

Generalized Fuzzy Neural Net and Application to Reinforcement Learning

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Generalized Fuzzy Neural Net and Application to Reinforcement Learning

P. Venkata Subba Reddy

Abstract—the information available to the system is incomplete in many applications particularly in control systems, medicine and business. Sometimes decision has to be taken with the incomplete information The fuzzy logic is useful in decision making. The fuzzy logic deals incomplete information with belief rather than likelihood (probability). Learning is necessary while deal with decision making, The fuzzy neural net is useful to take the decision because it needs threshold. In this paper, The fuzzy neural net is discussed. The fuzzy decision set is defined with two fold fuzzy set. The fuzzy inferences are studied with fuzzy neural network. The decision support system is given as application.

I. INTRODUCTION

he information available to many applications like Business, Medical, Geological, Control Systems etc are incomplete or uncertain. The fuzzy logic will deal incomplete information with belief rather than likelihood (probable). Zadeh formulated uncertain information as fuzzy set with single membership function. The fuzzy set with two membership function will give more evidence than single membership function. The two fold fuzzy set is with fuzzy membership functions "Belief" and "Disbelief". Usually, in Medical and Business applications, there are two opinions like " Belief" and "Disbelief" about the information and decision has to be taken under risk. For instance, in Mycin[1], the medical information is defined with belief and disbelief i.e. CF[h,e]=MB[h,e] - MD[h.e], where "e" is the evidence for given hypothesis "h". The fuzzy set is used instead of Probability to define fuzzy certainty factor.

The fuzzy neural networks are the one of the learning technique to study fuzzy problem. In the following, some methods of fuzzy conditional inference are studied through fuzzy neural network and before that preliminaries of fuzzy logic and neural network are discussed.

In the following fuzzy logic[10] and Generalized fuzzy logic[9] are studied briefly. The fuzzy Certainty Factor is studied and fuzzy Decision set is proposed. The fuzzy inference and fuzzy reasoning are studied for fuzzy Decision set. The Business applications are studied as applications of fuzzy Decision set.

II. FUZZY LOGIC

Various theories are studied to deal with imprecise, inconsistent and inexact information and these theories deal with likelihood where as fuzzy logic with belief. Zadeh[10] has introduced fuzzy set as a model to deal with uncertain information as single membership functions.. The fuzzy set is a class of objects with a continuum of grades of membership. The set A of X is characterized by its membership function $\mu A(x)$ and ranging values in the unit interval [0, 1]

 $\mu A(x)$: $X \rightarrow [0, 1]$, $x \in X$, where X is Universe of discourse. $A = \mu A(x1)/x1 + \mu A(x2)/x2 + ... + \mu A(xn)/xn$, "+" is union For example, the fuzzy proposition "x is young"

={.95/10+0.9/20+0.8/30+0.6/40+0.4/50+0.3/60+0.2/70+0.15/

80+0.1/90 } not young ={ 0.05/10 + 0.1/20 + 0.2/30+0.4/40 +0.6/50 +

not young = { 0.05/10 + 0.1/20 + 0.2/30+0.4/40 +0.6/50 + 0.8/60+0.7/70 +0.95/80+0.9/90 }

For instance "Rama is young" and the fuzziness of "young" is 0.8 The Graphical representation of young and Not young is shown in fig.1

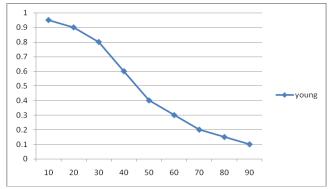


Fig.1: fuzzy membership function

The fuzzy set of type 2 "Headache" is defined as Headache = { 0.4/mild + 0.6/moderate+ 0.8/Serious}

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For example, Consider the fuzzy proposition "x has mild Headache"

. For instance "Rama has mild headache" with Fuzziness 0.4 The fuzzy logic is defined as combination of fuzzy sets using logical operators]21]. Some of the logical operations are given below

