
Deep Bhattacharjee
Bimeromorphic Equivalence: Taking compact Ricci-Flat Kähler Geometry with an extended relation between Morita Equivalence and Fujiki’s Class C Manifold
Taking Twisted K–Theory [New Version]

Deep Bhattacharjee

There’s a relation to this aspect of homological algebra in the bimeromorphic equivalent between Calabi-Yau and Fukaya Category with furthermore connectivity in Fujiki’s Class C Category. It is commonly known that Shift Operators play a nontrivial role in modelling the Abelian and the $A_\infty$ where the derived category (as there’s many equivalence in case of the triangulated category with derived categories) where a specific relation to K–Polystability can be found in case of a peculiar FANO surface which is on one hand associated with higher order J-Holomorphic Polygons and there’s the relation of HOMOLOGICAL MIRROR SYMMETRY (connecting the equivalence to Analytical with Algebro-Geometric Model). This helps in the development of A and B-Model and their relation in Supersymmetric string Theory. Any Fukaya Category for the (first Chern Class $X(C_1) = 0$ where “X” is the proven Calabi-Yau manifold: one can say that taking the equation $< 1 + 3(JX + Y) >^2$ with the minimal 1 and maximal 3 for the 3-fold of the Calabi-Yau as considered in String Theory: There’s lies a relation to the Weil cohomology and Hodge - de Rham Spectral Sequence whose degeneration is crucial for the J-Holomorphic Polygons along with the Atiyah - Hirzebruch Spectral Sequence in $E^d_p$ sheets for the value of 4 where it’s equal to Kähler manifold (the same when corresponds to Ricci flatness having the compact form then it’s safe to say) = Fukaya Category ≡ Morita Equivalence (for this $X(C_1)) = 0$. The nontrivial aspect to mention that in the construction of the hypercohomology when one finally arrives the Einstein – Kähler metric and proved the Calabi-Yau for the first Chern Class $X(C_1) = 0$ then for the $X$ which when established as the Fukaya Category then the bimeromorphic relation to that (Calabi-Yau and Fukaya Category) is only true when the Kähler current (big and nef) is taken to prove that the Kähler is in Fujiki’s Class C Manifolds. But the first construction must start with de Rham Complex (in Algebraic form) and then step-by-step taking this [derived category] with the degeneration leads to the first equivalence between de Rham Cohomology and Hodge - de Rham Spectral Sequence. There, the Hodge Diamond ($W, V$) can be easily seen in the Mirror Symmetrical Way through Homological Mirror SYMMETRY and dHYMT (deformed Hermitian Yang-Mills Theory) where we can use that [derived category] for $A_\infty$ to prove the further correspondence between A and B Model. Detailed computations regarding this has been made in the paper.
SECTION I: CONSTRUCTING BIMEROMORPHIC EQUIVALENCE

I. Fukaya Categories

For a Riemannian manifold $M$ where in the case of the first Chern class $c_1 \leq 0$ admits a Kähler – Einstein metric or not was first conjectured by Calabi and later proved by Yau. While for the case of $c_1 = 0$ and the vacuum EFE $T_{uv}dx^udx^v = 0$; the Riemannian manifold $M$ becomes the Calabi – Yau manifold such that $M = \rho$. This suffices a strong positivity condition for the Kähler potential $\sigma$ such that in the Kähler – Einstein formalism, by looking at the metric $\omega_g + 2^{-i}\partial\bar{\partial}\sigma$; the condition lies as $\sigma = +ve$. Now, for the Kähler manifold in real (1,1) – form, for the exact differential $\epsilon_1$ in the class $[\epsilon_1] \in H^{1,1}(\rho, \mathbb{R})$ one can define the potential to be $\sigma: \rho \rightarrow \mathbb{R}$ such that $[\epsilon_1\sigma] = [\epsilon_1]$ admits : $\epsilon_{1\sigma} = \omega_g + 2^{-i}\partial\bar{\partial}\sigma$ gives the deformed Hermitian Yang – Mills equation (dHYM) for $(\rho, \omega)$ where the real part can be given as $\text{Re}(\omega_g + 2^{-i}\partial\bar{\partial}\sigma) > 0$ with $\text{Im}(\omega_g + 2^{-i}\partial\bar{\partial}\sigma) = 0$ implying the positivity condition for the Kähler potential $\sigma$ such that in the Kähler – Einstein formalism in respect to the metric $\omega_g + 2^{-i}\partial\bar{\partial}\sigma$ where the dHYM provides a correspondence between the A – model and B – model of the equations of motion of D – Branes in string theory which can be seen in Hodge diamond for $V$ in $A$ – model and $W$ in $B$ – model termed as homological mirror symmetry [1-3].

