Analytical Method for Calculation of Foundation Plates for Dynamic Loads

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Abstract. The article presents the main provisions of the analytical method for calculating elasticity on the action of dynamic loads from industrial equipment. Simple dependencies make it possible to quickly and with a sufficient degree of accuracy calculate structures for the specified loads. Thin foundation slabs have a high load-bearing capacity and allow significant savings in material and simplification of their construction. The underlying floors of industrial buildings are widely considered as foundations for equipment, which in turn makes it possible to increase the versatility of buildings. The dependencies presented in the article make it possible to calculate the floors of industrial buildings as endless slabs on an elastic foundation. The analytical method is based on the construction of impulse transient and transfer functions using the known relationships between them. The axisymmetric problem is being solved. The equation of motion for a plate on an elastic foundation with additional masses is written in polar coordinates and reduced to a system with two degrees of freedom. As examples, the calculations of slabs on an elastic foundation for the load from vertical reciprocating compressors of several types are given. Deflections and bending moments were calculated as a function of time depending on the distance to the center of the track. The calculation results show that the installation of such equipment is possible directly on the underlying layers of the floors. The vibration isolation device significantly reduces the amplitude values of the displacements and accelerations of the plates, while increasing the movement of the equipment itself.

KEYWORDS

Analytical method, dynamic analysis, elastic slab, industrial equipment, influence function.

INTRODUCTION

The problem of strength and efficiency of machine foundations structures is one of the important areas of construction design. A fairly large volume of domestic literature is devoted to the study of this problem [1-3]. Analytical methods of dynamic calculation on an elastic foundation are presented in the works of B.G. Korenev and Chernov Yu.T. [3,4].

The underlying layers of floors as supporting structures are widespread, in particular in assembly shops of various industries: the automobile and the aviation industry in warehouses and many others, and in particular when the compressors are located at the enterprises of the production of power plants.

The use of floors of industrial buildings as supporting structures for vibroactive equipment is an effective engineering solution that allows you to obtain significant savings in material (concrete and reinforcement) compared to installation on separate foundations. In addition, the versatility of buildings is significantly increased, in which it is relatively easy to replace technological equipment.

In this case, a thin slab on an elastic foundation with attached masses can be considered as a design scheme (Fig. 2a). With slab thicknesses up to 30 cm, the boundary conditions already at a distance of 1-1.5 m practically do not affect the nature of the stress-strain state of the slab [5] in the area of equipment installation. Therefore, for most design cases, the slab can be considered as unlimited. Most often, rigid underlays of floors of industrial buildings are used as thin foundation slabs for equipment [6-8].

METHODS

The main dynamic loads from machines with crank mechanisms are unbalanced forces and moments of inertia of moving parts, represented as the sum of harmonics, the first of which has the main shaft rotation frequency, the second - double the main shaft rotation frequency. As a rule, the calculation of structures on which machines with crank mechanisms are based is carried out only taking into account the dynamic loads of the first and second order, neglecting the influence of dynamic loads of a higher order. A general view of the installation and a cross-section of the compressor are shown in Fig. 1.

(a) (b)

FIGURE 1. a) Two-cylinder vertical reciprocating compressors. b) A cross section using a crank mechanism.

The vertical and horizontal components of the dynamic loads developed by each cylinder are calculated by the formulas [9]:

$$Q_z = R\omega^2 \sum_{i=1}^{n} [(m_{ai} + m_{bi})\cos(\omega t + \beta_i) + \alpha_i m_{bi} cos2(\omega t + \beta_i)]; \quad (1)$$

$$Q_x = R\omega^2 \sum_{i=1}^{n} m_{ai} sin2(\omega t + \beta_i) \quad (2)$$

where n – number of linearly spaced cylinders; $\omega = \frac{2\pi}{60}N$ – rotational speed of the main shaft of the machine, glad/s; t – time, s;

m_a – the mass of the parts of the crank mechanism, reduced to the crank pin, determined by the formula:

$$m_a = \frac{R_1}{R}m_1 + \left(1 - \frac{L_1}{L}\right)m_2 \quad (3)$$

m_b – the mass of the parts of the crank mechanism, reduced to the crosshead, determined by the formula:

$$m_b = m_3 + \left(\frac{L_1}{L}\right)m_2 \quad (4)$$

β_i – wedging angle; $\alpha_i = \frac{R}{L}$ – corresponding number of crank mechanism; R – crank radius; R_1 – distance from the axis of rotation to the center of gravity of the crank; m_1 – crank mass; L – connecting rod length; L_1 – distance from the center of gravity of the connecting rod to the crank pin; m_2 – connecting rod weight; m_3 – mass of reciprocating moving parts.