Let A, B and C are fuzzy sets. The operations on fuzzy sets are

Negation

If x is not A

 $A'=1-\mu_A(x)/x$

Conjunction

x is A and y is $B \rightarrow (x, y)$ is A x B

A x B=min($\mu_A(x)$), $\mu_B(y)$ }(x,y)

If x=y

x is A and y is $B \rightarrow (x, y)$ is $A \wedge B$

AAB=min($\mu_A(x)$), $\mu_B(y)$ }/x x is A or y is B \rightarrow (x, y) is A' x B' A' x B' =max($\mu_A(x)$), $\mu_B(y)$ }(x,y)

If x=y

x is A and x is $B \rightarrow (x, x)$ is A V B

AVB=max($\mu_A(x)$), $\mu_B(y)$ }/x Disjunction

Implication

if x is A then y is $B = A \rightarrow B = \min\{1, 1 - \mu_A(x)\} + \mu_B(y)\}/(x,y)$ if x = y

 $A \rightarrow B = \min \{1, 1 - \mu_A(x)\} + \mu_B(y)\}/x$

If x is A then y is B else y is $C = A \times B + A' \times C$

The fuzzy proposition "If x is A then y is B else y is C" may be divided into two clause "If x is A then y is B " and "If x is not A then y is C" [15]

If x is A then y is B else y is C =A \rightarrow B= min {1, 1- μ A(x)+ μ B(y)}/(x,y)

If x is not A then y is B else y is $C = A' \rightarrow C = \min \{1, 1 - \mu_A(x)\}$ + $\mu_C(y)$ }/(x,y)

Composition

A o B= A x B=min{ $\mu_A(x)$, $\mu_B(y)$ }/(x,y)

If x = y

A o B==min{ $\mu_A(x)$, $\mu_B(y)$ }/x Composition

The fuzzy propositions may contain quantifiers like "Very", "More or Less". These fuzzy quantifiers may be eliminated as

Concentration

x is very A

 $\mu_{\text{very A}}(x)$, = $\mu_{A}(x)^{2}$

Diffusion

x is very A

 $\mu_{\text{more or less A}}(x) = \mu_{\text{A}}(x)^{0.5}$

III. GENERALIZED FUZZY LOGIC WITH TWO FOLD FUZZY SET

Since formation of the generalized fuzzy set simply as two fold fuzzy set and is extension Zadeh fuzzy logic.

The fuzzy logic is defined as combination of fuzzy sets using logical operators. Some of the logical operations are given below

Suppose A, B and C are fuzzy sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Since formation of the generalized fuzzy set simply as two fold fuzzy set, Zadeh fuzzy logic is extended to these generalized fuzzy sets.

Negation

 $A' = \{1 - \mu_A^{Belief}(x), 1 - \mu_A^{Disbelief}(x) \}/x$

Disjunction

 $\begin{array}{lll} AVB = \{ & max(\mu_A{}^{Belief} \ (x) \ , \ \mu_A{}^{Belief} \ (y)), \ max(\mu_B{}^{Disbelief} \ (x) \ , \\ \mu_B{}^{Disbelief} \ (y))\}(x,y) \end{array}$

Conjunction

AAB={ $min(\mu_A^{Belief}(x), \mu_A^{Belief}(y)), min(\mu_B^{Disbelief}(x), \mu_B^{Disbelief}(y)) }/(x,y)$

Implication

 $\mathbf{A} \rightarrow \mathbf{B} = \{ \min(1, 1 - \mu_A^{\text{Belief}}(\mathbf{x}) + \mu_B^{\text{Belief}}(\mathbf{y}), \min(1, 1 - \mu_A^{\text{Disbelief}}(\mathbf{x}) + \mu_B^{\text{Disbelief}}(\mathbf{y}) \} (\mathbf{x}, \mathbf{y})$

If x is A then y is B else y is $C = A \times B + A' \times C$

If x is A then y is B else y is $C = A \rightarrow B = \{\min(1, 1 - \mu_A^{Belief}(x) + \mu_B^{Belief}(y), \min(1, 1 - \mu_A^{Disbelief}(x) + \mu_B^{Disbelief}(y)\}/(x,y)$