For the complex manifolds having the holomorphic volume elements containing no such zeros; precisely the 3 – dim Calabi – Yau manifolds there exists a duality that in principle relates the Hodge numbers depending on the symmetry operations known as mirror symmetry; it has been stated that for the deg$n$ Calabi – Yau manifolds the correspondence is between the symplectic part ($A$ – model) to the complex part ($B$ – model) for $c_1 = 0$. The defined Hodge structure for the Kähler space $\rho$ defined as for $V$ and $W$ the equations are defined as,

$$H^{p+q=k}(\rho, \mathbb{Z}) \otimes \mathbb{C} \cong \bigoplus_{p+q=k} H^q(\rho, \Omega^p)$$

$$\text{deg}H^p(V, \Omega^p) = \text{deg}H^{n-p}(W, \Omega^q)$$

Let there be a triangulated category $d(A)$ for which the $A$ satisfies the Abelian to the category of $A$ – modules where $A$ is an associative algebra; the category $d(A)$ are the complex of free $A$ – modules and the associated morphisms are in equivalence for the homotopy classes in differential graded morphism of deg$0$. The derived bounded category $\mathcal{A}$ consisting of $A$ – module complexes with nonvanishing cohomology groups. For the categories [4-7],
\[ \text{Hom}_{d(A)}(M, N) := H^0 \left( \bigoplus_{p+q=k} \prod_i \text{Hom}_A(M^i, N^{i+k}) \right) \]

There are associated shift factors that shifts the degree of those complexes,

\[
\begin{cases}
M \to M[n], M[1]^{k+n} \\
(M[n])[m] = M[n+m], M[0] = M
\end{cases}
\]

This construction can be extended to the notions of twisted complex in the case of \( A_\infty \) – category \( A \) with a shift vector where for a one-sided twisted complex; there is a family of \( (M^{(i)})_{i \in \mathbb{Z}} \) for objects in \( A_\infty \) – category \( \forall i \exists j > i \) there is a collection of morphisms \( \tau_d^k \exists \Sigma \exists \gamma_{ij} \) for the shift,

\[
\text{Hom}_A(X[i], Y(j)) = \text{Hom}_A(X, Y)[j - i] \\
\exists \forall (M^{(i)})_{i \in \mathbb{Z}}
\]

for \( \tau_d^k \mid_{\mathbb{Z}} \cong \bigoplus_{k,j} \text{Hom}_A(M^{(j)}, N^{(j+k)}[-k]) \)

Let \( S \) be a closed symplectic manifold for \( c_1 = 0 \). For \( MS \) the space of pairs \( (y, M) \): where \( y \) is a point of \( M \) and \( S \) is a Lagrangian of subsets of \( T_y S \) for the space \( MS \) fibered over \( S \) for the fibres isomorphic to \( \mathbb{Z} \). As, \( M \) and \( S \) are subsets; thus, for the nontrivial purpose denoting with the Lagrangian operator \( M \) as \( \bar{L} \) and \( S \) as \( L \) with the notion of a Floer cochain complex \( A_x \), module generated by intersection points \( \bar{L} \cap L \) can be viewed as a set of morphisms from \( \bar{L} \) to \( L \) sufficing the equation,

\[
SA_x : (\bar{L}, L)
\]

This \( A_\infty \) – category can have higher composition maps for twisted complexes of \( A \) as,

\[
\bar{d}(A) := H(\tau_d^A)
\]

Returning to the equation \( 1 + 3(PX + Y)^2 \) where for the almost complex structure \( P \) on the symplectic manifold \( S \) for the generators of \( \tau_d^k \mid_{\mathbb{Z}} \); one can define \( J \) – holomorphic polygons for \( \Pi_{d-1,d} \in SA_x : (\bar{L}_{d-1}, L_d) \) and \( \tilde{\Pi}_{0,d} \in SA_x : (\bar{L}_0, L_d) \) forms the equation,

\[
\tau_d(\Pi_{d-1,d}, \ldots, \Pi_{0,1,d}) = \sum_{SA_x : (\bar{L}_0, L_d) \in \bar{L}_0 \cap L_d} n((\Pi_{d-1,d}, \ldots, \Pi_{0,1,d}) \cdot \tilde{\Pi}_{0,d})
\]

This sequence satisfies the \( A_\infty \) – categorical relations as the boundary of the \( J \) – holomorphic polygons corresponding the configuration space of degenerate polygons. This \( A_\infty \) – category is the use of homological mirror symmetry where the mirror conjecture will apply for \( (V, \omega) - dim_{2n} \) symplectic manifold for \( c_1 = 0 \) and \( W \) is its dual for \( dim_n \) complex algebraic manifold for the embedding of the Fukaya category
\( \bar{d}(F(V)) \) as a full triangulated category into \( \bar{d}(\text{Coh}(W)) \) for the isomorphism \( (\text{Hom}(M, N))^\times \approx \text{Hom}(N, M[n]) \) which for the \( \d_\infty \)–category is cyclically symmetric. Moreover, this duality can be extended to string theory for \( L \)–varieties as the local boundary conditions for \( A \)–model while holomorphic vector bundles as the local boundary conditions for the \( B \)–model. Whereas the \( J \)–holomorphic curve and the almost complex structure \( P \) in the equation \( 1 + 3(PX + Y)^2 \) is a trivial formalism for the parameter \( P \) and can be replaced by \( 1 + 3(JX + Y)^2 \) for the almost complex structure parameterization \( J \).

