



## Offline Measurement of Induction Motor Rotor Time Constant Based on Least Squares Algorithm

---

Cai Meidong and Zhao Yun

EasyChair preprints are intended for rapid dissemination of research results and are integrated with the rest of EasyChair.

November 7, 2020

# Offline Identification of Induction Motor Rotor Time Constant Based on Least Squares Algorithm

Meidong Cai<sup>—</sup> and Yun Zhao<sup>—</sup>

Hubei Key Laboratory of solar energy efficient utilization and storage operation control,  
Hubei University of Technology, Wuhan 430068, China  
Emile:15927305399@163.com

## Abstract

In the indirect rotor flux orientation scheme of induction motor, the control system based on current model has better dynamic and static characteristics, but the current model is very dependent on the rotor time constant. In order to realize the optimal control of induction motor, the off-line measurement method of rotor time constant of induction motor is presented in this paper. The off-line measurement method of rotor time constant based on recursive least squares (RLS) algorithm is analyzed in detail. Through the static characteristics of induction motor, the mathematical model of induction motor is simplified, and then the linear model of least square algorithm is derived. Through a large number of MATLAB simulation analysis, it shows that the results obtained by the recursive least squares method are more accurate than the traditional indirect offline measurement results. Finally, the error analysis of the main factors affecting the measurement results is given.

Keywords — induction motor; rotor time constant; off-line measurement; least square method; error analysis

## 1 Introduction

The vector control of induction motors based on the current model has the advantage of low speed and stability, which makes the current model widely used in induction motor control, but the current model is very dependent on the rotor time constant. The inaccuracy of the rotor time constant will lead to deviations in the field orientation, affecting the output torque of the motor, and then affecting the performance of the control system.

---

<sup>—</sup> Meidong Cai(1995~), male, Huangshi, Hubei, master's degree student of Hubei University of Technology, research direction is high-performance servo motor control algorithm.

<sup>—</sup> Zhao Yun (1985~), male, Wuhan, Hubei, Ph.D., associate professor of Hubei University of Technology, research direction is high-performance servo motor control algorithm.

Generally, the identification of the rotor time constant is divided into offline measurement and online correction<sup>[1]</sup>. The online correction is realized on the basis of offline measurement. The error of the rotor time constant is obtained by means other than the current model, and then the motor is running to eliminate it. Changes caused by factors such as temperature and magnetic field. This article mainly focuses on off-line measurement. Off-line measurement is a commonly used method in current drives. It is simple and reliable. It is an important step to complete the rotor time constant identification<sup>[2]</sup>. In the past ten years, the literature [3-4] has discussed many offline parameter measurement methods of induction motors in a stationary state. Literature [5-9] is a conventional off-line parameter measurement method. It uses DC experiments to measure resistance, single-phase experiments to measure rotor resistance and leakage inductance, and no-load experiments to measure mutual inductance, that is, before the motor runs, different types of currents are applied to the motor. And the voltage signal, and then detect the response of current and voltage, and calculate the required motor parameters according to the equivalent circuit relationship. However, the process of this measurement method is more complicated, and the motor parameters need to be measured step by step, and the measurement accuracy of resistance and inductance will also cause the rotor time constant to be inaccurate. Literature [10-15] uses the recursive least squares algorithm to measure the parameters of the induction motor, derives the estimated model from the dynamic model of the induction motor, and gives a detailed description of the specific measurement process. However, most of these methods use second-order filters or even third-order filters to convert the voltage and current signals, which increases the complexity of the algorithm. This paper mainly uses the recursive least squares algorithm and reduces the second-order filter to the first-order filter. This method only needs current and voltage sensors, does not need to know the motor parameters in advance, and does not need to perform too much coordinate transformation, effectively The complexity of the algorithm is reduced, and the effectiveness of the method is verified by simulation.

## 2 Mathematical Model of Induction Motor and Recursive Least Square Algorithm

### 2.1 Mathematical Model of Induction Motor

Fig.1 introduces the equivalent circuit of an induction motor in a two-phase stationary coordinate system. The  $\alpha$ -axis and  $\beta$ -axis equivalent circuits are coupled with each other.