If x is not A then y is B else y is $C = A' \rightarrow C = \min(1, \mu_A^{Belief}(x) + \mu_C^{Belief}(y), \min(1, \mu_A^{Disbelief}(x) + \mu_C^{Disbelief}(y))(x,y)$

Composition

A o $R = \{\min_x (\mu_A^{Belief}(x), \mu_A^{Belief}(x)), \min_x (\mu_R^{Disbelief}(x), \mu_R^{Disbelief}(x))\}/y$

The fuzzy propositions may contain quantifiers like "very", "more or less". These fuzzy quantifiers may be eliminated as

Concentration

"x is very A

 $\mu_{\text{very A}}(x) = \{\ \mu_A{}^{\text{Belief}}(x)^2, \, \mu_A{}^{\text{Disbelief}}(x)\mu_A(x)^2\ \}$

Diffusion

"x is more or less A"

 $\mu_{\text{more or less A}}(\mathbf{x}) = (\mu_{\text{A}}^{\text{Belief}}(\mathbf{x})^{0.5}, \mu_{\text{A}}^{\text{Disbelief}}(\mathbf{x})\mu_{\text{A}}(\mathbf{x})^{0.5}$

For instance, Let A, B and C are

 $A = \{ 0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5,$

 $0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 + 0.6/x_5\}$

$$\begin{split} B = \{ & 0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0..5/x_4 + \!0.6/x_5\,, \\ & 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + \!0.7/x_5 \} \end{split}$$

A V B = { $0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 + 0.6/x_5$,

 $0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.7/x_5$

A Λ B = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5$.

 $A A B = \begin{cases} 0.0/A_1 + 0.7/A_2 + 0.7/A_3 + 0.5/A_4 + 0.5/A_5 \\ 0.4/A_1 + 0.5/A_2 + 0.7/A_3 + 0.5/A_4 + 0.5/A_5 \end{cases}$

 $0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\}$

A' = not A= { $0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 + 0.5/x_5$, $0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 + 0.4/x_5$ }

 $A \rightarrow B = \{ 1/x_1 + 0.8/x_2 + /x_3 + 0.9/x_4 + 1/x_5,$

$$1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 + 1/x_5$$

A o B = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5$,

 $0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5$

 $\mu_{\text{Very A}}(x) = \{ \ \mu_{\text{A}}^{\text{Belief}}(x)^2, \, \mu_{\text{A}}^{\text{Disbelief}}(x)\mu_{\text{A}}(x)^2 \ \}$

= { $0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 + 0.25/x_5$, $0.16/x_1 + 0.09/x_2 + 0.16/x_3 + 0.49/x_4 + 0.36/x_5$

 $\mu_{\text{More or Less A}}(x) = \{ \ \mu_{A}^{\text{Belief}}(x)^{1/2}, \ \mu_{A}^{\text{Disbelief}}(x)\mu_{A}(x)^{1/2} \ \}$

 $= \{ 0.89/x_1 + 0.94/x_2 + 0.83/x_3 + 0.77/x_4 \}$

 $+0.70/x_5$,

 $0.63/x_1 + 0.54/x_2 + 0.63/x_3 + 0.83/x_4$

 $+0.77/x_5$

IV. FUZZY NEWRAL NETWORK

The neural network concept is taken from the Biological activity of nervous system. The neurons passes information to other neurons. There are many models described for neural networks. The McCulloch-Pitts model contributed in understanding neural network and Zedeh explain that activity of neuron is fuzzy process [13].

The McCulloch and Pitt's model consist of set of inputs, processing unit and output and it is shown in Fig.2

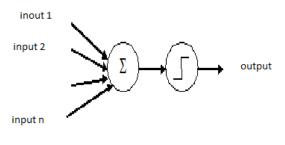


Fig2. McCulloch and Pitt's model

The fuzzy neuron model for fuzzy conditional inference for

if x_1 is A_1 and/or x_2 is A_2 and/or ... and/or x_n is A_n then B may be defined as set of individuals of the universe of discourse, fuzziness and computational functional function and shown in Fig.3.