The calculations were carried out taking into account the vertical component of the dynamic load, the main technical characteristics of the compressors are given in Table 1, and the diagram of the crank mechanism with the location of the masses is shown in Fig. 2b.

Compressor specifications

Table 1

	1	2	3	4	5	6	7
Load track radius, m	0.440	0.441	0.611	0.841	0.708	0.842	1.004
Compressor weight, kg	1900	2500	4700	5500	6100	9000	16000
Number of cylinders	2	2	2	2	3	3	4
Circular frequency of rotation of the main shaft of the machine, glad/s	104.67	104.67	88.97	78.5	78.5	78.5	78.5
Amplitude value of the vertical component of the load, kN	3.73	4.91	9.22	10.79	11.97	17.66	31.39

In cases where the compressor is installed on a rigid subfloor in an industrial building, the design scheme can be represented as an unlimited slab located on an elastic foundation. Let us consider in more detail the axisymmetric problem of calculating an unbounded slab with attached masses, which lies on the basis described by a model with one coefficient of subgrade resistance. We assume that the mass m_{np} is connected to the plate by rigid ties, uniformly distributed around the circumference of the radius a_{np}, and elastic to the mass m_m (Fig. 2a).

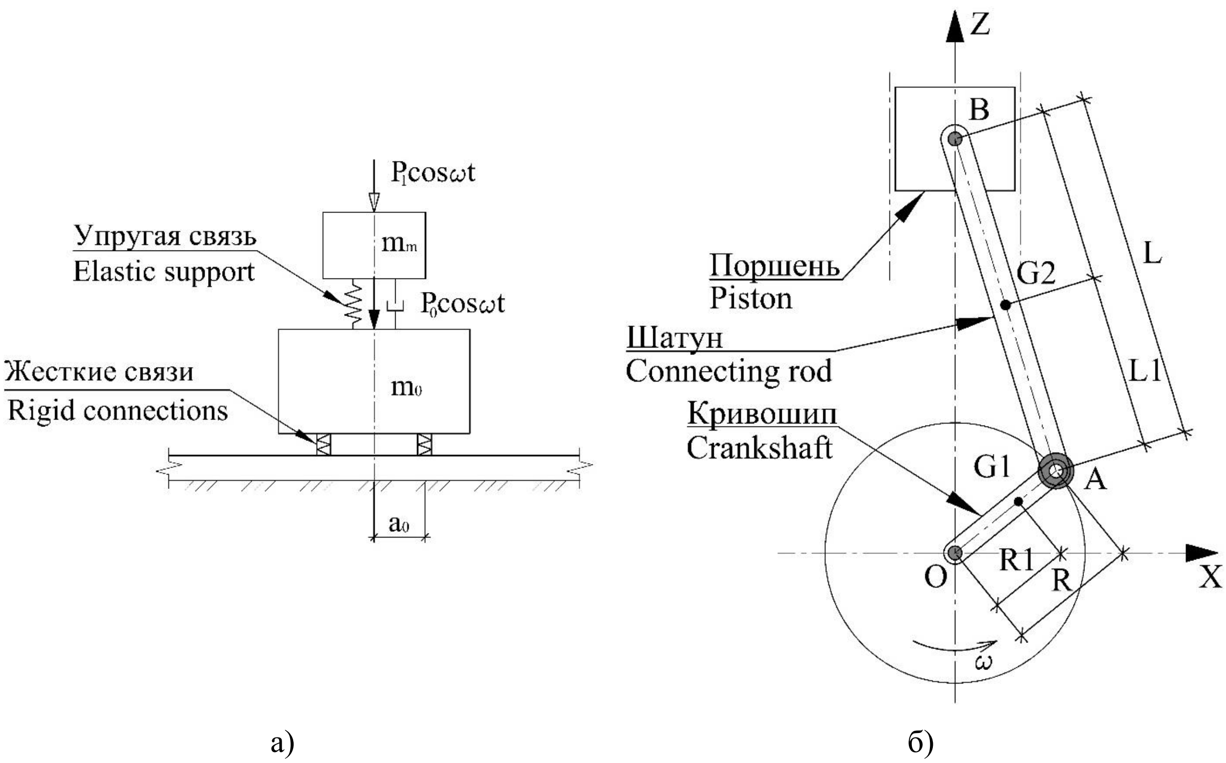

FIGURE 2. a) Accepted design scheme. b) Diagram of the moving parts of the compressor.

