Quantum heat engine (QHE) with Heisenberg XXZ model

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***Abstract*—Among of the most results in quantum thermodynamics is to prove the effects of quantum phenomena on the thermodynamics concept. In this work we propose a quantum heat engine by using as working substance (WS) two particles of spin-$\frac{1}{2}$ and spin-1 respectively. The proposed model is described by the Heisenberg XXZ model with the Dzyaloshinskii-Moriya (DM) interaction in the presence of magnetic field B. The analyse the influence on the efficiency and work of the (DM) interaction and the field is discussed.**

***Index Terms*—quatum heat engine, quantum Otto cycle, heisenberg models.**

I. Introduction

Quantum heat engines (QHEs) are a devicese which transforme energy from the heat form to work or power form like the classical engine , the diffenrece is that This machine have a quantum systems as working substance.

Recently after Scovil $et\ al$ idea about the three level spin refregiration [1] many peoples start to study quantum heat engines (QHEs) [2]–[4]. work produced from quantum heat engine using harmonic oscillator as a quantum system [5]–[8], the spins [9]–[17]. Ther is also some resarch about the quantum heat engine by using the cavity [18], [19],and systems like two level systems [20], meaning that quantum phenomena has an intersting role to produce the quantum thermodynamics quantities like entaglement which has many applications in quantum information domaine in particular in communication processing. This phenomina (entanglement) is introduced also in quantum thermodynamics speashly in quantum heat engine by Zhang et al. which they studied A quantum heat engine (QHE) with a working substance of two-particle (1/2,1) Heisenberg XXX model with Dzyaloshinskii-Moriya (DM) interaction in the external magnetic field B [**?**]

Our work is to study a QHEs with a two-qubit isotropic Heisenberg XXZ . we want to analyse the efficiency η and work W and also the performance of the machine in the presence of magnetic field B and the DzyaloshinskiiMoriya (DM) interaction.

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The work is organized as follows: In Sec II the proposed model. In sec III we guive the detailes of the cycle of the machine . In Sec IV we discuss the results and finaly in Sec V The conclusion.

II. The proposed model

In this model we consider two particles, the first one have the spin -1- and the second one have the spin-$\frac{1}{2}$- present the working substance (WS). the two particles are coupled by the Heisenberg XXZ model with the Dzyaloshinskii-Moriya (DM) interaction and also the inhomogeneous magnetic field B. The Hamiltonien of the model can be written as:

$$\begin{aligned} H = J_x S_1^x S_2^x + J_x S_1^y S_2^y + Jz S_1^z S_2^z + D(S_1^x S_2^y \\ -S_1^y S_2^x) + (B+b) S_1^z + (B-b) S_2^z, \end{aligned} \tag{1}$$

where Jx and Jz are the constants coupling , with S_1^β,S_2^β where $\beta = x, y, z$ are the the spin operators of the spin-$\frac{1}{2}$ and spin 1. D and b are the strength of the DM-interaction and the degree of inhomogeneity of the magnetic field B respectively. The eigenvalues of the Hamiltonian (1) are given as

$$\begin{aligned} E_{1,2} &= \frac{1}{2}(\mp b \mp 3B + J_z) \\ E_{3,4} &= \frac{1}{4}\left(2b + 2B - J_z \mp \omega\right) \\ E_{5,6} &= \frac{1}{4}\left(-2b - 2B - J_z \mp \phi\right). \end{aligned} \tag{2}$$

where $\omega = \sqrt{8(D^2 + J_x^2) + (-4b + J_z)^2}$ and $\phi = \sqrt{8(D^2 + J_x^2) + (4b + J_z)^2}$ and in the thermal equilibrium between the working substance and the baths the density matrix can be written as

$$\varrho(T) = \sum_i p_i |\psi_i><\psi_i| \quad , \quad p_i = \frac{e^{\frac{E_i}{k_B T}}}{Z} \tag{3}$$

where $|\psi_i>$ are the eigenstates of the hamiltonian H, p_i is the probability of occupation of each state ,k_B is Boltzmann's constant, T the temperature T and Z is the partition function of the system, $Z = \sum_i e^{\frac{E_i}{k_B T}}$.

