2\,simulated}$, °C, calculated by two-dimensional lognormal distribution law, from observed points of annual maximal amplitude of average monthly temperatures is $\sigma(\Delta t_{2\,simulated})$ = 1.682 °C. Difference between $\sigma(\Delta t_{2\,real})$ = 2.227 °C and $\sigma(\Delta t_{2\,simulated})$ = = 1.682 °C is 24.5%.

Deviation of amplitudes of average monthly temperatures $\Delta t_{2\,simulated}$, °C, calculated by two-dimensional lognormal distribution law, from amplitudes of average monthly temperatures $\Delta t_{2\,real}$, °C, calculated by transformation into conditional normal distribution law, is $\sigma = 2.11$ °C.

4 Conclusions

Taking into account the great diversity of distribution laws of random variables of natural-climatic factors connected by correlation dependencies, and mathematical complexity of construction of joint distribution laws, method which is based on transformation of distribution laws into normal form has advantage in the further use. Assessment of accuracy of the results obtained by different procedures is performed. The results can be used in probabilistic calculations of reliability of hydraulic structures and cascades of hydroschemes.

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