We write the system of resolving equations in the form:

$$m_m \frac{\partial^2 w_m}{\partial t^2} + k_m (1 + \mu_m \frac{\partial}{\partial t})(w_m - w_a) = p_m(t);$$
$$D(1 + \mu_{пл} \frac{\partial}{\partial t}) \nabla\nabla w_{пл} + \rho_0 \frac{\partial^2 w_{пл}}{\partial t^2} + k_0 (1 + \mu_0 \frac{\partial}{\partial t}) w_{пл} = p_{пл}(t), \quad (5)$$

where $\nabla = \frac{\partial^2}{\partial r^2} + \frac{\partial}{r \cdot \partial r}$ - Laplace's operator in polar coordinates, $w_{пл}(r,t)$ - slab deflection, $w_a(t)$ - slab deflection for $r = a_0$, $w_m(t)$ - mass transfer m_m, k_0 - coefficient of subgrade resistance, ρ_0 - mass per unit area of the slab, $\mu_m, \mu_{пл}, \mu_0$ - coefficients characterizing the energy dissipation in an elastic bond, plate and base, D - flexural rigidity, $p_m(t)$ - equipment load on mass m_m, $p_{пл}(t)$ - load transmitted to the slab and distributed around the circumference of the radius a_0.

When deriving the calculated dependences, it is convenient to determine the dissipative coefficients μ in accordance with the hypothesis of frequency-independent internal friction.

The load acting on the slab from the attached mass is determined from the equations of the system with two degrees of freedom:

$$p_{пл}(t) = \frac{1}{2\pi a_0} \left[p_0(t) - m_0 \frac{\partial^2 w_a}{\partial t^2} + k_m (1 + \mu_m \frac{\partial}{\partial t})(w_m - w_a) \right], \quad (6)$$

где $p_0(t)$ - equipment load transferred to the connected mass; m_0 - added mass.

Accept $p_m = P_m e^{i\omega t}$, $p_{пл} = P_{пл} e^{i\omega t}$, where $P_{m(пл)}$ - resultant forces acting on the plate and mass, $w_{пл} = W_{пл}(r) e^{i\omega t}$, $w_m = W_m e^{i\omega t}$ and substitute these dependencies in (5).

After transformations, we represent the first equation of the system in the form:

$$(1 + i\omega\mu_m) k_m (W_m - W_a) = F_m(i\omega)(P_m + m_m \omega^2 W_a), \quad (7)$$

where $F_m(i\omega) = (1 + i\omega\mu_m) / (1 + i\omega\mu_m - \varphi_m^2)$; $\varphi_m^2 = \omega^2 m_m / k_m$.

Let's introduce a new variable: $\xi = r / l_1$, где $l_1^4 = l^4 / \pm(1 - \varphi^2)$, $l = \sqrt[4]{D / k_0}$ and given (7), we rewrite the second equation of the system (5):

$$(1 + i\omega\mu_{пл}) \nabla\nabla W_{пл} \pm (1 + i\omega\mu_0^*) W_{пл} = \frac{l_1^4}{2\pi a_0 D} \left[P_0 + m_0 \omega^2 W_a + F_m(i\omega)(P_m + m_m \omega^2 W_a) \right] \delta(\xi, a_0), \quad (8)$$

where $\mu_0^* = \mu_0 / \pm(1 - \varphi^2)$; $\varphi^2 = \frac{\omega^2 \rho_0}{k_0}$, δ - Dirac's delta function, ρ_0 - mass of one square meter of the slab.