2. Fujiki Class \( C \) Manifolds

Let us denote a compact Kähler manifold through a parameterization \( (\bar{\rho}, b_k \omega^k_n) \) whose real closed forms are \( \epsilon_1, \epsilon_2 \) or \( \alpha, \beta \) respectively is of \((1,1)\)–forms; where there exists a cohomology class that is nef. Then for the class \([\epsilon_i]\) or taken respectively here \([\alpha] \) is big iff \( V_\alpha > 0 \) where \( V \) is the volume. The Kähler current can be taken as \( J \geq b \omega_k \) for some \( b > 0 \). Then there for \( V_\alpha > 0 \) exists a closed positive current by definition \( \bar{J} \) in the class \([\alpha] \) gives the equation for the manifold \((\bar{\rho}, b_k \omega^k_n)\); for the Kähler current \( J \geq \int_{\bar{\rho}} J^n \geq b^n \int_{\bar{\rho}} \omega^n > 0 \) is,

\[
\int_{\bar{\rho}} J^n = \frac{\int_{\bar{\rho}} J^n \geq b^n \int_{\bar{\rho}} \omega^n > 0}{2} > 0
\]

For the closed positive current \( \bar{J}_k \in [\alpha] \) there are analytic singularities with \( J_k \geq -b_k \omega \) when \( b_k = 0 \) and with \( J_k \rightarrow \bar{J}_k \) as \( k \rightarrow \infty \) then we get,

\[
\lim_{k \to \infty} \inf_{\bar{\rho}} \int_{\bar{\rho}} (J_k + b_k \omega)^n \geq \bar{J} \forall k \text{ large};
\]

\[
J_k \geq -b_k \omega
\]

\[
\int_{\bar{\rho}} (J_k + b_k \omega)^n \geq \bar{\Sigma} > 0
\]

For \( \bar{\Sigma} > 0 \); a resolution of the singularities for \( J_k \) can be taken as \( \bar{\Sigma}_k : \bar{\rho}_k \rightarrow \bar{\rho} \exists \Sigma_k \) is a composition of blows up of smooth centers for \( \bar{\rho}_k \) proved to be a Kahler given in terms of \( \bar{\Sigma}_k J_k \) as \( \bar{\Sigma}_k J_k = -E_k \bar{\Sigma}_k^\omega + [\epsilon_k] \) with \( \bar{\Sigma}_k J_k \) comprises of \([\epsilon_k]\) as the effective \( \mathbb{R} \)–divisor and \( -E_k \bar{\Sigma}_k^\omega \) is the closed real \((1,1)\)–form for the Bott – Chern class \([\alpha] \in H^{1,1}_{BC}(\bar{\rho}, \mathbb{R}) \). Thus, we have the equation,

\[
\int_{\bar{\rho}_k} (J_k + b_k \omega)^n = \int_{\bar{\rho}_k} (\bar{\Sigma}_k J_k + b_k \bar{\Sigma}_k^\omega)^n \approx \int_{\bar{\rho}_k} (-E_k \bar{\Sigma}_k^\omega + \bar{\Sigma}_k^\omega)^n \geq \bar{\Sigma} > 0
\]

Thus, the nef class \((-E_k \bar{\Sigma}_k^\omega + \bar{\Sigma}_k^\omega)^n\) on \( \bar{\rho}_k \) shows \( \beta = \bar{\Sigma}_k^\omega \) for a closed positive curvature \( J'_k \) on \( \bar{\rho}_k \) gives,
\[
J'_k \geq \frac{\int \rho_k \left( -E_k \tilde{\Sigma}_k^* \omega - \tilde{\Sigma}_k^* J_k \right)^n}{n \int \rho_k \left( -E_k \tilde{\Sigma}_k^* \omega - \tilde{\Sigma}_k^* J_k \right)^n \wedge \tilde{\Sigma}_k^* \omega}
\]

Which is bounded and follows that there is a constant \( \tilde{\Sigma}' > 0 \) \( \forall k \) there is \( J'_k \geq \tilde{\Sigma}' \tilde{\Sigma}_k \omega \) for the value of \( k \) there is \(( \tilde{\Sigma}_k )^* ( J_k' + [ E_k ] ) - b_k \omega \geq \omega 2^{-1} \in [ \alpha ] \) and is a Kähler current containing analytic singularities and any non–Kähler locus \( E_\alpha \) is intersection of all loci in \([ \alpha ]\); then \( E_\alpha \subset \tilde{\rho} \forall [ \alpha ] \neq \text{big} ; E_\alpha \equiv \tilde{\rho} \). Thus, for the cohomology class \( \text{nef} \) and \( \text{big} \) in \([ \alpha ]\) for \(( 1,1 \)–form real closed; one gets a union of \( N_{\tilde{\alpha}} \rightarrow \tilde{\rho} \) for the equation with the analytic singularities,

\[
E_\alpha = \bigcup_{\int \alpha_{\dim N}=0} N_{\tilde{\alpha}}
\]

Now, for the null locus of the set on the class \([ \alpha ]\); the existence of a \( \text{big} \) class on \( \tilde{\rho} \Rightarrow \tilde{\rho} \) is in Fujiki’s class \( C \) manifold \( \text{compact Kähler manifold } [8-11] \).

