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# Topological Characteristics of digital models of geological core

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**Abstract.** This work is devoted to an assessment of the possibility of using stochastic approaches to core modeling using tools of Topology. In addition, the research shows the prospects of applying topological characteristics to describe the core and search for analogs. Later, by connecting these topological characteristics with the reservoir properties (possibly with the application of machine learning methods), we will be able to obtain them for new core samples without carrying out expensive and long-term filtration experiments.

**Keywords:** Topological characteristics, Betti numbers, digital core, geological modelling.

Nowadays the process of oil fields exploitation needs a continuous information support. This is especially necessary in the context of a shift in emphasis in the development, planning and monitoring of oil and gas fields on very highly dissected and low-permeability reservoirs.

Correct estimation of economic efficiency and optimal placement of production wells are able only in the presence of a quantitative geological picture of the oil field. It means that mathematical measure of the geological modelling of such objects is needed.

These estimation processes include a geological and hydrodynamic modelling. Adjustment of models includes:

- static well data – logging curves, core analysis, drilling data etc.;
- dynamic data – well flow, bottomhole pressure etc.

In practice, the adjustment of models occurs iteratively – by the numerical simulation of series of direct resource-intensive tasks. The speed of such an adjustment depends on geological model quality. To evaluate the quality of such a model, a mathematical method is needed to describe its "heterogeneity" and "internal complexity". It is necessary to find a proper mesh size, variogram radius, experimental data correlation etc.

Similar problems also exist in the digital modelling of core samples – rock samples extracted from the well by means of a specially designed drilling. In modern practice, there are some tools for evaluation of the heterogeneity of such models [1]:

- construction of dissection maps;
- spectral modelling of logging curves.

However, these methods do not provide numerical characteristics (metric, measure) of a constructed 3-D model. It is suggested to use topological characteristics of these models as such a measure [2].

Such an approach is used in the study together with digital models of geological cores. Obtained results are compared with topological characteristics of geological models.

## 1 Topological characteristics of three-dimensional digital solid body models

Topological Characteristics of three-dimensional digital solid body models in the study are defined with the same mathematical apparatus as in [3]. These models represent ordered sets of elementary cubes – cubic complexes. Observed solid bodies are considered topologically equivalent if their topological characteristics are the same. The Betti numbers  $b_0$ ,  $b_1$  and  $b_2$  are used as such topological characteristics.

The meaning of these characteristics is very natural:

- $b_0$  is the number of connected components;
- $b_1$  is the number of handles;
- $b_2$  is the number of holes (cavities).

In this paper, two elementary cubes are considered to belong to the same linear component only in case of intersecting each other by a joint face. Intersecting by a joint vertex or edge does not make them belong to the same linear component.

If we start from a solid cube, remove  $k$  holes from its interior and attach  $l$  handles to the cube we obtain the body  $X$  for which  $b_0=1$ ,  $b_1=l$ ,  $b_2=k$ .

Calculating of Betti numbers for these cubic complexes is implemented via the numerical algorithm [3].

## 2 Digital core model analysis

### 2.1 Digital core model description

The digital core model is a model obtained as a result of computer tomography. Computer tomography - a method of nondestructive layered analysis of the internal structure of the object, was proposed in 1972 by Godfrey N. Hounsfield and Allan M. Cormack, awarded for this development of the Nobel Prize. Method based on computer-processed combinations of many X-ray measurements taken from various angles to produce cross-sectional images of a scanned object, allowing to see inside the

object without destruction [4]. Today, computed tomography is an evolving method for studying the petrographic properties of rocks. The X-ray tomography method allows to solve a huge number of geological problems, such as modelling cavities (fracture, caverns, pores), calculating the porosity, studying rock heterogeneity, analysis of reservoir properties, measurement as core's volumes as all its voids and solids.

X-ray tomography is used in oil industry since 80-s years [5]. The first studies were conducted in Australia, USA, and Great Britain [6].

The result of the X-ray tomography of a core sample is a set of grayscale snapshots representing corresponding virtual sections [7]. Each snapshot point shows its radiodensity. Combination of these grayscale snapshots is used then to generate 3-dimensional radiodensity distribution of the sample in the volume [8].