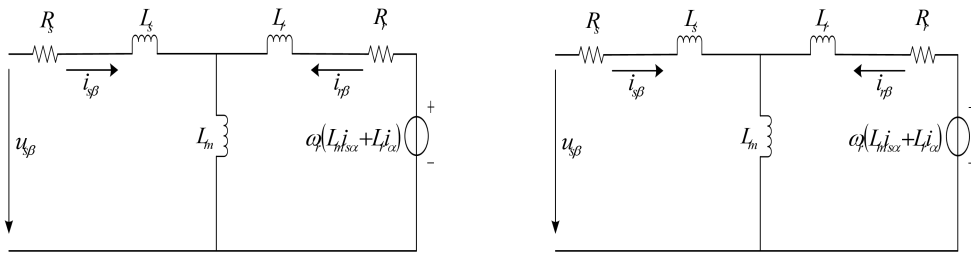


Fig.1 The equivalent circuit diagram of the  $\alpha$ -axis and  $\beta$ -axis of an induction motor

The mathematical model of the induction motor in the two-phase stationary coordinate system is:

$$u_{s\alpha} = (R_s + L_s p) i_{s\alpha} + L_m p i_{r\alpha} \quad (1)$$

$$u_{s\beta} = (R_s + L_s p)i_{s\beta} + L_m p i_{r\beta} \quad (2)$$

$$0 = L_m p i_{s\alpha} + \omega_r L_m i_{s\beta} + (R_r + L_r p)i_{r\alpha} + \omega_r L_r i_{r\beta} \quad (3)$$

$$0 = L_m p i_{s\beta} - \omega_r L_m i_{s\alpha} + (R_r + L_r p)i_{r\beta} - \omega_r L_r i_{r\alpha} \quad (4)$$

$$T_e = \frac{n_p L_m}{L_r} (\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha}) \quad (5)$$

Among them,  $R_s$  and  $R_r$  are stator and rotor resistances respectively,  $L_m$  is mutual inductance,  $L_s$  and  $L_r$  are stator and rotor inductances respectively,  $i_{s\alpha}$  and  $i_{s\beta}$  are the stator currents of  $\alpha$  axis and  $\beta$  axis respectively, and  $u_{s\alpha}$  and  $u_{s\beta}$  are the stators of  $\alpha$  axis and  $\beta$  axis respectively Voltage,  $i_{r\alpha}$ ,  $i_{r\beta}$  are the rotor currents of the  $\alpha$  axis and  $\beta$  axis, respectively,  $\omega_r$  is the motor speed, and  $p = d/dt$  the differential operator.

Since the rotor current cannot be measured, in order to eliminate the rotor current, equations (6) and (7) can be obtained:

$$-p^2 i_{s\alpha} = \frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{1}{T_s} \right) p i_{s\alpha} + \frac{1}{\sigma T_s T_r} i_{s\alpha} - \frac{1}{\sigma L_s} p u_{s\alpha} - \frac{1}{\sigma L_s T_r} u_{s\alpha} \quad (6)$$

$$-p^2 i_{s\beta} = \frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{1}{T_s} \right) p i_{s\beta} + \frac{1}{\sigma T_s T_r} i_{s\beta} - \frac{1}{\sigma L_s} p u_{s\beta} - \frac{1}{\sigma L_s T_r} u_{s\beta} \quad (7)$$

Where  $T_r = L_r / R_r$  is the rotor time constant,  $T_s = L_s / R_s$  is the stator time constant,  $\sigma$  is the leakage inductance coefficient.

In equations (6) and (7), the coefficients of voltage, current and their derivatives can be defined as follows:

$$K_1 = \frac{1}{\sigma} \left( \frac{1}{T_r} + \frac{1}{T_s} \right), \quad K_2 = \frac{1}{\sigma T_s T_r} \quad (8)$$

$$K_3 = \frac{1}{\sigma L_s}, \quad K_4 = \frac{1}{\sigma L_s T_r}$$

Thus, equations (6) and (7) can be rewritten as equation (9):

$$\begin{cases} -p^2 i_{s\alpha} = K_1 p i_{s\alpha} + K_2 i_{s\alpha} - K_3 p u_{s\alpha} - K_4 u_{s\alpha} \\ -p^2 i_{s\beta} = K_1 p i_{s\beta} + K_2 i_{s\beta} - K_3 p u_{s\beta} - K_4 u_{s\beta} \end{cases} \quad (9)$$

