



## Symmetry Energy from Multifragmentation

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# Nuclear symmetry energy from Multifragmentation

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**Abstract** We present the results of a molecular dynamics approach, namely, Isospin-dependent Quantum Molecular Dynamics (IQMD) model which is a transport theory formulated using a computational method of Monte-Carlo Simulations. The model is used to describe Nuclear Multifragmentation, a phenomena, which is observed in heavy-ion collisions at intermediate energies. Fragmentation of nuclear matter has been observed to be a good probe of nuclear equation of state, in particular, nuclear symmetry energy. The results on the yield of various mass fragments and light charged particles are presented for neutron-rich colliding partners. Our findings revealed that the ratio of relative yield of light charged particles poses better candidate to probe the density dependence of nuclear symmetry energy. The fragmentation pattern has been further divided into Gas/Liquid phases (low/high density phases) and relative distribution of nucleons into these phases vary with the beam energy. A cross over energy is obtained where Gas/Liquid content becomes equal and this energy is again observed to be a good probe of nuclear symmetry energy.

## 1 Introduction

Heavy-ion Physics offers a unique possibility of exploring the properties of nuclei and their interactions. Reactions using heavy-ions can easily be carried out in terrestrial

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labs and field has witnessed a lot of growth since inception. At the same time, various theoretical tools are also being devised to understand the detailed mechanism of these collisions. Thus one relies on various computational algorithms which are incorporated in these models. One of the most widely used approach in heavy ion physics at intermediate energies is molecular dynamics approach. Here, one follows the time evolution of each nucleon under the influence of nucleon-nucleon interactions (potential and scattering). Intermediate energy physics offers various phenomena such as nuclear fragmentation, collective flow, nuclear stopping, particle production etc. [1, 2, 3, 4] Here we will study some aspects of nuclear fragmentation.

Nuclear Multifragmentation is an intermediate mechanism between low-energy fission dynamics and complete vaporization at high energies. During this process, the production of various fragments ranging from heavy mass fragments to free particles takes place. It offers a unique opportunity to understand the nature of equation of state of isospin asymmetric nuclear matter, particularly its isospin-dependent term i.e. the density dependence of nuclear symmetry energy [5]. The knowledge about nuclear symmetry energy is important as it has implications on the dynamics of neutron-rich nuclei as well as in the physics of neutron stars. The rapid advancements in radioactive ion beam facilities has led to a tremendous growth in the field of nuclear symmetry energy. Nuclear symmetry energy is not a directly measurable quantity and thus has to be obtained from various indirect observables. The value of nuclear symmetry energy at normal nuclear matter density is around 30 MeV, as inferred from various studies carried out in recent past. However, its behaviour at densities other than normal matter density is poorly known.

The energy per nucleon of an isospin asymmetric nuclear matter can be approximated in terms of symmetric part and additional term responsible for the isospin asymmetry  $\alpha$  as [5]:

$$E_{sym}(\rho, \alpha) = E_{sym}(\rho, 0) + E_{sym}(\rho)\alpha^2, \quad (1)$$

where,  $E_{sym}(\rho, 0)$  and  $E_{sym}(\rho)$  signify the energy per nucleon of symmetric nuclear matter and symmetry energy, respectively. The understanding of the density dependence of nuclear symmetry energy gets feeble as one moves away from the normal nuclear matter density and  $\beta$ -stability line [6]. Hence, below mentioned equation provides parametrization for the density dependence of nuclear symmetry energy:

$$E_{sym}(\rho) = E_{sym}(\rho_0) \left( \frac{\rho}{\rho_0} \right)^\gamma, \quad (2)$$

where  $E_{sym}(\rho_0)$  is the nuclear symmetry energy at normal nuclear matter density and parameter  $\gamma$  depicts the stiffness of the nuclear symmetry energy at densities below and above the saturation density.