SECTION II: CONSTRUCTING MORITA AND FUJIKI’S CLASS C MANIFOLD EQUIVALENCE

I. Morita Equivalence for Hilbert \( C^* \)-Module

Any map from a domain to a codomain with the mapping parameter \( \theta : \zeta \rightarrow \zeta' \) can provide a continuous set of functions when \( \zeta \) and \( \zeta' \) is endowed with a metric which when attempt for any representation of a Topological structure considering two sets \{ \zeta \} and \{ \zeta' \} there norms even a bijection\(^1\) between them \( \zeta \leftrightarrow \zeta' \) which for a defined function \( f \) over a value of \( f(x) \) there involves a structure of a vector space with concerned operations through a continuous linear transformation, that space for that function carries a Topology best known as Hilbert space. The specified module that carries the \( c^* \)– algebra for that space is defined as \( c^* \)– Hilbert modules\(^3\) through the inner product.

For any group \( ^\wedge \) with a subgroup \( \ell \) the representations \( \Gamma^\Lambda \) makes it easier to construct new representations through the subgroup or the smaller group \( \ell \) over certain parameters that when categorize through the constructive modules of Hilbert’s \( c^* \) then this extent the \( c^* \)– module to \( c^* \)– algebras through the non–commutative formulations.

Furthermore, any derived pathway to construct the noncommutative geometry provides a framework for the moulder category to represent an equivalence over (\( \text{left} \) – \( \text{right} \)) – symmetric rings as established afterwards with rings \( R \) and \( R' \); then for the \( \text{ring} \) – \( \text{representations} \), studying the category of those modules; there exists Morita equivalence for the isomophic commutative form or in general norms in the case of \( \text{non–commutative} \) rings.
For the constructions of $KK - Theory$; Morita equivalence is an important tool to $c^*-$ algebras where for the inequality on the two modules $A$ and $B$; for the moulder form $E$ on $A$ and $B$ for the moulder form $E$ on $A$ and $E \cdot$ on $B$ (as appeared later in the paper) a homotopy invariant bifunctor can make a Morita equivalence for the $KK - Theory$ through $KK(A,B)$ and $KK(B,C)$ for $A, B, C$ as $c^*-$ algebras; there’s for the modular form $E$ having elements $\epsilon, \epsilon$ the inequality represents the form $< \epsilon, \epsilon > < \epsilon, \epsilon >$ where for the $A - module$; the above relation holds and taking the $B - module$ representing the $c^*-$ algebraic pair $KK(A,B)$ and $KK(B,C)$ where one finds the combined form over the composition product representing $KK(A,C)$ and the Morita equivalence to be represented in a specific formulation as to be proved throughout the paper$^{[12,13]}$.

Over the compact Hausdorff spaces$^{[14]}$ and considering the Fredholm modules of Atiyah–Singer Index Theorem$^{[15]}$ for a relatable definition of $A, B, C$ in $c^*-$ algebras the Kasparov’s product $KK(A,C)$ for $KK(A,B)$ and $KK(B,C)$ will be established over an elliptic differential operator $\phi^0_M$ or $\phi^0_M$ for $s - smoothness$ or $n - dim$ and through extensive analysis of that operator which indeed suffice the Fredholm module making a relatable framework for $K - Homology$ and $K - Theory$ $^{[16,3,5,6]}$; The Thom isomorphism is established for the Chern Character $Ch$ over a mapping parameter $t$ through a rank $- n$ vector bundle $v_{1(n)}$ with $v_2$ having the first related to a unit sphere bundle. This in turn induces the categorical correspondence between a relational establishment over noncommutative geometry and noncommutative topology taking the function $f$ over a bounded structure through linear transformations that bounds the concerned subsets $I$ and $J$ for a mapping parameter $\rho_n$ in the same Hilbert space $H$.

This will deduce for a much more concrete formalism of the $K - Theory$ to $K - Homology$ with an extension of $c^*-$ algebras to reduced $c^* - algebra$ for parent group ($^*$) that defined the $\ell^2$ norm of Hilbert space taking into consideration the $KK - Theory$ with Gromov’s $a - T - menable$ property for all the necessary formulations concerned before except Morita equivalence that when established through 5 $parameters$ through an assembly mapping parameter $7$ over discrete torsions gives the ultimate relation of $KK - Theory$ in $Baum - Connes$ conjecture taking into account both the $Novikov$ $conjecture$ and $Kadison - Kaplansky$ $conjecture$ for injectivity and surjectivity respectively connecting to noncommutative topology.