## **2.2 Digital core model creation process**

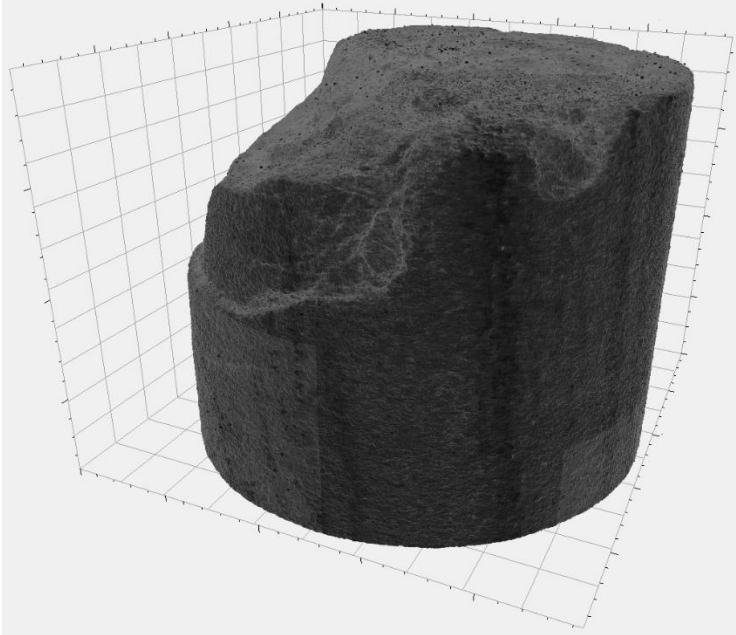
A core sample share with a diameter of 46 mm and 7.8 mm high was used in the study.

Tomography snapshots are taken with 0.025 mm interval for inline and crossline sections and with 0.004 mm interval for vertical sections.

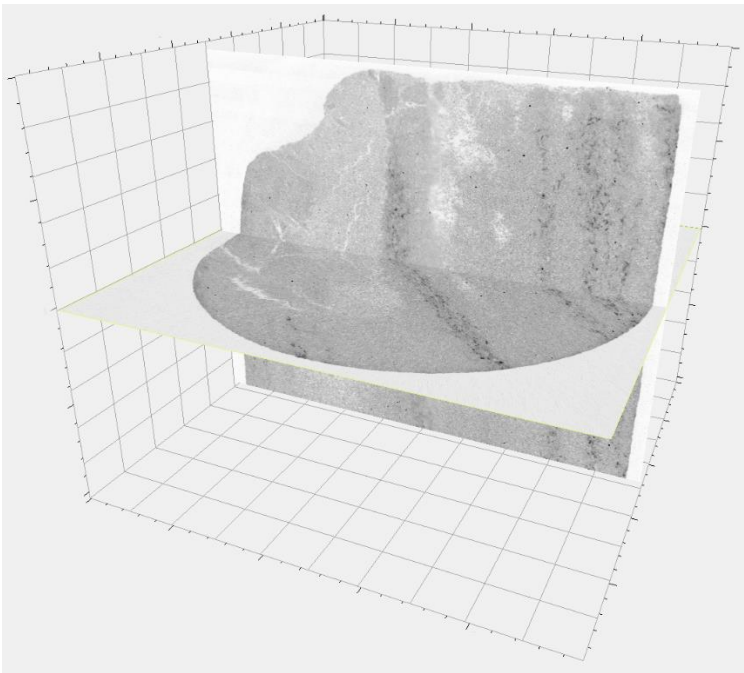
After the tomography scan, the result was stored in the SEG-Y format, commonly used to encode the results of seismic studies. This format is convenient for further analysis, because allows you to import the results in the form of voxels: a three-dimensional array, where each element corresponds to the value of the radiodensity. (see. Hounsfield scale [9])

The value of radiodensity was taken in conventional units of radiodensity - certain number given by the device for computed tomography.

