A Study on Ultrasonic Shear Horizontal Waves in Composite Structures

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Abstract. Composite materials, which are generally lighter, stronger and more durable than conventional materials, have been increasingly used in aerospace and automotive industries in recent decades. Ultrasonic guided waves have been proved to be an advanced technique for characterizing composite materials and structures since they can cover a large inspection area with a low attenuation effect. This paper discusses the motions of shear horizontal (SH) guided waves generated by a time-harmonic source in multilayered composite structures. We propose explicit expressions of SH waves in composite structures based on the transfer matrix method. The amplitudes of SH wave motions due to the time-harmonic loading are then theoretically derived using reciprocity theorems. Afterward, the phase and group velocity dispersion curves obtained by solving the characteristic equation are superimposed by quantitative amplitude spectra. The improved dispersion curves can enhance the wave mode and frequency selection in the ultrasonic evaluation of composite structures.

Keywords: Shear-horizontal waves, Laminated composites, Reciprocity theorem, Dispersion curves.

1 Introduction

The majority of materials used in the aerospace industry nowadays are dominated by composites due to their superior properties such as good stiffness to weight ratio, high durability, resistance to corrosion, mildew, noise, or reducing electromagnetic responses. These characteristics make composites suitable for commercial aircraft, spacecraft, or even military jets. Specifically, Boeing 787 Dreamliner, the company's newest product line, is 80% composite by volume and 50% by weight [1]. Composite structures, however, are facing the issue of degradation due to micro-crack or delamination. That problem raises a high demand for inspecting and monitoring such structures. Ultrasonic guided waves have proved their efficiency in the large-area inspection of aerospace composites, see [1-3]. Nondestructive evaluation and structural health monitoring are the two mainstream topics that have been considered by numerous scholars and engineers in the field.

Research on ultrasonic guided waves in composite materials are comprehensively presented in several textbooks, e.g., [1, 4-6]. The transfer matrix method (TMM) for modeling guided waves in multilayered structures was first provided in [7] and then corrected in [8]. However, TMM must deal with numerical instability when considering high frequency and thickness. To avoid this numerical problem, the global matrix method is introduced in [9]. The newest matrix technique called the stiffness matrix method (SMM) is proposed in [10]. The SMM has the speed of TMM while maintaining stability like GMM. Although if only SH waves are concerned, the TMM is the most suitable because it does not contain the exponential terms in the diagonal line of the matrix thus the numerical instability does not exist. Moreover, the TMM is straightforward to implement and write the explicit solution. To obtain the full solution of guided waves, several analytical approaches have been developed, such as residue theorem [11], normal mode expansion [12], or reciprocity theorem [13]. However, to the best of the authors' knowledge, only the reciprocity theorem holds for closed-form solutions and has a simple implementation for multilayered anisotropic systems, see [14-20].

This paper first presents the explicit expressions of SH guided waves in composite plates. The dispersion curves are obtained using a root-finding algorithm. Based on the explicit expressions, closed-form solutions of SH waves generated by a time-harmonics source are derived. The amplitude spectra are then integrated into the dispersion curves to highlight the magnitude of wave signals.

2 Shear-horizontal waves in a laminated composite plate

We first consider the propagation of SH guided waves in an orthotropic layer of thickness h, see **Fig. 1**, which has the stiffness matrix C. The equation of motions of the guided waves in the plate can be written as

$$\sigma_{ij,j} = \rho \ddot{u}_i \quad (i, j = x, y, z) \tag{1}$$

where σ_{ij} denote stress components, u_i represents displacement components, and ρ is the mass density. In pure shear-horizontal waves, only the motion u_y is considered, and it should also be independent of y. Thus, the equation of SH guided waves can be expressed as

$$C_{66} u_{y,xx} + C_{44} u_{y,zz} = \rho \ddot{u}_y \tag{2}$$

where C_{44} and C_{66} are components of the stiffness matrix C. The general solution of wave motion propagating in the x-direction can be written as

$$u_y = U e^{ik(x+\alpha z-ct)} \tag{3}$$

where U is the unknown amplitude, k stands for the wavenumber, c indicates phase velocity, and α is the ratio of wavenumber between z and x direction.