- Extensions have been made in the operator and Topological aspects in the cohomology class where several classifiers are shown with distinct property to suffice the $Sp_c - Structure$ and the Atiyah – Hirzebruch spectral sequence for the Type II (II-A and II-B) as concerned on the complex Topology space $T^*$ where the Atiyah – Singer Index Theorem taking the Fredholm modules as necessary for K-Theory with Bott – Periodicity is taken and a channelization is made to Grothendieck – Riemann – Roch; for the transition of KK-Theory to Strings; Hodge dual, Gauge symmetry, charge density for the required Lagrangian in RR-fields through D-Brane Potential, De Rham Cohomology, and GSO – Projections are shown. P-form electrodynamics with P-Skeleton are considered for the purpose. NS 3-form
and its relation to RR-flux in both D-Brane charge density and supergravity is established. The spectral sequence of Atiyah-Hirzebruch is taken and operator over $E_n^{p,q}$ for $n$ taking the values $2, 3, \infty$ over a consideration of several orders of K-Theory as such Topological, Algebraic, and Twisted. \`etale cohomology and its representation is shown for Algebraic K-Theory and the K"ahler (without any specific consideration of compact and Ricci flatness) has been shown in general terms for K-Theory in a Twisted formalism in $E_i$ for $i = 4 = \infty$.

For a Hilbert space $H$ with a $c^*$-module $H_c$ one can define a $c^*$-algebra for the metric $g$ on a Riemann manifold $M$ (having the form $M_g$) with a vector bundle $V$ there exists a compact neighbourhood being locally variant on a small patch; over an isomorphism of the Hilbert space of that vector bundle $V$ in a continuous way for a commutative $c^*$-algebra through the vanishing infinity.

For the modular form of $c^*$-algebra the Hilbert module for the non-commutative form is the generalized norm taking the algebra over a topological field $T$ in unital formulation for the unit parameter $i$ as such for every $\epsilon$ in the algebra there exists $\epsilon = i\epsilon = \epsilon i$.

Representing over the induced form for any finite group $^\ell$ with $\ell \subset ^\ell$ for the vector bundle $V$ on the Hilbert space $H$, any construction can be defined over the $k$-elements of the group $^\ell$ over $L$ defined a parameter $P$ as,

$$P = \sum_{k=1}^{n} L_k$$

This gives for each $k$, the induced representation through group $^\ell$ in the same $L_k^+ \in L_k$ for $\ell \subset ^\ell$ through the vector representation $V$ of subgroup $\ell$ being $\ell \subset ^\ell$ in Hilbert space $H$ parametrized through,

$$X_{(\pi, V)}$$

Thus, one gets,

$$\bigoplus_{k=1}^{n} L_k^+ V$$

there is,

$$\sum_{k=1}^{n} L_{(1, \ldots, n)k} \pi (L_k^+) \mathcal{E}_k$$

Representing $\mathcal{E}_k \in V$, three non-trivial actions can be noted for the constructions,

1. $\mathcal{E}_k \in V$
2. $L_k^+ \in L_k \forall \ell \subset ^\ell$
3. $\ell \subset ^\ell$
This takes a pre-Hilbert Hausdorff space to construct $c^*$-algebra satisfying the operations of an inner product through the Hilbert $\mathbb{A}$–module being non–negative and self-adjoint. Taking the inner product of the complex manifold representing $\mathcal{M}^*$ through,

$$\mathcal{M}^* \times \mathcal{M}^* \rightarrow A$$

Thus, for any sequence of set that is countable over the Topological space $T$ with a proper representation for the previously encountered manifolds $\mathcal{M}^T$ taking $k^{th}$ countable order of infinity,

$$\{\mathcal{M}_k^T\}_{k=1}^{\infty}$$

When merged with the unital form taken before $\epsilon = i\epsilon = \epsilon i$ such that for every unit parameter $i$ there exists $\epsilon$ in the algebra; where for any $c^*$-algebra there holds the Banach–algebra for a compact $\mathcal{F}$, that if provided there exists three forms taking $B_0(\mathcal{F})$\cite{3,19,20,22},

1. **Typical form** – For the complex space $\mathcal{M}^*$; the locally compact Hausdorff space for vanishing infinity norm gives $B_0(\mathcal{F})$ for continuous functions on $\mathcal{M}^*$. If is commutative

2. **Unital** – \begin{align*}
&\text{identity element of having norm 1} \\
&\mathcal{F} \text{ in } B_0(\mathcal{F}) \text{ is compact}
\end{align*}

3. For Point [2] to have a congruent transformation, there is Banach algebra $B_0(\mathcal{F})$ in $\mathbb{A}$–form where the congruent transformation is unital for a closet set $[A]$.