The core sample was imported into the Petrel (Schlumberger's oil engineering software package) (see Fig. 1). SEG-Y format is also useful to show virtual cross-sections of a digital via Petrel (see Fig. 2).



**Fig. 1.** Terrigenous core sample – 3D view of the sample (vertical axe is scaled).

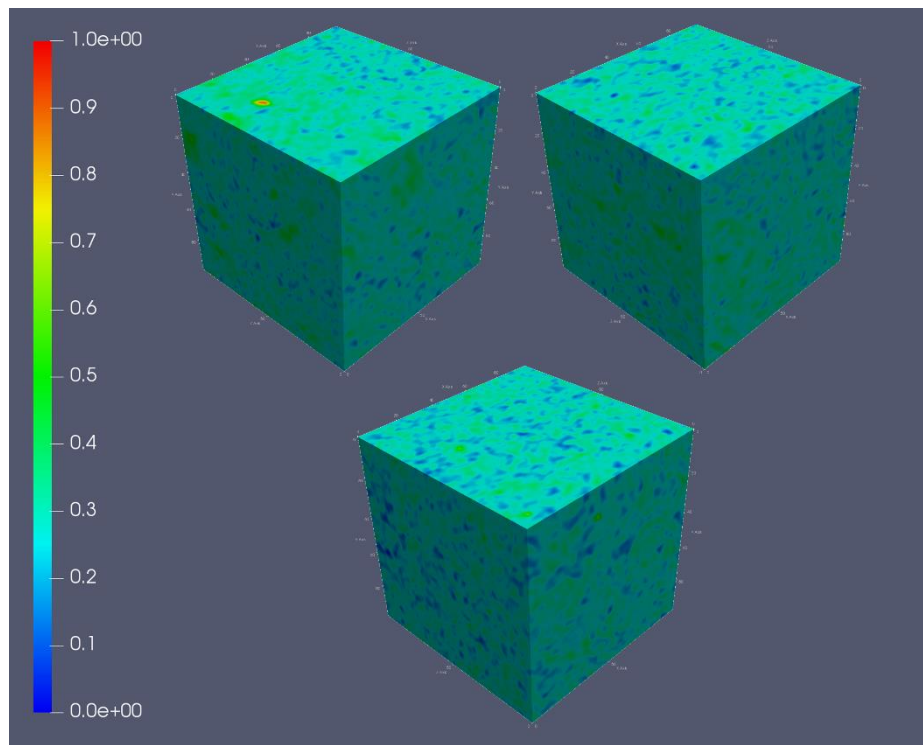


**Fig. 2.** . Terrigenous core sample inline cross-section (vertical axe is scaled).

### 2.3 Digital core model processing

Three 3D cube-samples with the size of 100x100x100 voxels are cut from the core model (Fig.2). The principle of "indicator formalism" is applied to the radiodensity values contained in voxels of cut cubes [10]. The range of voxel values is linearly mapped to the segment [0,1], so that the smallest of the radiodensity values is mapped to 0, the largest value is set to 1. These values are hereinafter referred to as "normalized radiodensity".

Theses cube-samples are transformed to a .grdecl format to evaluate their topological characteristics and to a .vtk format to visualize them using the Paraview software.