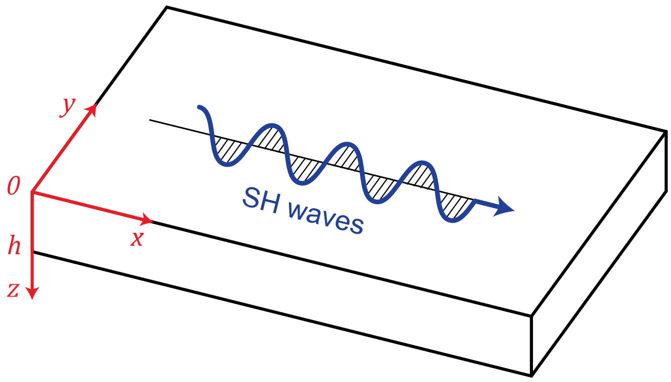


Fig. 1. SH guided waves in an orthotropic layer

Substituting Eq. (3) into Eq. (2) yields

$$\alpha_1 = -\alpha_2 = \sqrt{\frac{\rho c^2 - C_{66}}{C_{44}}} \tag{4}$$

where α_1 and α_2 are two solutions of α.
Then the solution of SH waves can be rewritten in the form of two partial waves as

$$u_y = \left(U_1 e^{ik\beta z} + U_2 e^{-ik\beta z}\right) e^{ik(x-ct)} \tag{5}$$

where $\beta = \alpha_1$. Using Hooke's law, the stress expression is described as

$$\sigma_{yz} = ikC_{44}\beta\left(U_1 e^{ik\beta z} - U_2 e^{-ik\beta z}\right) e^{ik(x-ct)} \tag{6}$$

The relation of stress and displacement between the lower surface ($z = 0$) and upper surface ($z = h$) of the orthotropic layer can be expressed as

$$\begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_{z=h} = [T] \begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_{z=0} \tag{7}$$

where $[T]$ is the transfer matrix given by

$$[T] = \begin{bmatrix} 1 & 1 \\ ik\beta C_{44} & -ik\beta C_{44} \end{bmatrix} \begin{bmatrix} e^{ik\beta h} & 0 \\ 0 & e^{-ik\beta h} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ ik\beta C_{44} & -ik\beta C_{44} \end{bmatrix}^{-1} \tag{8}$$

If we consider an N-layer composite plate, see **Fig. 2**, consisting of several orthotropic laminae in the same direction or the cross-ply direction, the boundary at interfaces should indicate the continuity of stress and displacement, i.e.,

$$\begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_{z=0}^{(n)} = \begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_{z=h}^{(n-1)} \tag{9}$$

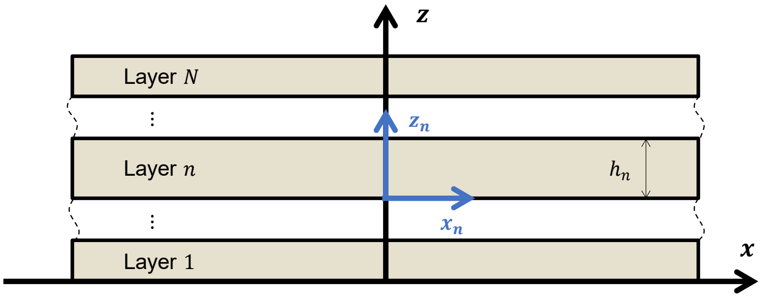


Fig. 2. Laminated composite plate

Therefore, we may derive the relation between the bottom and top layers as

$$\begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_{z=h}^{(N)} = [TT] \begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_{z=0}^{(1)} \tag{10}$$

where TT is the total transfer matrix which can be computed as

$$\begin{bmatrix} TT_{11} & TT_{12} \\ TT_{21} & TT_{22} \end{bmatrix} = \prod_{i=1}^{N} [T]^{(i)} \tag{11}$$

By invoking the traction-free boundary conditions at the lower and upper surfaces of the plate, we may write

$$TT_{21} = 0 \tag{12}$$

Equation (12) is called the characteristic equation which describes the dispersion of SH waves. Using a root-searching algorithm, we then obtain the dispersion curves for both phase and group velocity. The dispersion figures will be shown in the result section.

Invoking the boundary condition at $z = 0$, we may write

$$U_1^{(1)} - U_2^{(1)} = 0 \tag{13}$$

or

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}^{(1)} = A \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}^{(1)} \tag{14}$$

where

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{15}$$

and A is an arbitrary constant.
Thus, we have

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}^{(n)} = \prod_{i=1}^{n} [T]^{(i)} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}^{(1)} \tag{16}$$

The expressions of displacement and stress fields for an arbitrary layer now can be rewritten as

$$u_y = A\left(d_1 e^{ik\beta z} + d_2 e^{-ik\beta z}\right) e^{ik(x-ct)} = AU(z) e^{ik(x-ct)} \tag{17}$$

$$\sigma_{yz} = ikC_{44} A\left(\beta d_1 e^{ik\beta z} - \beta d_2 e^{-ik\beta z}\right) e^{ik(x-ct)} = ikC_{44} A\Sigma(z) e^{ik(x-ct)} \tag{18}$$

These equations have only one unknown relative amplitude A. The solutions of SH waves generated by a time-harmonics source will be obtained using the reciprocity theorem. This approach has shown to be an efficient analytical tool for wave propagation problems [15, 17, 21].