Applying to the equation (8) Hanckel's integral transformation [10] we represent its solution in the form (complex amplitude of plate vibrations):

$$W_{пл}(\xi) = \left[P_0 + P_m F_m(i\omega) + \omega^2 (m_0 W_a + m_m F_m(i\omega) W_a) \right] \cdot I(\xi, \alpha_0), \quad (9)$$

where integral $I(\xi, \alpha_0)$ obtained according to the scheme outlined, in particular [4] and, using the theorem of addition of cylindrical functions for a unit load, uniformly distributed over a circle of reduced radius α_0:

$$I(\xi,\alpha_0)=\frac{l_1^2}{2\pi D(1+i\omega\mu_{пл})}\int_0^\infty \frac{\alpha\cdot I_0(\alpha\alpha_0)\cdot I_0(\alpha\xi)}{\alpha^4\pm\beta^4}d\alpha, \tag{10}$$

where $I_0(\alpha\xi)$ - Bessel's function of the first kind, zero order; $\beta^4=\frac{1+\omega^2\mu_{пл}\mu_0^*+i\omega\mu^*}{1+\omega^2\mu_{пл}^2}$; $\mu_0^*=\frac{\mu_2}{(1-\varphi^2)}$; $\mu^*=\mu_0^*-\mu_{пл}$.

Assuming in (9) $\xi=\alpha_0$ we will define W_a and will substitute the resulting value back into (9). We write the transfer function of the system (dynamic function of influence) in the form:

$$W_{пл}(\xi,\omega)=\frac{P_0+P_mF_m(i\omega)}{1-\omega^2 I(\alpha_0)\cdot(m_0+m_mF_m(i\omega))}\cdot I(\xi,\alpha_0), \tag{11}$$

where

$$I(\alpha_0)=\frac{l_1^2}{2\pi D(1+i\omega\mu_{пл})}\int_0^\infty \frac{\alpha\cdot I_0(\alpha\alpha_0)\cdot I_0(\alpha\alpha_0)}{\alpha^4\pm\beta^4}d\alpha, \tag{12}$$

The mark (+) in formulas (10, 12) refers to the basic design case, when the frequencies of natural or forced vibrations of the system (plates with attached masses) are lower than the frequency of natural vibrations of the plate as a rigid stamp.

Modern means of computer mathematics allow calculating integrals (10) и (11) numerically, without resorting to further analytical calculations.

Let's consider special cases in more detail.

a) Plate with attached non-vibration-insulated mass.

Assuming $m_m=0$ and $P_m=0$ of (11) we write down the formula for determining the complex amplitudes:

$$W_{пл}(\xi,\omega)=\frac{P_0}{1-\omega^2 I(\alpha_0)m_0}\cdot I(\xi,\alpha_0) \tag{13}$$

After some transformations, the formulas for determining $I(\xi,\alpha_0)$ и $I(\alpha_0)$ take the form:

$$I(\alpha_0)=\frac{l_1^2}{2\pi D(1-\omega^2\mu_{пл}^2)}\int_0^\infty \frac{\alpha\cdot I_0^2(\frac{\alpha\cdot a_0}{l_1})}{\alpha^4\pm\beta^4}d\alpha, \tag{14}$$

$$I(\xi,\alpha_0)=\frac{l_1^2}{2\pi D(1-\omega^2\mu_{пл}^2)}\int_0^\infty \frac{\alpha\cdot I_0(\frac{\alpha\cdot a_0}{l_1})\cdot I_0(\frac{\alpha\cdot r}{l_1})}{\alpha^4\pm\beta^4}d\alpha, \tag{15}$$

where a_0 - radius of the circumferentially distributed load transmitted to the plate; r - coordinate of the point in question in the polar coordinate system.

Radial and annular bending moments are determined by the formulas:

$$M_r=-D(\frac{\partial^2 W_{пл}}{\partial r^2}+\frac{\nu}{r}\frac{\partial W_{пл}}{\partial r}),\;\; M_\varphi=-D(\nu\frac{\partial^2 W_{пл}}{\partial r^2}+\frac{1}{r}\frac{\partial W_{пл}}{\partial r}) \tag{16}$$

Let's take into account that $r=\xi\cdot l_1$ and rewrite the dependencies (16)

$$M_r=-\frac{D}{l_1^2}(\frac{\partial^2 W_{пл}}{\partial \xi^2}+\frac{\nu}{\xi}\frac{\partial W_{пл}}{\partial \xi}),\;\; M_\varphi=-\frac{D}{l_1^2}(\nu\frac{\partial^2 W_{пл}}{\partial \xi^2}+\frac{1}{\xi}\frac{\partial W_{пл}}{\partial \xi}) \tag{16*}$$

b) Plate with attached vibration-insulated mass.