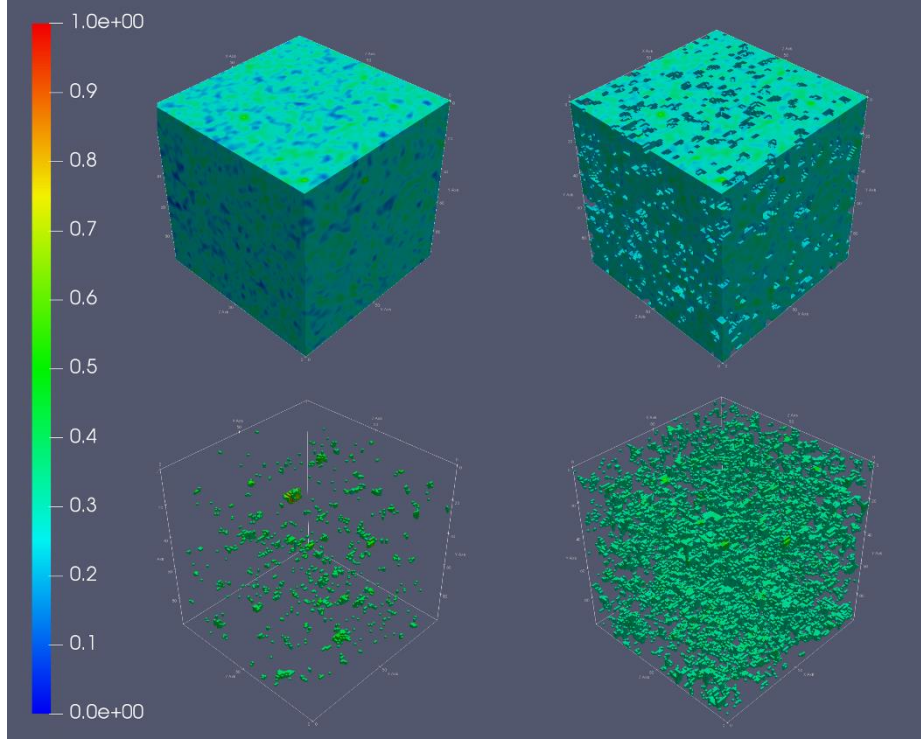


**Fig. 3.** Voxel cubes cut from the digital core model to calculating topological invariants (clockwise - 1, 2 and 3 cube).

To calculate the model's topological characteristics, it is necessary to classify voxels - to divide the them according to the "skeleton of the rock" - "void". Voxels, marked as "skeleton of the rock", are then considered equivalent to elementary cubes from item 1.

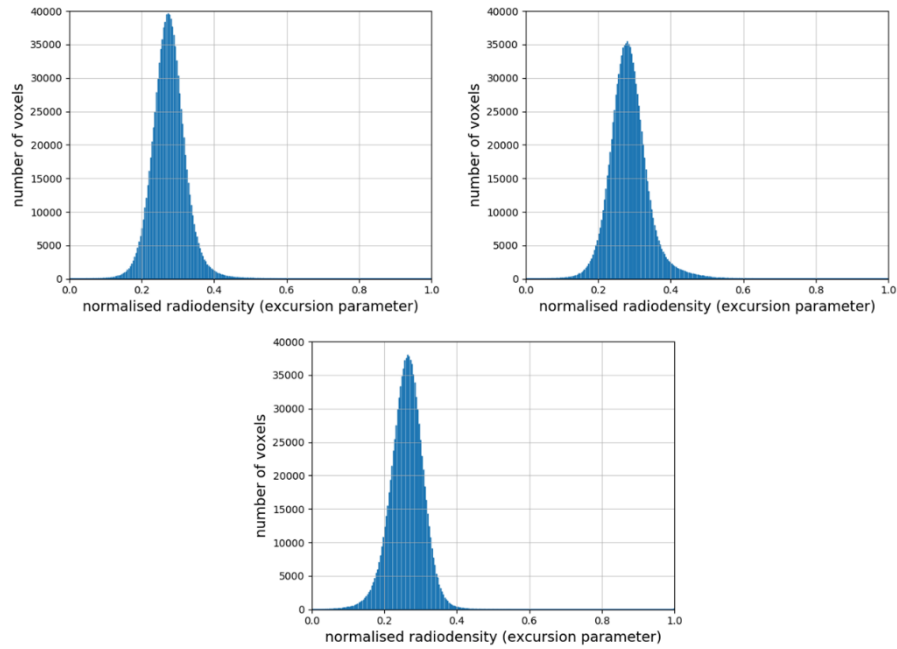
The built-in material classifier of the tomography apparatus is not used in the study. For this purpose, used an excursion parameter  $\alpha$ , the value of which varies from 0 to 1 [3] (Fig.3).

Application of the excursion parameter  $\alpha$  to the voxel model generates a cubic complex, where an elementary cube is a voxel which normalized radiodensity value is greater than  $\alpha$ .