Basically, a reciprocity theorem describes a relation between two elastodynamics states. One state can be considered as actual state $\mathcal{A}$ which describes the SH guided waves generated by a time-harmonics load. The other one is the propagation of free SH waves, called virtual state $\mathcal{B}$. The formula of the reciprocity theorem for a multi-body system can be written as, see [13],

$$\sum_{n=1}^{N} \int_{V_n} \left(f_j^{\mathcal{A}} u_j^{\mathcal{B}} - f_j^{\mathcal{B}} u_j^{\mathcal{A}}\right) dV_n = \int_{\bar{S}_n} \left(\tau_{ij}^{\mathcal{B}} u_j^{\mathcal{A}} - \tau_{ij}^{\mathcal{A}} u_j^{\mathcal{B}}\right) n_i d\bar{S}_n \tag{19}$$

Where f is body force, n is the normal unit vector, $\bar{S}_n$ $(n = 1,2, \dots, N)$ are external boundaries, which are contours around V_n without the interface with other bodies.

The body load applied at (x_0, z_0) can be expressed in forms of Delta function as

$$f_y = P.\delta(x - x_0)\delta(z - z_0) e^{-ikct} \tag{20}$$

Considering layer n, the expressions of state $\mathcal{A}$ can be proposed as a combination of all modes as

$$u_y^{\mathcal{A}} = \sum_{i=1}^{m} A^{(m)} U^{(m)}(z) e^{ik^{(m)}(x - c^{(m)} t)} \tag{21}$$

$$\sigma_{yz}^{\mathcal{A}} = \sum_{i=1}^{m} A^{(m)} \Sigma_{yz}^{(m)}(z) e^{ik^{(m)}(x - c^{(m)} t)} \tag{22}$$

$$\sigma_{yx}^{\mathcal{A}} = \sum_{i=1}^{m} A^{(m)} \Sigma_{yx}^{(m)}(z) e^{ik^{(m)}(x-c^{(m)}t)} \tag{23}$$

whereas state $\mathcal{B}$ can be chosen as mode p of free waves, i.e.,

$$u_y^{\mathcal{B}} = B^{(p)} U^{(p)}(z) e^{-ik^{(p)}(x+c^{(p)}t)} \tag{24}$$

$$\sigma_{yz}^{\mathcal{B}} = B^{(p)} \Sigma_{yz}^{(p)}(z) e^{-ik^{(p)}(x+c^{(p)}t)} \tag{25}$$

$$\sigma_{yx}^{\mathcal{B}} = A^{(p)} \Sigma_{yx}^{(p)}(z) e^{-ik^{(p)}(x+c^{(p)}t)} \tag{26}$$

where $(m), (p)$ denote mode orders, and

$$\Sigma_{yz}^{(j)} = ik^{(j)} C_{44} \left(\beta d_1 e^{ik^{(j)}\beta z} - \beta d_2 e^{-ik^{(j)}\beta z} \right) \tag{27}$$

$$\Sigma_{yx}^{(j)} = ik^{(j)} C_{66} \left(d_1 e^{ik^{(j)}\beta z} + d_2 e^{-ik^{(j)}\beta z} \right) \tag{28}$$

It should be noted that, in state $\mathcal{A}$, the propagation direction is positive x direction while the waves in state $\mathcal{B}$ are traveling in the opposite direction. The indicator n has been neglected for simplicity.

Substituting Eqs. (19)-(25) into Eq. (18), after some manipulations, we have

$$A_P = \frac{P U_y(z_0) e^{-ik_p x_0}}{2I} \tag{29}$$

where

$$I = \sum_{n=1}^{N} ik_p C_{66} \int_{z^{(n)}}^{z^{(n+1)}} \Sigma_{yx}^{(n)} U_y^{(n)} dz \tag{30}$$

It is worth to mention that the solution in Eq. (28) stands for the forward field. For the backward field, the result remains the same in magnitude but opposite sign in the term $ik_p x_0$.