Assuming that the stator leakage inductance is equal to the rotor leakage inductance, some parameters of the induction motor are as follows:

$$T_r = \frac{K_3}{K_4}, R_r = \frac{K_4 K_1 - K_2 K_3}{K_3 K_4} \quad (10)$$

$$L_m = \frac{K_4 K_1 - K_2 K_3}{K_4^2} \sqrt{1 - \frac{K_4^2}{K_3 (K_4 K_1 - K_2 K_3)}}$$

$$L_{lr} = \frac{K_4 K_1 - K_2 K_3}{K_4^2} \left( 1 - \sqrt{1 - \frac{K_4^2}{K_3 (K_4 K_1 - K_2 K_3)}} \right)$$

## 2.2 Recursive Least Square Algorithm

In the recursive least squares algorithm, the control model needs to be written in the form of a regression equation.

$$\hat{y}(\hat{\theta}|t) = \hat{\theta}^T(t) \quad (11)$$

Among them  $\hat{y}(\hat{\theta}|t)$  is the prediction vector,  $\Gamma(t)$  is the regression matrix, and  $\hat{\theta}$  is the estimated parameter vector. The regression model can be derived as follows:

$$\hat{y}(\hat{\theta}|t) = i_{s\alpha} \quad (12)$$

$$\Gamma(t) = [\Gamma_1(t) \quad \Gamma_2(t) \quad \Gamma_3(t) \quad \Gamma_4(t)] \quad (13)$$

$$\hat{\theta} = [k_1 \quad k_2 \quad k_3 \quad k_4] \quad (14)$$

The recursive least squares algorithm is as follows:

$$\hat{\theta}(N+1) = \hat{\theta}(N) + M_{N+1} [y(N+1) - \Gamma(N+1)\hat{\theta}(N)] \quad (15)$$

$$M_{N+1} = \frac{P_N \Gamma^T(N+1)}{\lambda + \Gamma(N+1) P_N \Gamma^T(N+1)} \quad (16)$$

$$P_{N+1} = \frac{P_N - M_{N+1} \Gamma(N+1) P_N}{\lambda} \quad (17)$$

In the formula,  $M_{N+1}$  is the gain matrix;  $P_N$  is the covariance matrix, usually the initial value  $P_0 = 10^\beta I$ ,  $I$  is the identity matrix, and  $\beta$  takes a larger positive integer;  $\lambda$  is the forgetting factor ( $0 < \lambda < 1$ ), which can solve data saturation. In this case, the previous data is gradually "forgotten" through exponential decay to show the effect of the new data. Generally take  $\lambda \in [0.8, 1]$ , and take  $\lambda = 1$  for offline identification.

## 3 Implementation of the offline measurement algorithm for the rotor time constant

The offline measurement structure diagram of the rotor time constant of the induction motor is shown in Fig.2. It includes four parts: single-phase rotor test module, digital filter, least square algorithm estimation module and parameter calculation module.

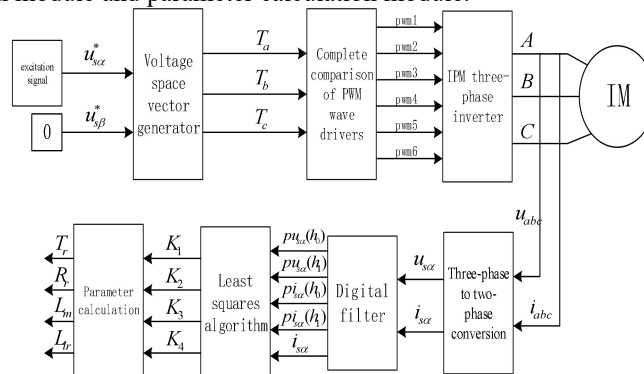


Fig. 2 Offline measurement structure diagram based on least square method

### 3.1 Design of digital filter

Laplace transform on both sides of the state equation (9), the transfer function (18) can be obtained. The transfer function is not only conducive to the evolution of linear models, but also suitable for analyzing state variables in the frequency domain.

$$\frac{i_{s\alpha}}{u_{s\alpha}} = \frac{K_3s + K_4}{s^2 + K_1s + K_2} \quad (18)$$