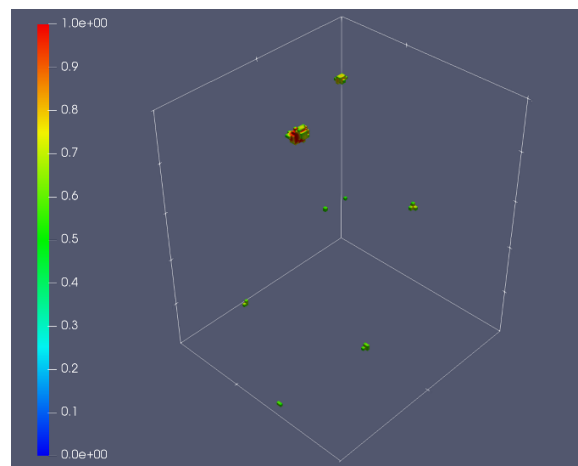
**Fig. 4.** An example of a core model separation of the "rock skeleton" - "void" for the first voxel cube cut from the digital core model (clockwise, the excursion parameter is 0, 0.2, 0.3, 0.4)..

Histograms of the normalized radiodensity distribution for core samples show that the number of voxels with values greater than 0.6 for all samples is smaller than other values. It was expected that the topological characteristics obtained with these values of the excursion parameter will be negligibly small (Fig. 5).



**Fig. 5** Histograms of the normalized radiodensity distribution for core samples (clockwise - 1, 2 and 3 cube).

That is because if the excursion parameter is greater than 0.6, voxels marked as “rock skeleton” belong to local consolidations with the highest density values (Fig 6.).



**Fig. 6.** Local consolidations inside the digital core sample.

Also these histograms show the difference between second cube dispersion:



$$D_{2\text{nd cube}} = 2.86 \cdot 10^{-3} \quad (1)$$

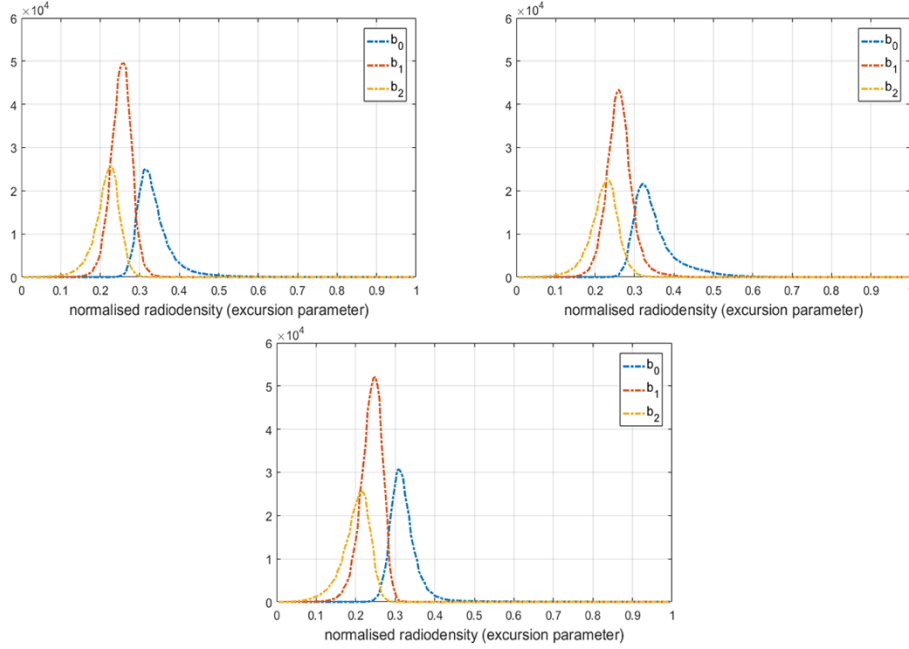
and first, and third cube dispersion:

$$D_{1\text{st cube}} = D_{3\text{rd cube}} = 2.15 \cdot 10^{-3} \quad (2)$$

## 2.4 Topological Characteristics of digital core models evaluation

Topological Characteristics of three-dimensional digital core models in a paper are evaluated similarly to digital geological models example in [3].