Multifragmentation has been widely used to understand the behaviour of nuclear symmetry energy for past many decades. Various observables related to it such as isospin diffusion, isospin fractionation, isospin mixing, yield of light particles etc. [7, 8, 9] have been proposed to gain information about symmetry energy. Despite of continuous research in this area, no consensus has been made about the behaviour

of nuclear symmetry energy at densities far from normal matter density. One, thus requires deeper insight and more probes of nuclear symmetry energy and nuclear physics community around the globe is greatly involved in it. With the aim to put forward new observables which can be sensitive to nuclear symmetry energy, we studied the fragmentation pattern of various isotopic, isobaric colliding partners and yield of various fragments such as intermediate mass fragments (IMFs;  $5 \leq A \leq 30\%A_{P/T}$ , where  $A$  is the fragment mass and  $A_{P/T}$  is the mass of projectile/target nuclei), light charged particles (LCPs;  $2 \leq A \leq 4$ ) and free nucleons (FNs;  $A = 1$ ) are investigated. It would also be of interest to investigate the ratio of relative yields of various fragments in isotopic and isobaric colliding pairs and to see if it can serve as a better probe (in comparison to bare yield of fragments) to pin down the nuclear symmetry energy or not. Secondly, these various fragments and free particles are divided into liquid/gas phase and transition among these phases is also investigated as a function of beam energy. We shall address these questions using isospin-dependent quantum molecular dynamics (IQMD) model [10], which is a transport approach based on event-by-event simulation and the obtained phase space of nucleons is clusterised using Minimum Spanning Tree (MST) method [11]. The method is based on spatial constraints among nucleons and two nucleons are assumed to form a fragment if the centroids of two nucleons are less than a certain distance,  $r_{min}$ , where  $r_{min}$  can vary between 2 fm and 4 fm.

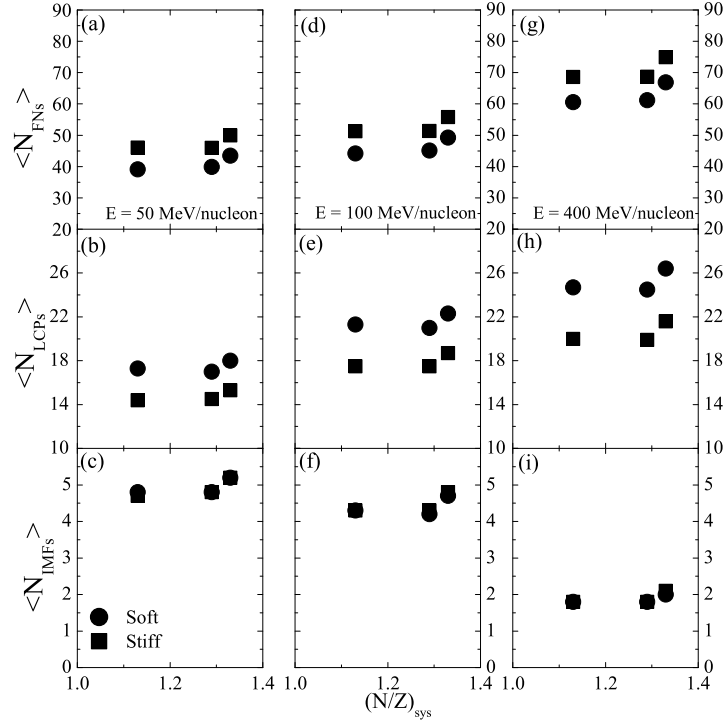
## 2 Results and Discussions

For the first part of the study, the reactions of  ${}^{64}_{28}\text{Ni} + {}^{64}_{28}\text{Ni}$ ,  ${}^{64}_{30}\text{Zn} + {}^{64}_{30}\text{Zn}$ ,  ${}^{70}_{30}\text{Zn} + {}^{70}_{30}\text{Zn}$  are simulated between incident energies of 50 and 400 MeV/nucleon for central ( $\hat{b} = 0.2-0.4$ , where  $\hat{b}$  is the reduced impact parameter  $= b/b_{max}$ ) colliding geometry with Soft-Momentum-Dependent equation of state. Two forms of symmetry energy have been used to study the influence of density dependence of symmetry energy, i.e.  $E_{sym}(\rho) = E_{sym}(\rho_0)(\rho/\rho_0)^\gamma$  with  $\gamma = 0.5$  (soft symmetry energy) and  $\gamma = 1.5$  (stiff symmetry energy).