III. CYCLE OF THE MACHINE

The cycle of the machine is composed by four step two adiabatic processes and two isochoric process.

In the first step the working substance (WS) is coupled with the hot bath at temperature $T1$, after few time of the contact some energy as heat form is transferred from the hot bath to the working substance (WS) which makes the initial occupation probability P_{0i} changes to P_{1i}, such that the eigenenergies E_{1i} are fixed with the magnetic field $B = B_1$.

In the second step the working substance (WS) is decoupled from the hot bath which means no heat is transferred, only the work is performed. The adiabatic process changes the eigenenergy from E_{1i} to E_{2i}, with the magnetic field is $B = B_2$ and the occupation probability is kept fixed P_{1i}.

In the step number three the working substance (WS) is coupled to the cold bath at the temperature $T2$, during the contact the occupation probability changes from P_{1i} to P_{2i} with the eigenenergy E_{2i} and the magnetic field B_2 are kept fixed.

And finaly the step number four the working substance (WS) is disconnected to the cold bath, the occupation probability from the cycle $P_{2i} = P_{0i}$ is kept fixed, then the changed eigenenergy returns to the initial value from E_{2i} to E_{1i}.

Now we can get the eigenvalues of the heat bath from the equation (2) as:

$$\begin{aligned} E_{11,12} &= \frac{1}{2}(\mp b \mp 3B_1 + J_z) \\ E_{13,14} &= \frac{1}{4}\left(2b + 2B_1 - J_z \mp \omega\right) \\ E_{15,16} &= \frac{1}{4}\left(-2b - 2B_1 - J_z \mp \phi\right). \end{aligned} \tag{4}$$

and the eigenvalues of the cold bath

$$\begin{aligned} E_{21,22} &= \frac{1}{2}(\mp b \mp 3B_2 + J_z) \\ E_{23,24} &= \frac{1}{4}\left(2b + 2B_2 - J_z \mp \omega\right) \\ E_{25,26} &= \frac{1}{4}\left(-2b - 2B_2 - J_z \mp \phi\right). \end{aligned} \tag{5}$$

where $\omega = \sqrt{8(D^2 + J_x^2) + (-4b + J_z)^2}$ and $\phi = \sqrt{8(D^2 + J_x^2) + (4b + J_z)^2}$
the heat absorbed from the hot bath (step 1) and the heat realized to the cold bath (step 3) are giving as

$$Q_{abs} = \sum_{i=1}^{6} E_{1i}(P_{1i} - P_{2i}) \quad \text{and} \quad Q_{rea} = \sum_{i=1}^{6} E_{2i}(P_{2i} - P_{1i}) \tag{6}$$

and from the first law of thermodynamics (balance energie) the net work produced is given as

$$W = \sum_{i=1}^{6} (E_{1i} - E_{2i})(P_{1i} - P_{2i}) \tag{7}$$

and the efficiency η of the machine is

$$\eta = \frac{W}{Q_{abs}} = \frac{Q_{abs} + Q_{rea}}{Q_{abs}} = 1 + \frac{Q_{rea}}{Q_{abs}}. \tag{8}$$

IV. RESULTS AND DISCUSSION

In the firt we studied the impact of the strength coupling on the efficiency η, we also plot the efficiency as function of the stength coupling J_z with taking different values of D (D=0,D=2,D=4 and D=6). As shown in fig.2 when the coupling is week we note that there is no efficiency meaning that the machine can note operate, and for $J_z \geq 0,5$ the efficiency emerge, and for the strong coupling the efficency η get tha maximum value. Also from the graph we can see the influence of the (DM) interaction such that it's increase correspond to the rapid stability of the efficiency η.
In fig.3 we plot the efficiency η and work W as fuction of the (DM)interaction strength D. we find that when D is smaler, D $\in$[0;0,6] for the XXZ model and D $\in$[0;1,2] for the XX model, the working substance can note produce the efficiency η and work W. And also from the graph we note that the increase of D correspond of the increase of both of them so we can say that the (DM) interaction have apositive rool for the efficency and work.In the other side when we compare the XXZ and XX models we find that (QHE) operate rapidly for the XXZ model than the XX model when we vary the parametre D.