Taking in (11) $m_0=0$ и $P_0=0$ we write down the formula for determining the amplitude values for this case:

$$W_{пл}(\xi,\omega)=\frac{P_m F_m(i\omega)}{1-\omega^2 I(\alpha_0)\cdot m_m F_m(i\omega)}\cdot I(\xi,\alpha_0) \quad (17)$$

Bending moments are determined by the formulas (16).

RESULTS

As an example of the application of the obtained dependencies, the calculations of vertical reciprocating compressors of a well-known brand were carried out, the manufacturer is located in Switzerland. The vertical component of the load is calculated using formula (1), depending on the specific technical characteristics of the machine.

The compressors are installed on reinforced concrete subfloors of industrial buildings with a thickness of 30 cm, made of concrete of class B25, the base must be prepared and well compacted. Coefficient of subgrade resistance - k = 25000 kN/m^3. Coefficients of inelastic resistance of reinforced concrete and elastic foundation are taken respectively 0,05 и 0,3.

The calculations were carried out for two options for installing equipment - vibration-insulated and non-vibration-insulated, for seven different types of compressors, the technical characteristics of which are shown in Table 2 and Table 3.

Calculation results for non-vibration-insulated compressors

Table 2

	1	2	3	4	5	6	7
Amplitude of vertical vibrations, mm	0.01261	0.01695	0.02913	0.02955	0.03517	0.05103	0.09271
Amplitude of accelerations, cm / m^2	13.8	18.6	23.1	18.2	21.7	31.4	57.1
Amplitude of the bending moment at the place of equipment installation, kN * m	0.43	0.65	0.758	0.81	1.01	1.559	1.562

Calculation results for vibration-insulated compressors

Table 3

	1	2	3	4	5	6	7
Total stiffness of vibration isolators, kN / m	2313	3043	4133	3766	4177	6162	10955
Amplitude of vertical vibrations, mm	0.003795	0.005026	0.008533	0.008802	0.01036	0.01463	0.02486
Amplitude of accelerations, cm / m^2	4.2	5.5	6.8	5.4	6.4	9.0	15.3
Amplitude of the bending moment at the place of equipment installation, kN * m	0.133	0.179	0.233	0.236	0.293	0.387	0.49

DISCUSSION

The results obtained indicate that the vibration amplitudes of plates with compressors of various masses installed on them meet the requirements of SP 26.13330.2012 “Foundations of machines with dynamic loads. Updated edition of SNiP 2.02.05-87 "and do not exceed the established maximum amplitudes of 0.1 mm. But the amplitude values of accelerations given in the tables are somewhat higher than the recommended ones [1], [11], which can cause additional settlements of the foundation soils associated with the vibration creep of the soil. Since the given calculated dependencies do not take this factor into account, it is recommended to reduce the acceleration amplitudes to a level of 15 cm/s^2 for soft soils and up to 30 cm/s^2 - in dense soils.

Calculation results show that installation of some lightweight compressors is possible directly on floors, even without vibration isolation devices.

As the comparison of Table 2 and Table 3 shows, the vibration isolation device can significantly reduce (3-4 times) the levels of vibrations and accelerations in the plate. The vibration-isolated version of the installation meets all the requirements for any of the above characteristics of the compressors. With more detailed calculations, it is possible to reduce the thickness of the underlying layer from 30 to 20-25 cm.

CONCLUSIONS

Applying the above dependences for calculating such structures for dynamic loads, it is possible to obtain the necessary results when choosing the characteristics of the slab and performing work.

The above formulas allow calculating thin unlimited slabs located on an elastic base with vibration-insulated and non-vibration-insulated options for installing vibroactive equipment.

The existence of the possibility of installing this type of equipment with dynamic loads directly on the underlying floors of industrial buildings has been demonstrated. The vibration isolation device allows installing heavier equipment without introducing additional structures (metal frames, local increase in floor thickness, etc.) install heavier equipment.

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