### 2.1 Yields of fragments

Figure 1 displays the yields of FNs (top panels), LCPs (middle panels) and IMFs (bottom panels) as a function of N/Z of colliding system  $[(N/Z)_{sys}]$  at incident energies of 50 (left panels), 100 (middle panels) and 400 (right panels) MeV/nucleon. The calculations with soft and stiff symmetry energy are displayed by circles and squares. We observed higher yields of FNs and LCPs at higher energies because of violent nature of collisions, which, on the other hand, will reduce the yields of IMFs, as observed. As evident from the figure, FNs and LCPs numbers are quite sensitive to density dependence of symmetry energy whereas yield of IMFs do not exhibit this

sensitivity. Similar results for IMFs have also been reported earlier [12] where their production do not show sensitivity to density dependence of symmetry energy. We also noticed that higher production of LCPs takes place with soft symmetry energy compared to stiffer one and this sensitivity towards symmetry energy increases at higher beam energies. Thus, we conclude that LCPs production can serve as a good candidate to probe supra-saturation behavior of symmetry energy. It is worth men-



**Fig. 1** The yields of FNs (upper panels), LCPs (middle panels) and IMFs (bottom panels) as a function of  $N/Z$  of colliding system  $[(N/Z)_{sys}]$  at incident energies of 50 (left panels), 100 (middle panels) and 400 (right panels) MeV/nucleon.

tioning that previous studies using QMD-type models also reported enhanced yields of light clusters with soft symmetry potential [13, 14], though contrary behavior is reported for studies using BUU-type models [15]. The yield of free nucleons follows opposite trend with stiffer symmetry energy resulting in more emission. We also observe that yield of LCPs (and FNs) is almost similar for the reactions of  $^{64}_{30}\text{Zn} + ^{64}_{30}\text{Zn}$  and  $^{64}_{28}\text{Ni} + ^{64}_{28}\text{Ni}$ , i.e. going from  $(N/Z)_{sys} = 1.13$  to 1.29. This indicates that light cluster production is insensitive to  $N/Z$  of the colliding system in isobaric pairs.

However, yields of LCPs increases for the reaction of  ${}^{70}_{30}\text{Zn} + {}^{70}_{30}\text{Zn}$  [ $(N/Z)_{\text{sys}} = 1.33$ ] indicating that it is governed by the total colliding mass (or neutron content) of the system in isotopic pairs.

## 2.2 Relative yields ‘ $R_N$ ’

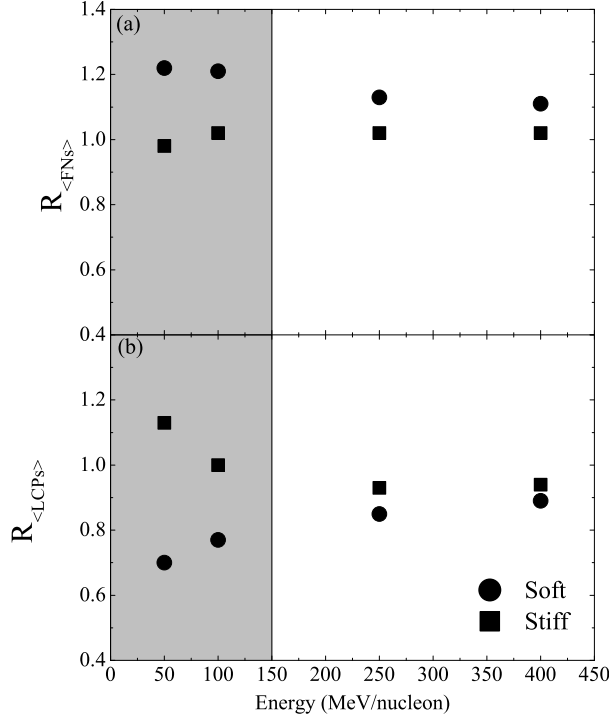
Next, we also calculated the ratio of relative yields of FNs and LCPs in the reactions of  ${}^{64}_{28}\text{Ni} + {}^{64}_{28}\text{Ni}$ ,  ${}^{64}_{30}\text{Zn} + {}^{64}_{30}\text{Zn}$  and  ${}^{70}_{30}\text{Zn} + {}^{70}_{30}\text{Zn}$ . The relative yields have been calculated for free nucleons and light clusters only as intermediate mass fragment’s production is not sensitive to symmetry energy. The relative yield ratio is defined as per Ref. [16] as:

$$R_{\langle N \rangle} = \frac{\langle N \rangle_{({}^{64}_{30}\text{Zn}+{}^{64}_{30}\text{Zn})} - \langle N \rangle_{({}^{70}_{30}\text{Zn}+{}^{70}_{30}\text{Zn})}}{\langle N \rangle_{({}^{64}_{28}\text{Ni}+{}^{64}_{28}\text{Ni})} - \langle N \rangle_{({}^{70}_{30}\text{Zn}+{}^{70}_{30}\text{Zn})}}, \quad (3)$$

where  $\langle N \rangle$  can be the yield of either FNs or LCPs. In figure 2, we display the results of  $R_{\langle FN_s \rangle}$  (upper panel) and  $R_{\langle LCP_s \rangle}$  (bottom panel) as a function of incident energy. Symbols have same meaning as in Fig.1.  $R_{\langle FN_s \rangle}$  and  $R_{\langle LCP_s \rangle}$  are sensitive to density dependence of symmetry energy at lower beam energies, however, dominance of nucleon-nucleon scattering at higher energies reduces the sensitivity towards symmetry energy. About 10-20% sensitivity of  $R_{\langle FN_s \rangle}$  and bare free nucleons yield is observed. However,  $R_{\langle LCP_s \rangle}$  show up to 60% sensitivity compared to  $\sim 15$ -20% sensitivity shown by bare yield of LCPs. Therefore, the ratio of relative yield of LCPs can act as better candidate to constrain the density-dependent behavior of symmetry energy in Fermi energy region (shown by shaded area) [17].

## 2.3 Cross-over energy

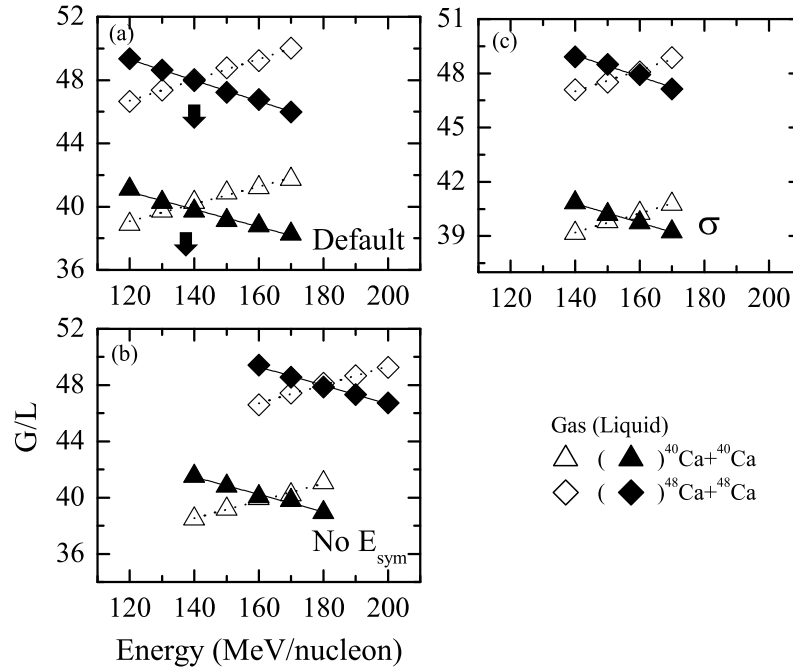
Next, we will try to put forward another probe of symmetry energy using nuclear multifragmentation. One of the phenomena which has been reported to be sensitive to symmetry energy is isospin fractionation, which is an unequal partitioning of neutrons and protons into high and low density phases. Extensive work on isospin fractionation have demonstrated that it is energetically favorable for an asymmetric system to partition itself into a neutron-rich gas phase compared to less neutron-rich liquid phase. At the same time, there are many ways reported in the literature to define gas/liquid phase. Some studies treated the free particles emitted during simulation as gas phase and particles with  $Z > 2$  as liquid. Others used the concept of density and quantified the gas (liquid) phase as nucleons with densities less (greater) than 1/8th (Li et al. [18]) or 1/10th (Guo et al. [19]) of normal nuclear matter density. As QMD-type models can easily identify fragments with after-burners at the freeze-out