In the fig.4(a,b)we plot the efficiency η and work W as function of yhe magnetic field B_1 and B_2. As the figure shown the efficiency and work are positive for the all cases $B_1 > B_2$ and $B_1 < B_2$.

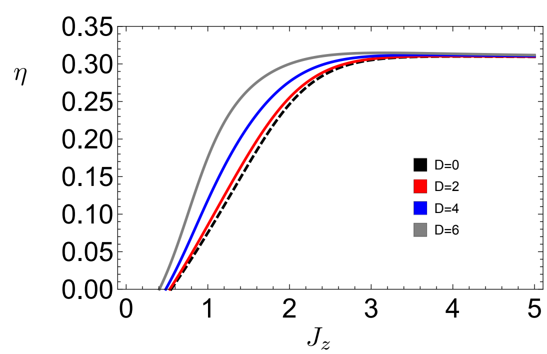

Fig. 1. the efficiency as function of the coupling strength J_z with the parametres ($B1 = 1$, $B2 = 0.6$,$b = 0.1$, $J_x = 5$, $T1 = 1.6, T2 = 1$) for the Heisenberg XXZ model.

V. CONCLUSION

We have studied a quatum heat engine (QHE) with the XXZ heisenberg model which the working substance (WS) is constituted by two paricals (Spin-1 , Spin-$\frac{1}{2}$). We have analysed the efficiency η by varying the constante coupling

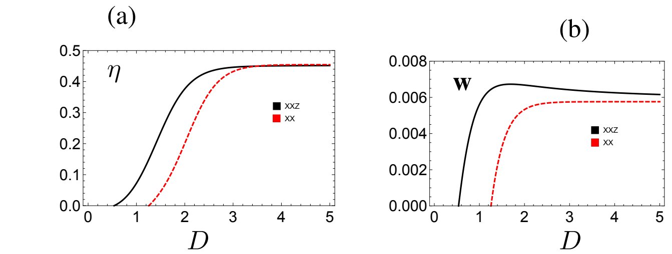

Fig. 2. the efficiency η and the net work W (a),(b)respectively as function of the (DM) interaction strength D with the same values of parametres of fig.2 exept ($B_2 = 0,5$,$J_x = 3$, $J_z = 5$,$T_2 = 2$) for the XXZ model and the similar for the XX model just $J_z = 0$.

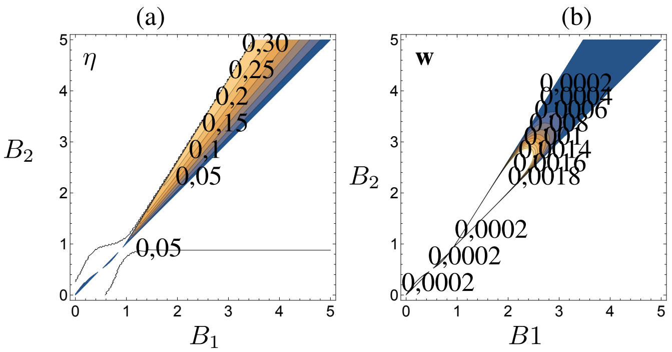

Fig. 3. the efficiency η (a) and the net work W (b) , as function of the magnetic field B_1 and $B2$ for the parametres ($D = 0,5$,$b = 0,1$,$J_x = 2$, $J_z = 5$,$T_1 = 1,6$, $T_2 = 1$) for the XXZ model .

in the direction z J_Z and the (DM)interaction strength D by showing that the (DM) interaction have an importante rool in improving the efficiency η and work W specially when the constante coupling is strang, also by comparing the Heisenberg XXZ model with the Heisenberg XX model we find that the efficiency and work within the for the firt one the machine can operate rapidly than the second one when we varied the (DM)interaction strength D . And finaly we we sse the impact of the magnetic field on the efficency η and work W such that we note that both of them can produced positively in $B_1 > B_2$, $B_1 < B_2$ cases .

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