To perform the least squares recursive operation on the transfer function (18), the form needs to be transformed into a linear model. Generally, the voltage and current differentials are obtained in the voltage and current signals, which will introduce very strong noise, which requires filtering. Since the equation is second-order, it may be better to use a second-order filter first. Select the filter  $G(s) = (s + h_0)(s + h_1)$ , in order to reduce the complexity of the algorithm, the second-order filter is simplified to the first-order filter, defining  $z = s^2 i_{s\alpha} / G(s)$ , according to the transfer function equation (18), the formula (19) can be obtained:

$$z = \begin{bmatrix} -K_1 & -K_2 & K_3 & K_4 \end{bmatrix} \begin{bmatrix} si_{s\alpha} / G(s) \\ i_{s\alpha} / G(s) \\ su_{s\alpha} / G(s) \\ u_{s\alpha} / G(s) \end{bmatrix} \quad (19)$$

According to the definition of z:

$$i_{s\alpha} = z + \frac{(h_0 + h_1)s + h_0h_1}{G(s)} i_{s\alpha} = \frac{(h_0 + h_1 - K_1)s + (h_0h_1 - K_2)}{G(s)} i_{s\alpha} + \frac{K_3s + K_4}{G(s)} u_{s\alpha} \quad (20)$$

After derivation, the linear parameter model can be obtained as follows:

$$i_{s\alpha} = \hat{\theta}^T \Gamma(t) \quad (21)$$

Where,

$$\hat{\theta}^T = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} \frac{K_4 - K_3h_1}{h_0 - h_1} \\ \frac{K_3h_0 - K_4}{h_0 - h_1} \\ \frac{-K_2 + K_1h_1 - h_1^2}{h_0 - h_1} \\ \frac{K_2 - K_1h_0 + h_0^2}{h_0 - h_1} \end{bmatrix}, \quad \Gamma^T(t) = \begin{bmatrix} \Gamma_1(t) \\ \Gamma_2(t) \\ \Gamma_3(t) \\ \Gamma_4(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{s + h_1} u_{s\alpha} \\ \frac{1}{s + h_0} u_{s\alpha} \\ \frac{1}{s + h_1} i_{s\alpha} \\ \frac{1}{s + h_0} i_{s\alpha} \end{bmatrix}$$

The vector  $\hat{\theta}^T$  is the parameter vector to be identified, and  $\Gamma^T(t)$  is the input voltage and current signals. The relationship between these parameters and the actual parameters is as follows:

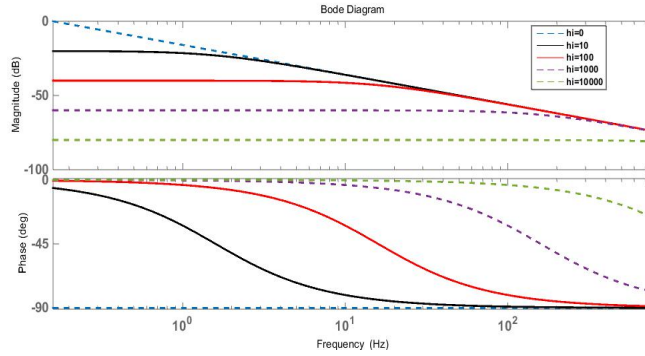
$$\begin{aligned} K_1 &= h_0 + h_1 - k_3 - k_4 \\ K_2 &= h_0h_1 - h_0k_3 - h_1k_4 \\ K_3 &= k_1 + k_2 \\ K_4 &= h_0k_1 + h_1k_2 \end{aligned} \quad (22)$$

In order to realize the algorithm on the digital controller, the system and the filtering link need to be discretized. So when it comes to the selection of the sampling period, if the sampling period is too short, the correlation of the sampling data will increase, and then relying on these data to solve the

parameter estimates, will produce many ill-conditioned equations; if the sampling period is too long, the input cannot be fully utilized. Information in the signal excitation process, which in turn affects parameter identification. Combining the above situation, the sampling period selected in this article is 0.0001s, and because the first-order filter parameters  $h_0$  and  $h_1$  are related to parameter calculations, in principle  $1/h_i (i=0,1)$  is greater than the sampling period. In addition, the first-order filter  $1/(s+h_i)$  is subjected to a bilinear transformation, so that the discretization model of the first-order filter can be obtained:

$$H(z) = \frac{Tz + T}{(2 + Th_i)z - 2 + Th_i} x \quad (i = 0,1) \quad (23)$$

Among them,  $H(z)$  is the output of the filter,  $x$  is  $i_{sr}$  or  $u_{sr}$ , and  $T$  is the sampling period. The Bode plot of this filter is shown in Fig.3.