3 Results and discussions

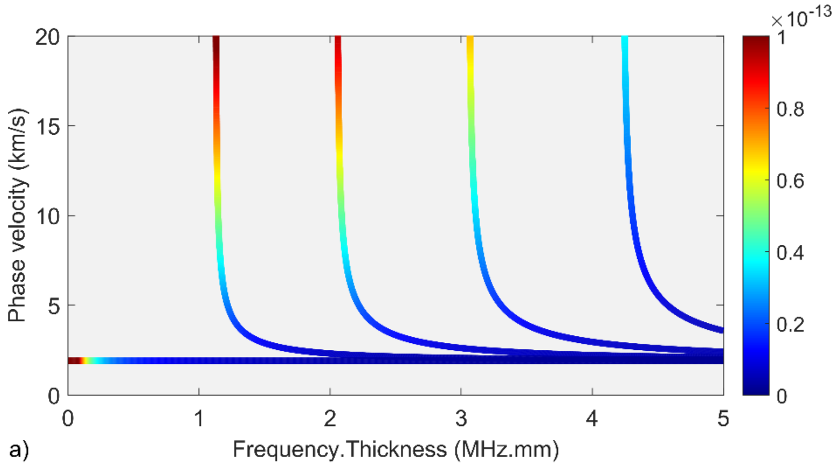
In this section, we present the results of dispersion curves and reciprocity calculation for an eight-layer cross-ply laminate which is commonly used in aerospace structures [22]. The stacking sequence of fiber orientations is $(0/90/90/0/0/90/90/0)$ degree. The thickness of a single lamina is $0.2\ mm$, thus the total thickness is $1.6mm$. Material properties of the composite plate are listed in **Table 1**.

Table 1. Material properties of the lamina.

Material	Stiffness matrix (GPa)	Mass density (kg/m^3)
T300/914	$\begin{bmatrix} 143.8 & 6.2 & 6.2 & 0 & 0 & 0 \\ 6.2 & 13.3 & 6.5 & 0 & 0 & 0 \\ 6.2 & 6.5 & 13.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.7 \end{bmatrix}$	1560

Based on the closed-formed solution obtained in Section 2, the amplitude spectra of SH guided wave modes are immersed in the original dispersion curves. The improved dispersion curves are called the dispersion curves superimposed by amplitude spectra. These dispersion curves of phase velocity and group velocity are shown in **Fig. 3**.

As illustrated in the dispersion diagrams, the lowest mode (SH_0) is nondispersive and has the velocity of transverse waves. This mode is usually used in experiments or practical inspections due to the simplicity in observing and processing signals. Based on the amplitude spectra, the domains colored red have a big displacement magnitude and vice versa for the blue domains. According to this, if the SH_0 is excited in the low-frequency zone $(< 0.1MHz)$ the mode signals will get the highest magnitude. Using the proposed dispersion diagram, the practitioners can instantly choose a suitable mode and excitation frequency to optimize the calibration and post-processing processes. Therefore, the presented diagram may serve as a reliable reference for nondestructive evaluation applications.



a)

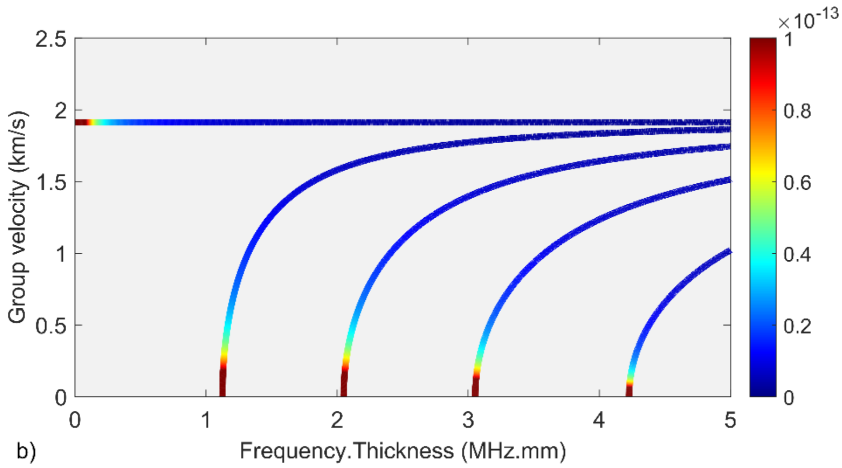


Fig. 3. Dispersion curves

4 Conclusions

This article discusses the motion of SH guided waves generated in multilayered composite structures under time-harmonic ultrasonic sources. The explicit expressions of guided waves have been introduced based on the transfer matrix method. Using the elastodynamic reciprocity theorems, we have calculated the amplitudes of SH guided waves generated by the time-harmonic load. The results of the phase and group velocity dispersion curves superimposed by amplitude spectra have been presented and discussed. These modified dispersion curves are critical to the selection of suitable wave modes and frequencies in the ultrasonic inspection by nondestructive evaluation techniques.

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