For the compact Hausdorff (here parameterizing $\mathcal{F}_0^+$) with vector bundles $\mathcal{V}$ for the labeling of $\mathcal{F}_0^+ - 0$ for positive to extend over Bott Periodicity with $+$ as adjoint through $8$–periodic homotopy groups from $\pi_0$ to $\pi_7$ such that\cite{12},

$$\pi_{0,1,2,3,4,5,6,7} \text{ gives } 3 \text{ - category tables in unitary } U, \text{ orthogonal } O, \text{ symplectic } Sp,$$

<table>
<thead>
<tr>
<th>$U$</th>
<th>$O$</th>
<th>$Sp$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_k \rightarrow \pi_{k+2}$</td>
<td>$\pi_{k+4}$</td>
<td>$\forall k = 0,1...$</td>
</tr>
<tr>
<td>$\pi_{k+4}$</td>
<td>$\pi_{k+8}$</td>
<td>$\pi_{k+8}$</td>
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Thus, for Hausdorff $\mathcal{F}$; the underlying K-Theory $K(\mathcal{F})$ there is\cite{12,23};

I. **Topological K-Theory** $\Rightarrow$ on $\mathcal{M}^T$ for $K(\mathcal{F})$

II. **Reduced K-Theory** $\Rightarrow K_{red}(\mathcal{F})$ for $S^n \exists n > 0$ relates the Bott for positive $0$ for $\mathcal{F}_0^+$ and adjoint $+$ in Hausdorff $\mathcal{F}$ for $K_{red}(\mathcal{F}_0^+)$ in non-commutive form.

Where Point [I] relates the Banach–algebras for the locally compact Hausdorff over a abelian module on any sequence of set countable over Topological space $T$ (as previously mentioned) on $c^*$-algebras for bivariant forms suffice the proper framework for the Hilbert $c^*$-module on rings $\mathcal{R}$ and $\mathcal{R}'$ for modular homeomorphisms on $\mathcal{R}$ such that the biproduct exists in finitary over a defined functor $\delta$ preserving equivalence and additive properties,
\[ \delta : \text{mod} - R \longrightarrow \text{mod} - R' \]
\[ \delta' : \text{mod} - R' \longrightarrow \text{mod} - R \]
\[ \delta' : \text{mod} - R' \longrightarrow \text{mod} - R \]
\[ \delta' : \text{mod} - R' \longrightarrow \text{mod} - R \]
\[ \{ \text{suffice Morita Equivalence (strong)} \} \]
\[ \{ \text{for } * - \text{operations on } c^- \text{-algebras} \} \]
\[ |_{\text{Morita}}^{|R_{\text{Morita}}} \approx |_{R_{\text{Morita}}} \]

For the naturally induced isomorphism for functors \( \delta \) and \( \delta' \) for a finite module ring \( R \) for the bi-module \((R, R')\) suffice the natural isomorphism iff for \( X_{(R, R')} \) and \( Y_{(R', R)} \) there is,

\[
(R, R') - \text{bimodule} \Rightarrow X_{(R, R')} \otimes_{R'} Y_{(R', R)} \cong R
\]

\[
(R', R) - \text{bimodule} \Rightarrow Y_{(R', R')} \otimes_{R} X_{(R, R)} \cong R'
\]

Moreover, if we consider \( A, B \) and \( C \) as \( c^- \)-algebras then if there is a Hilbert \( B \)-module that is fully countably generated in the form of \( E \), then for that \( c^- \)-subalgebras of \( B \) there exists a strong Morita equivalence between \( A \) and \( B \) provided for the \( B \) module there is \( \varphi (E) \cong A \) and for \( A \) module there is \( \varphi (E \cdot ) \cong B \) where for the \( c^- \)-algebraic pair \((A, B)\), over a homotopy invariant bifunctor the constructions can be taken for \( A, B \) and \( C \) in such a way that for the defined abelian group \( KK(A, B) \) and combining it with \( KK(B, C) \) a strong Morita equivalence can be established in the form,

\[ KK(A, B) \cong KK(A, C) \exists \]

Combining the elements of \( KK(A, B) \) AND \( KK(B, C) \), there exists the product and the non-trivial assumptions that \( B \) and \( C \) are strongly Morita equivalent.

### 2. Type II Strings relation with Calabi-Yau through Twisted K-Theory

The \( K \)-Theory for the operator and Topological aspects in the cohomology class; there exists distinct classifiers for the \( D \)-\textit{Branes} or Dirichlet Branes in the Ramond–Ramond (RR)– Sector of Type II-B Strings sufficing the \( 3 - \text{dim} \) integral class property. There is the cohomology class for the transformation–twist giving the \( \text{mod} - 2 \) torsion quantum corrections considering the Freed–Witten discrepancies as and when considered in the peculiar \( K \)-\textit{Theory} in the reconciled aspects over Atiyah–Hirzebruch spectral sequence.