**Fig.3.** The amplitude-frequency and phase-frequency response curves of the filter

According to the amplitude-frequency and phase-frequency response curves of the first-order filter, when  $h_i$  is between 10 and 100, the response speed and dynamic performance of the filter are better, so this article  $h_0=40, h_1=90$ .

## 3.2 Conditions to be met by the excitation signal

In order to make the identified parameters converge to the true value, it is necessary to ensure that the input voltage and current signals are continuously excited and bounded, where continuous excitation means that the frequency spectrum of the signal is sufficient.  $u_{sr}^*$  is the excitation signal,  $u_{sr}$  and  $i_{sr}$  obtained through the output of the motor, and the bounded situation completely depends on the excitation signal. In most cases, the excitation signal contains at least  $n/2$  ( $n$  is the number of identification parameters,  $n=4$  in this article) sine waves of different frequencies. Finally, a linear combination of a DC signal and two sinusoidal signals of different frequencies is selected as the excitation signal we need.

## 4 Simulation Verification

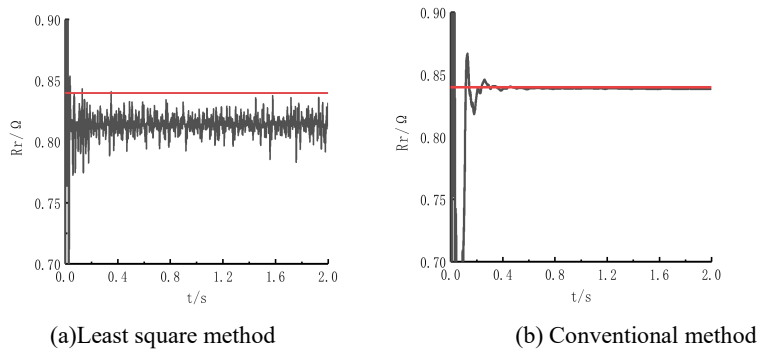
Using MATLAB to carry on simulink simulation, this paper has made simulation verification on the off-line measurement of rotor time constant based on the recursive least square algorithm and the

conventional method in literature [9]. The parameters selected in the simulation are based on a 3.7KW induction motor, the number of pole pairs is 2, the Rated current is 9.1A, Rated voltage is 380V, and Rated speed is 1500rpm, the stator resistance is  $1.029\Omega$  and the rotor resistance is  $0.84\Omega$ , the mutual inductance is  $0.1198\text{H}$ , stator leakage inductance is  $5.51\text{mH}$  and the rotor leakage inductance is  $5.51\text{mH}$ .

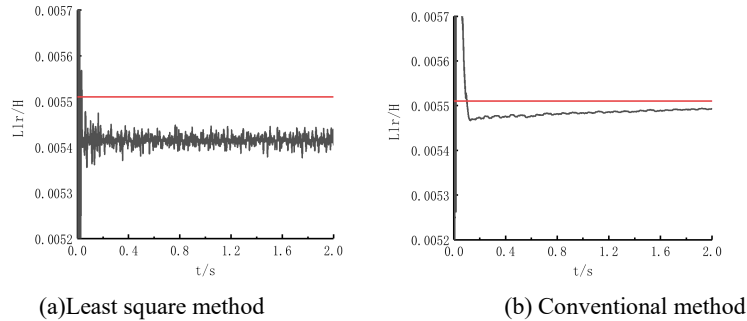
From the given parameters, the rotor time constant can be calculated as:

$$T_r = \frac{L_m + L_{lr}}{R_r} = 0.1492\text{s}$$

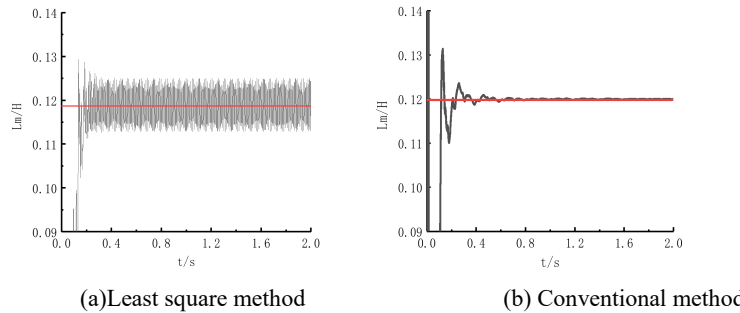
The simulation parameter identification curve is shown in Fig.5-Fig.8, and red line is reference value, black line is Identification value.



**Fig.5**  $R_r$  identification value under different algorithms

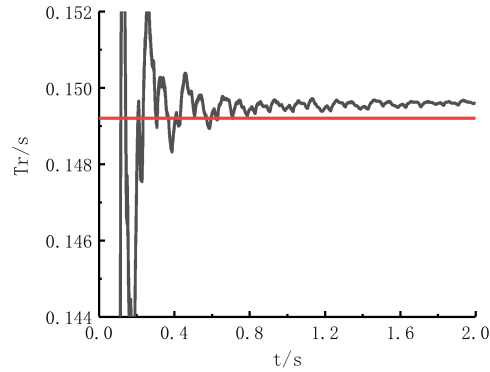


**Fig.6**  $L_{lr}$  identification value under different algorithms



**Fig.7**  $L_m$  identification value under different algorithms





**Fig.8** Tr identification value under the least square method

The error values measured based on the least square method and the conventional method are shown in Table 1:

parameter	$R_r/\Omega$	$L_m/H$	$L_{lr}/H$	$T_r/s$
Leastsquares method	0.8387	0.120	0.005493	0.1496
error	0.155%	0.167%	0.309%	0.268%
Conventional method	0.8274	0.1164	0.005405	0.1472
error	1.50%	2.838%	1.906%	1.340%

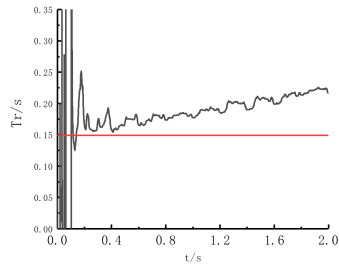
**Table 1:** The error values

According to Table 1, the error value measured based on the least square method is smaller than the error value measured by the conventional method. Regarding the error produced by the least square method measurement, this article makes the following analysis:

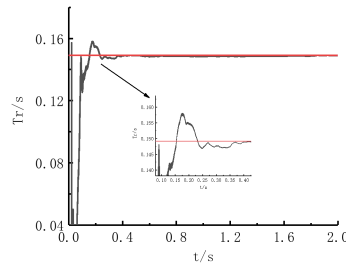
Error 1: the influence of excitation signal

For the above situation, to ensure that the parameters completely converge to the true value, the excitation signal needs to be fully excited, that is, the excitation signal with enough frequency points is required.

Fig.9 shows the rotor time constant identification curve of the excitation signal without the DC component, and Fig.10 shows the  $T_r$  identification curve of the sum of the sinusoidal signal with the DC component and four frequency points.



**Fig.9.** without DC component



**Fig. 10.** four different frequency point

It can be seen from the figure that when the DC component is not added, a relatively large rotor time constant error is obtained. After the DC component is added, the more frequency points the excitation voltage contains, the closer the measured rotor time constant is to its true value. Although the parameters converge to the true value in a long time, with the increase of the excitation signal frequency, the rotor time constant changes greatly in a short time, and the jitter with different frequencies will be more serious.

Error 2: The filter design is inappropriate

Through a lot of simulation analysis, the rotor time constant measurement method based on the least square method is more sensitive to the cut-off frequency of the filter. Within a certain range, the increase of the cut-off frequency may make the measured value of the rotor time constant smaller; the cut-off frequency may decrease Increase the measured value of the rotor time constant.

Error 3: Reference value and calculation error

The reference value in the simulation is obtained through the identification result of the high-performance inverter and the high-precision LCR meter, and there may be a certain error with the real value. In addition, the error caused by parameter calculation will also affect the accuracy of the simulation results.