Where $B=f(A_1,A_2,...A_n)$

This fuzzy neuron fit for where the relation between president part and consequent part of fuzzy conditional inference is not known

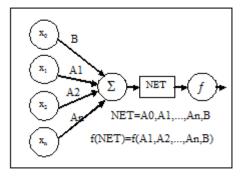


Fig.3.Fuzzy neuron model

The multilayer fuzzy neural net work is shown in Fig.3

The fuzzy neuron for Defuzzification for Centre of Gravity (COG) is shown in fig.4

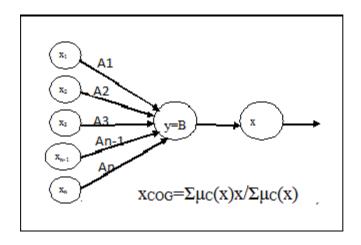


Fig.4. Defuzzification

V. FUZZY DECISION SET

Zadeh[10] proposed fuzzy set to deal with incomplete information. Generalized fuzzy set with two fold membership function μ A $(x) = \{ \mu$ A Belief (x), μ A Disbelief (x) $\}$ is studied [18]

The fuzzy Certainty Factor may be defined as (FCF) $\mu_A^{FCF}(x) = \mu_A^{Belief}(x) - \mu_A^{Disbelief}(x)$, where

fuzzy Decision set R is defined based on convex fuzzy set [10]

R= {A, $\mu_A^{FCF}(x) \ge \alpha$ }, where $\alpha \in [0,1]$

For instance,

Demand = $\{0..8/x1+0.7/x2+0.86/x3+0.75/x4+0.88/x5, 0.2/x1+0.3/x2+0.25/x3+0.3/x4+0.2/x5\}$

 $\mu_{Demand}^{FCF}(x) =$

0.6/x1 + 0.4/x2 + 0.61/x3 + 0.45/x4 + 0.68/x5

The Generalized fuzzy set for Demand for the Items and fuzzy certainty factor is shown in Fig5.

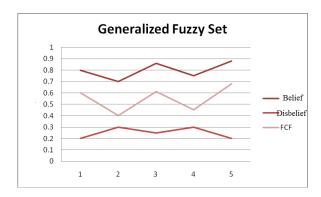


Fig.5 Generalized fuzzy set Suppose fuzzy Decision set is defined $\mu_{Demand}^{FCF}(x) \ge 0.5$ =1/x1+0/x2+1/x3+0/x4+1/x5

Decision may be taken under Decision shown in Fig.6.



Fig.6 fuzzy Decision set

The fuzzy logic is combination of logical operators. Consider the logical operations on fuzzy Decision sets r1, R2 and R3

Negation

If x is not R1

 $R1'=1-\mu_{R1}(x)/x$

Conjunction

x is R1 and y is R2 \rightarrow (x, y) is R1 x R2

R1 x R2=min($\mu_{R1}(x)$), $\mu_{R2}(y)$ }(x,y)

If x=

x is R1 and y is R2 \rightarrow (x, y) is R1 Λ R2

R1 Λ R2=min($\mu_{R1}(x)$), $\mu_{R2}(y)$ }/x x is R1 or y is R2 \rightarrow (x, y) is R1' x R2'

R1' x R2' = $\max(\mu_{R1}(x)), \mu_{R2}(y)$ }(x,y)

If x=y

x is R1 and x is R2 \rightarrow (x, x) is R1 V R2

R1VR2= $max(\mu_{R1}(x)), \mu_{R2}(y)$ }/x Disjunction

Implication

if x is R1 then y is R2 =R1 \rightarrow R2 = min{1, 1- $\mu_{R1}(x)$ } + $\mu_{R2}(y)$ }/(x,y)

if x = y

 $R1 \rightarrow R2 = min \{1, 1 - \mu_{R1}(x)\} + \mu_{R2}(y)\}/x$

Composition

R1 o R2= R1 x R2=min{ $\mu_{R1}(x)$ }, $\mu_{R2}(y)$ }/(x,y)