The non–trivial aspect to discuss in high energy physics for the Topological K–Theory taking the Type–II (II-A and II-B) superstrings is to consider the RR–fields in \( P - \text{form} \) electrodynamics considering the \( 10 - \text{dim Supergravity} \) for the potential \( U^* \) over \( \Omega_{p+1} \) – \textit{field} defined through the Hodge duals \( *_{d} \) in the form \( \Omega_{d-\mu}^* \) there exists \( 4 - \text{classifiers} \) that will ultimately result the approach of \( K \)-\textit{Theory} in the complex Topological space \( T^* \) on manifold \( M \) over a representation \( M_7 \) relates not only the Atiyah–Singer Index Theorem (for the Fredholm modules, Bott–Periodicity as taken earlier) but also gives the Grothendieck–Riemann–Roch Theorem on bounded
complex $\Lambda^*$ on sheaves $S_\sigma$ over a relation $S_\sigma^{\Lambda^*}$ taking the morphism $\sigma_m : X \rightarrow Y$ for $\sigma_m : A(X) \rightarrow A(Y)$ over the Tangent sheaf $T_{\Lambda^*}$ of $\Lambda^*$ on $\sigma_m$ ! to suffice $ch(\sigma_m ! \Lambda^*)$ gives,

$$\Lambda^*_{\sigma_m} \left[ch(S_\sigma)Td(T_{\sigma_m})\right]$$

All suffice through the 4 – classifiers as mentioned above[27,28],

1. Hodge dual $*_d$
2. Gauge symmetry $g_{P-form}$
3. Equations of motion $\partial \ast g^\ast = \ast J$ for $J_{P-vector}$
4. Charge density $C_p$ through the Lagrangian for $\xi_{C_p}$ in $RR - fields$ for $\sigma_{10-P}$ through the D–Brane potential $(10 - P)$ gives the equations of motion $S_{\xi}$ for $(10 - P)$having a replacement order of $P$ to $(7 - P)$ for the previously taken charge density $C_p$ giving two non–trivial relations[29,30],

A. De Rham Co homology with $H – twist$ for the exterior derivative $\partial$ with charge density $C_p$ for the parameter $\chi$ gives,

$$\partial \chi_{9-P} + H \times \Omega_{9-P}$$

$$= \partial \chi_{P+1}$$

$$= \partial^2 \omega_{7-P}$$

$$= C_{9-P}$$

B. The action for Type II (II–B being both T and S–dual to itself) for non–invariant GSO – projections in subdomains where for the existence of 32–supercharges in Type II–B $(\mathbb{R}^{8,1} \times S^1)$ the action $S_\sigma$ of $P$–form electrodynamics on a manifold $M$ through gauge symmetry can be represented by $g_{P-form}$ gives,

$$S_\sigma = \int_M \left[ \frac{1}{2} g^\ast \chi \ast g^\ast + (-1)^P B\chi \ast J \right]$$

Which gives the nilpotent potential in manifold $M$ over a spacetime coordinates $(\sigma, \tau)$ as,

$$\partial \Omega_{P+1} + \chi_{9-P} + \Omega_{(\sigma,\tau)}$$

$$= \partial \Omega_{P+1} (\sigma,\tau)$$

$$= \partial^2 \omega_P (\sigma,\tau)$$

$$= 0 \ (\sigma,\tau)$$

All of these suffice for $Sp_c$ in the extension of Poincare duality in a generalized norm of orientability of homology theory taking the Thom Isomorphism in complex form of Topological $K – Theory$ relating Atiyah–Singer Index Theorem and Fredholm

Additionally, to discuss furthermore about the Type II Superstrings formalism as associated with supergravity for a homology class there is a relation between the Dirac quantization conditions and RR–fields where in the Lie group structure,

\[ U(1) \times SU(2) \times SU(3) \subseteq SU(5) \subseteq SO(10) \subseteq E(8) \]

The Photon being represented by \( U(1) \) the related methodology of the charge quantization and the magnetic monopoles where their independent nature relates the breaking of gauge group from \( D(1) \) heavy branes when the distance is infinite for a path \( v \) suffice the relation,

\[
\prod_v \left( 1 + ieA_j \frac{dx^j}{d(v)} d(v) \right) = \exp \left( ie \int A \cdot d(v) \right)
\]

\[ \exists e \oint_{\partial \Sigma} A \cdot d(v) = \int_{\Sigma} B d(v) \]

Considering a cycle \( \sigma_{cy} \) in the homogeneous Lie group, the movement can ultimately results in lifting the Lie group that originates over identity structures through,

\[ 2 - \text{times}(\sigma_{cy}) \text{ and } 3 - \text{times}(\sigma_{cy}) \]

Where the \( 2 - \text{times}(\sigma_{cy}) \) where a covering parameter \( 3 \) for \( SO(2) \) can maintain the Type II superstring actions over the Twisted K – Theory (over Topological norms).