**Fig. 2**  $R_{<FNS>}$  (top panel) and  $R_{<LCPs>}$  (bottom panel) as a function of incident energy with soft and stiff forms of symmetry energy. Various symbols are explained in the text.

stage, here, we will treat all free nucleons with  $A = 1$  as gas and  $A > 1$  bound nucleons as liquid. Note that similar definitions of Gas/Liquid content has also been used in Ref. [19]. For this part of analysis, we simulated semi-central collisions of  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$  at various beam energies using Soft-Momentum-Dependent Equation of state. The obtained fragmentation pattern of various mass fragments and free nucleons is then divided into gas and liquid phase as mentioned above.

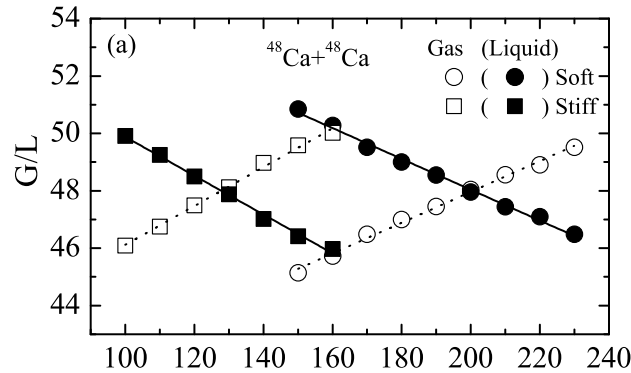
In Figure 3 we display the yield of gas and liquid content as a function of incident energy for the reactions of  $^{40}\text{Ca} + ^{40}\text{Ca}$  (triangles) and  $^{48}\text{Ca} + ^{48}\text{Ca}$  (diamonds). Open and solid symbols represent Gas and Liquid content, respectively. From the figure, we observe that gas content increases with beam energy as expected, with a corresponding decrease in liquid content. A cross-over is obtained at a particular incident energy where gas and liquid content becomes equal. We observe that cross-over is happening at nearly same beam energy for both  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$ . This behaviour is not what one expects according to total colliding mass. Heavier system should require more energy to break and thus cross-over should happen at higher beam energies. To further investigate this behaviour, we performed the calculations without symmetry potential and results are displayed in the middle



**Fig. 3** The energy dependence of Gas and Liquid content for the reactions of  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$ . Symbols are explained in the text.

panel. From the figure, we noticed that cross-over gets shifted to higher values, enhanced shift for  $^{48}\text{Ca} + ^{48}\text{Ca}$  compared to  $^{40}\text{Ca} + ^{40}\text{Ca}$ . This is due to repulsive nature of symmetry potential, as in the absence of repulsions more energy is required to break the system. Thus higher role of repulsive symmetry potential in  $^{48}\text{Ca} + ^{48}\text{Ca}$  pushes the cross over to happen at nearly same energy as for  $^{40}\text{Ca} + ^{40}\text{Ca}$ . Another factor which can govern the isospin dynamics is the scattering cross-section, which by default, is isospin-dependent; as neutron-proton collision cross section is three times as of proton-proton (neutron-neutron) cross sections. To study its influence, we performed the calculations with isospin independent cross-section and results are displayed in the bottom panel. We again observe that cross-over energy is shifted to higher values as isospin independent cross-section is reduced one and thus breakage will happen at higher values of beam energy. However, the cross over is increased to a higher extent in the absence of symmetry potential rather than with isospin independent cross-section. Thus, one can say that cross-over energy is a sensitive probe of nuclear symmetry energy.