If x = y

R1 o R2==min{ $\mu_{R1}(x)$, $\mu_{R2}(y)$ }/x

The fuzzy propositions may contain quantifiers like "Very", "More or Less". These fuzzy quantifiers may be eliminated as

Concentration

x is very R1

 $\mu_{\text{very R1}}(x)$, = $\mu_{R1}(x)^2$

Diffusion

x is very R1

 $\mu_{\text{more or less R1}}(x) = \mu_{R1}(x)^{0.5}$

Zadeh[10] fuzzy conditional inference is give as

If x_1 is R1 and x_2 is R2 and ... and x_n is Rn then y is B

=f(R1,R2,...,Rn,B)=min(1,1-R1+R2+...+Rn+B) The fuzzy neural network for Zadeh is shown in fig.7

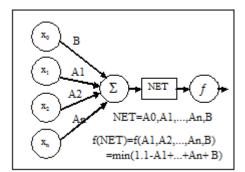


Fig.7 Zadeh fuzzy conditional inference

Mamdani[2] fuzzy conditional inference is give as

If x_1 is R1 and x_2 is R2 and ... and x_n is Rn then y is B = f(R1,R2,...,Rn,B)=min(R1,R2,...,Rn,B)

The fuzzy neural network for Mamdani is shown in fig.8

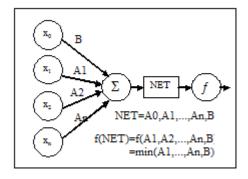


Fig8 Mamdani fuzzy conditional inference

Reddy[8] fuzzy conditional inference is give as If x_1 is R1 and x_2 is R2 and ... and x_n is Rn then y is B =f(R1,R2,...,Rn,B)=min(R1,R2,...,Rn)
The fuzzy neural network for Reddy is shown in fig.9

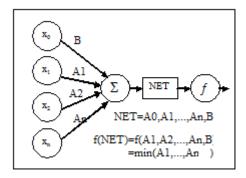


Fig9 Reddy fuzzy conditional inference

VI. FUZZY NEURAL NETWORK AND APLLICATION TO DECISION MAKING

The biological neuro system has threashold. For instance, The pain will be recognized after some threshold. The neural network has threshold. Similarly, the fuzzy neural netwot has threshold. The threshold of fuzzy neural net is defined as decision set.

EXAMPLE1

The Decision management is to take the decision with in Decision Support Systems. The fuzzy neural net will help in taking the decision.

Consider Business rule
If x is Demand of the product then x is High Price
Let x1, x2, x3, x4, x5 be the Items.
The Generalized fuzzy set

Demand = $\{0.56/x1+0.48/x2+0.86/x3+0.36/x4+0.88/x5,$ 0.06/x1+0.04/x2+0.07/x3+0.03/x4+0.2/x5 } $\mu_{Demand}^{FCF}(x) =$ 0.5/x1 + 0.44/x2 + 0.79/x3 + 0.33/x4 + 0.68/x5High Price = 0.49/x1+0.52/x2+0.35/x3+0.4/x4+0.3/x5, 0.09/x1+0.02/x2+0.06/x3+0.02/x4+0.1/x5 } $\mu_{High\ Price}^{FCF}(x) =$ 0.4/x1+0.5/x2+0.29/x3+0.38/x4+0.2/x5 Zadeh inference is given as $A \rightarrow B = min\{1, 1-\mu_A(x) + \mu_B(x)\}$ $\mu_{Demand} \rightarrow_{High Price} FCF(x) =$ 0.9/x1+1/x2+0.5/x3+1/x4+0.52/x5 $\mu_{Demand} \rightarrow_{High\ Price} FCF(x) \ge 0.6$ = 1/x1+1/x2+0/x3+1/x4+0/x5Mamdani inference is given as $A \rightarrow B = \min\{\mu_A(x), \mu_B(x)\}\$ $\begin{array}{l} \mu_{\text{ Demand}} \rightarrow_{\text{High Price}} \ ^{FCF}(x) = \\ 0.4/x1 + .44/x2 + 0.29/x3 + .33/x4 + 0.2/x5 \end{array}$ μ Demand \rightarrow High Price FCF $(x) \ge 0.6$ = 1/x1+1/x2+0/x3+1/x4+0/x5Mamdani inference is given as $A \rightarrow B = \min\{\mu_A(x)\}\$ μ Demand \rightarrow High Price FCF (x) =0.4/x1+.44/x2+0.29/x3+.33/x4+0.2/x5 μ Demand \rightarrow High Price FCF $(x) \ge 0.6$ = 1/x1+1/x2+0/x3+1/x4+0/x5EXAMPLE2