As expected from histograms (Fig .6), topological characteristics barely change if the excursion parameter is greater than 0.6. That is because topological characteristics are close to constant while the excursion parameter is increasing (Fig. 7).

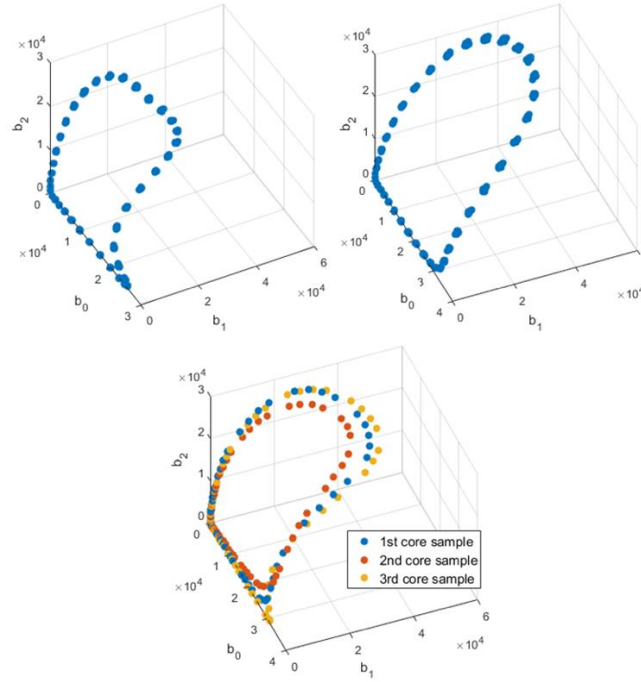


**Fig. 7.** Relationship between the Betti numbers and the excursion parameter (clockwise - 1, 2 and 3 cube).

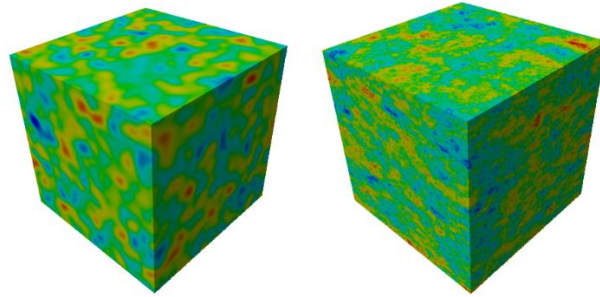
Counted Betti numbers allow to consider digital core models as realizations of random fields – the same way as stochastic digital geological models [3]. It is shown on a 3-D plot of topological characteristics which axes are  $b_0$ ,  $b_1$ , and  $b_2$ , and a parameter is the excursion (Fig. 8). Shape of the obtained “curves” for the first and the third cube is similar to a “curve”, obtained for the Gaussian variogram realization of a geological random field (Fig. 9).

Analogically for the second cube - shape of the obtained “curve” is similar to a “curve”, obtained for the exponential variogram realization of a geological random field (Fig. 9).

In both cases variogram radiuses are little in comparison of linear size of a model: variogram radius does not exceed 2% of linear size of a model. It shows a high compartmentalization of these models.



**Fig. 8.** Topological characteristics in the Betti number axes (clockwise – digital geological model with a Gaussian variogram, digital geological model with an exponential variogram, digital core).



**Fig. 9.** Realization of a geological random field with a Gaussian (left) and exponential (right) variograms.

Obtained differences in these “curves” shape indicate an internal core heterogeneity. This difference in the inner complexity of the second cube from the first and third can be caused by an acid treatment carried out on this core sample.

### 3 Conclusion

Analysis of topological characteristics of digital core models allows to:

- find regions of internal core heterogeneity;
- consider digital core models as realizations of digital geological stochastic models. That is how a developed geostatistics methodology can be used in digital core analysis;

The inner complexity and betti numbers dependencies for digital core models are going to be researched with a larger amount of cut cubes. The assessment by a specialist is required in terms of their inner structure - porosity, the presence of cracks and caverns.

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