**Fig. 4** The Gas and Liquid content as a function of incident energy for the reactions of  $^{48}\text{Ca} + ^{48}\text{Ca}$  with soft and stiff forms of symmetry energy.

Next, we investigate the role of density dependence of nuclear symmetry energy on the cross-over between gas/liquid content. Here we use two forms as used earlier as well; soft and stiff forms of symmetry energy. The results are displayed in Figure 4. Circles and Squares represent soft and stiff forms of symmetry energy, whereas open and solid symbols are for Gas and Liquid content, respectively. From the figure, we notice that cross-over occurs at lower energy in case of stiff symmetry potential compared to soft symmetry potential. This happens because effective strength of nuclear symmetry energy is more for stiff symmetry potential at supra-saturation densities, which in turn, leads to stronger repulsive forces and thus system breaks at lower energies. These findings thus reveal that cross-over energy is a good probe to understand nuclear symmetry energy and its density dependence [20]. Our detailed study thus reflects that nuclear fragmentation can be used as a tool to understand nuclear symmetry energy.

### 3 Summary

Nuclear fragmentation is explored in detail to understand the behaviour of nuclear symmetry energy. We here present the results of molecular dynamics approach which is used to simulate the neutron-rich reactions. Our findings revealed that relative yield of light charged particles can be good probe of nuclear symmetry energy. The observed fragments/free particles are divided into Liquid/Gas phases and energy dependence of these phases is noticed. The cross-over energy where the Gas/Liquid content becomes equal is also sensitive to nuclear symmetry energy and

its density dependence. Using these observables, one can thus further understand the behaviour of symmetry energy in supra-saturation density region.

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## References

1. H. Sorge et al., Phys. Rev. Lett. 78, 2309 (1997).
2. B. Jakobsson et al., Nucl. Phys. A 509, 195 (1990).
3. G. E. Cooper, Nucl. Phys. A 661, 362 (1999).
4. J. Aichelin and C. M. Ko, Phys. Rev. Lett. 55, 2661 (1985).
5. B. A. Li, L. W. Chen and C. M. Ko, Phys. Rep. 464, 113 (2008).
6. C. Xu et al., Phys. Rev. C 82, 054607 (2010).
7. M. B. Tsang et al., Phys. Rev. Lett. 92, 062701 (2004).
8. M. Farine et al., Z. Phys. A 339, 363 (1991).
9. L. Shi and P. Danielewicz, Phys. Rev. C 68, 064604 (2003).
10. C. Hartnack et al., Eur. Phys. J A 1, 151 (1998).
11. R. Kumar et al., Phys. Rev. C 89, 064608 (2014).
12. J. Y. Liu et al., Phys. Rev. C 63, 054612 (2001).
13. Q. Li et al., Phys. Rev. C 72, 034613 (2005).
14. Y. Zhang and Z. Li, Phys. Rev. C 71, 024604 (2005).
15. L. W. Chen et al., Nucl. Phys. A 729, 809 (2003); L. W. Chen et al., Phys. Rev. C 68, 017601 (2003).
16. Z. Kohley et al., Phys. Rev. C 82, 064601 (2010).
17. M. Kaur, S. Gautam and R. K. Puri, Nucl. Phys. A 955, 133 (2016).
18. B. A. Li, Phys. Rev. Lett. 85, 4221 (2000).
19. W. M. Guo et al., Phys. Lett. B 738, 397 (2014).
20. P. Bansal, S. Gautam and R. K. Puri, Phys. Rev. C 98, 024604 (2018).