Consider Medical Diagnosis

If x has infection in the leg then surgery

Let x1, x2, x3, x4, x5 are the Patients.

The fuzzy set

$$\mu_{\text{Infection}}^{\text{FCF}}(x)$$

$$= 0.76/x1+0.78/x2+0.46/x3+0.86/x4+0.58/x5$$

$$0.16/x1+0.12/x2+0.06/x3+0.14/x4+0.05/x5$$

$$= 0.6/x1 + 0.64/x2 + 0.4/x3 + 0.72/x4 + 0.53/x5$$

$$\mu_{Surgery}^{FCF}(x)$$

$$= 0.59/x1+0.26/x2+0.55/x3+0.24/x4+0.35/x5$$

$$0.09/x1+0.06/x2+0.05/x3+0.04/x4+0.03/x5$$

$$= 0.5/x1+0.2/x2+0.5/x3+0.2/x4+0.32/x5$$

Using inference rule A \rightarrow B= min{1, 1- μ_A (x) + μ_B (x)}

$$\mu$$
 Infection \rightarrow Surgery FCF (x)

$$= 0.9/x1+0.56/x2+0.9/x3+1/x4+1/x5$$

$$\mu$$
 Infection \rightarrow Surgery $^{R}(x) = 1$ μ Infection \rightarrow Surgery $^{FCF}(x) \ge 6$

$$0 \quad \mu \text{ Infection} \rightarrow \text{Surgery} \quad ^{FCF}(x) < 6$$

The fuzzy risk set R is

$$1/x1+0/x2+1/x3+1/x4+1/x5$$

The decision is to take for surgery is Yes for x1, x3,x4, x5 and No for x2.

The fuzzy reasoning under Risk management in Decision Support Systems may be

Consider the fuzzy rule and fuzzy fact

If x has Infection of the product then x is to go for Surgery

x has very Infection

x is very Infection o Infection→Surgery

$$\mu_{\text{Infection}} \rightarrow_{\text{Surgery}} FCF(x)$$

$$= 0.9/x1+0.56/x2+0.9/x3+1/x4+1/x5$$

$$\mu_{\text{ very Surgery}}\,^{FCF}\left(x\right.)=$$

$$0.25/x1+0.2\04/x2+0.25/x3+0.04/x4+0.1/x5$$

x is very Demand o Demand→Increase Price

$$=0.35/x1+0.66/x2+0.35/x3+0.04/x4+0.1/x5$$

Suppose Fuzzy risk set for $\alpha \ge .5$, the decision is Yes for x2 and No for x1, x4, x4 and x5.

VII. CONCLUSION

The decision has to be taken with incomplete information in many applications like Control systems ,medicine and business.

The fuzzy neural net is helpful for decision making because it can be represented with threshold. The fuzzy threshold is decision. The fuzzy neural net is discussed with two fold fuzzy set. The fuzzy inference is studied with fuzzy neural net. The fuzzy decision set is defined. Applications are discussed for fuzzy decision set.

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