One category of Type II superstrings (Type II-B) which has been extended to \( 12 – \text{dim} \) where in the t’Hooft limit, for Yang–Mills \( N = 4 \), F-Theory being encountered under \( SL(2, \mathbb{Z}) \), the D–Brane analogy being extended where there exists some non–trivial aspects being existent over RR–Fields and its relation to the Twisted K–Theory making up these points,

1. \( GSO \) – Projections for an eliminated Tachyon and preserved Supersymmetry.
2. Distinct classifiers for Type II into \( I_{H=1}^{11-4} \).
3. \( SL(2, \mathbb{Z}) \) for a CFT for a worldsheet periodicity as concerned for Fermion–projections giving 3 sub–relations,
   a. Invariance over \( SL(2, \mathbb{Z}) \).
   b. Modular diffeomorphisms as expressed on Torus for Point [3] to get rid of gravitational anomalies.
      i. This in turn establishes the integral for \( Kalb – Ramond (K – R) \) field with the relation to the \( B \) field for \( \lambda \) as,

\[ -\int_{KR} \lambda^i \lambda^j B_{ij} \]

Thus, for the correspondence to \( K \) \( N S \) – \( N S B – field \) ; a far more concrete relation can be attained for \( H – \text{flux}_{\text{NS-brane}} \text{ where the } P – \text{form for } P – \)
skeleton represents a complicated structure later but for the cohomology integral coefficients for a D-Brane absent RR–flux the relation can be stated over,

$$\text{NS } 3 \text{ - form } \otimes \text{RR–flux } \equiv \text{charge density of D-Brane}$$

$$+ \text{RR–flux } \equiv \text{equations of motion (supergravity)}$$

Extending Type II for Type II–B the representation when made for a manifold $M$ for the group operators $Og$ in the quotient space $q$ with $q^{\delta–re\text{scalling}}$ Type II–B represents the Orientifold over the operator relation where $\partial$ in $\partial – re\text{scalling}$ being taken trivially for the involution parameter, the non–empty operator represents the orientifold for the operator $Og_P^2$ such that for the operator $P~$ there is Type II–B for,

$$\partial(P~)$$

Where through the splitting another structure represents $II – A$ for the $(1 – 1) – form$.

The $P – skeleton$ as stated above in turn gives the Topological K–Theory over $B$ for the fibre $f$ in the cohomological space $M$. over a Serre fibration parameter $S_f: M. \rightarrow B$ in the $(p, q) – norm$ representing the cohomolgy pair $(M_{(p)}, M_{(p,q)})$ for $k^{th} – co \text{ hom o log y group}$ through,

$$\bigotimes_{p,q} H^k(M_{(p)})$$

$$\bigotimes_{p,q} H^k(M_{(p)}, M_{(p,q)}, M_{(p–1)})$$

For the Atiyah–Hirzebruch taking the space $M$. and the spectral sequence associated with it for the fibres $f$. there exists the $E_n – sheet$ taking $(p,q) – norms$ for $E_n^{P,q}$ for $n$ taking the values $2, 3, \infty$; the spectral sequence can be in respect of the differentials $E_d$ where there is,

1. Atiyah–Hirzebruch spectral sequence
2. Twisted K–Theory
3. Topological K–Theory
4. Algebraic K–Theory
5. Complex $\delta$
6. $E_n$ for different values of $n$ providing;
   a. Serre spectral sequence for $E_1$
   b. Topological K–Theory for $E_2^P$
   c. Twisted K–Theory for $E_3$ over the differential $E_{3,(d)}$ such that for the $E_n^{p,q}$
      ; $n$ takes an equality for $E_2 and E_3$.
   d. For [Point b] in complex parameter $\delta = 2k + 1$ denoting complex projective $\mathbb{C}P^\delta$ there exists two foundations,
      i. Collapsing for even $2k$
      ii. Non collapsing for odd $2k + 1$
1. Where Topological K–Theory as associated with Atiyah–Hirzebruch for $2k + 1$ over space $M$; a nice relation can be expressed in $E^{p,q}_{2} \delta (M)$.

7. For the Kähler where any compact Kahler having Ricci flatness is a Calabi–Yau for all the threefold being non–trivial in superstring theory, any Kähler (without any consideration of being compact) can give the twisted formalism of K–Theory for $E_{i}$ such that $i \equiv 4 = \infty$.

8. Algebraic K–Theory having a relation to the étale cohomology for the scheme $M_{T}^{e}$ where $M_{T}^{e'}$ is a Topological space; any representation can be done in the local isomorphism such that for the category taking $M_{T}^{e}$ for étale representation et ($M_{T}^{e}$) suffice isomorphism for the Topological space $T(\text{or } M_{T})$ which provides the Atiyah–Hirzebruch spectral sequence for $E_{2}$ in $(p, q) – \text{norms}$ thereby establishing the Quillen–Lichtenbaum conjecture for $E_{2}^{pd}$ with the étale cohomology $M_{T}^{e}$ [2,3,5].

REREFENCES

[1] Bhattacharjee, D. (2022d). Establishing equivalence among hypercomplex structures via Kodaira embedding theorem for non–singular quintic 3-fold having positively closed (1,1)-form Kähler potential $\bar{\partial} \partial^{*} \rho$. Research Square (Research Square). https://doi.org/10.21203/rs.3.rs-1635957/v1