## 5 Conclusions

This article analyzes and discusses each link in the rotor time constant measurement process based on the least square method. By reducing the second-order filter to the first-order filter, avoiding the second derivative of the filtered signal, and introducing the design of the filter and the selection of the excitation signal. Then, compare the rotor time constant measured values of the conventional method and the least square method. At the same time, a number of factors affecting the measurement results of the least squares method are analyzed and discussed, which enhances the practicability of the method. The measurement method used in this article has the following advantages:

- 1) No need to measure each motor parameter step by step, you can directly measure the rotor time constant;
- 2) The measurement method is relatively simple, only involves the first-order filter, and the amount of calculation is small;
- 3) The measurement method has small error and high accuracy. Compared with the reference value, the error of the measurement results made in this article are all within 0.4%.

## References

- [1] Cheng Chao, Cheng Shanmei. Motor parameter identification and self-tuning strategy based on vector control [J]. Micromotor, 2000,33(6):19~21.
- [2] Yanhui He, Yue Wang, et al. Parameter Identification of an Induction Machine at Standstill Using the Vector Constructing Method[J]. IEEE Transactions on Power Electronics, 2012, 27(2): 905-915.
- [3] Shen G T, Wang K, Yao W X, et al. DC Based Stimulation Method for Induction Motor Parameters Identification at Standstill Without Inverter Nonlinearity Compensation [C] //2013 IEEE Energy Conversion Congress and Exposition (EC-CE). Denver CO, 2013, 9: 5123-5130

- [4] M. Carraro, M. Zigliotto. Automatic parameter identification of inverter-fed induction motors at standstill [J]. IEEE Transactions on Industrial Electronics, 2014, 61(6): 4605-4613.
- [5] Fan Shengwen, Ding Maoqi, Li Zhengxi, et al. Simulation study on off-line identification of vector control induction motor parameters [J] . Metallurgical Automation, 2012, 36(4): 60-63.
- [6] Wang Mingyu, Xian Chengyu, Hui Yaqian. Off-line identification technology of induction motor vector control parameters[J]. Journal of Electrotechnical Technology, 2006, 21(8): 90-96.
- [7] Dai Liang, Huang Zhiyuan, Chen Yeming, et al. Offline identification method of asynchronous motor parameters in a static state [J] . Micro Motor, 2014, 42(2): 33-36.
- [8] Luo Hui, Liu Junfeng, Wan Shuyun. Offline identification of induction motor parameters [J]. Electric Drive, 2006, 36(8):16-21.
- [9] He Yanhui, Wang Yue, Wang Zhaoan. Improved algorithm for off-line identification of asynchronous motor parameters[J]. Journal of Electrotechnical Technology, 2011, 26(6): 74-80.
- [10] M. Cirrincione, M. Pucci, and G. Vitale, A least-squares based methodology for estimating the electrical parameters of induction machine at standstill,in Proc. IEEE Int. Symp. Ind. Electron. (ISIE), 2002, vol. 2, pp. 541–547.
- [11] Cirrincione M, Pucci M, Cirrincione G, et al. A new experimental application of least-squares techniques for the estimation of the induction motor parameters[J].IEEE Transactions on Industry Applications, 2003, 39(5): 1247-1256.
- [12] Wang Kaiyu , Chiasson J , Bodson M , et al . A nonlinear least-squares approach for identification of the induction motor parameters[J] . IEEE Transactions on Automatic Control , 2005, 50(10): 1622-1628.
- [13] Vieira R P , Azzolin R Z , Grundling H A . Parameter identification of a single-phase induction motor using RLS algorithm[C]//IEEE International Conference on Power Electronics. Bonito-Mato Grosso do Sul, Brazilian: IEEE, 2009: 517-523.
- [14] L. Yang, X. Peng, and Z. Li, Induction motor electrical parameters identification using RLS estimation, in Proc. IEEE Int. Conf. Mechanic Autom. Control Eng.2010, pp. 3294–3297.
- [15] Zhang Hu, Li Zhengxi, Tong Chaonan. Offline identification of induction motor parameters based on recursive least squares algorithm [J]. Proceedings of the Chinese Society for Electrical Engineering, 2011, 31(18): 79-86.