An Integrated Model and Enhanced Quantum Annealing Algorithm for Berth Allocation and Quay Crane Scheduling Problem

Zhen Li and Shurong Li

May 23, 2021

An integrated model and enhanced quantum annealing algorithm for berth allocation and quay crane scheduling problem

Zhen Li[1] and Shurong Li[1*]

School of Artificial Intelligence, Beijing University of Posts and Telecommunications, Beijing 100876, China

Abstract. This paper is concerned with the modeling and optimization for berth allocation, quay crane assignment and scheduling problem (BACASP). A serial integrated linear programming model is first established, which includes berth allocation and quay crane assignment problem (BACAP) as Sub-model 1 and quay crane scheduling problem (CSP) as Sub-model 2. Compared with most of the existing research, a more comprehensive summary for BACASP is provided by the proposed integrated model. Considering that the BACAP is a large-scale integer optimization with multiple local minimums, an enhanced quantum annealing algorithm (EQA) with strong global searching ability is then developed, in which a threshold compensation mechanism is designed to improve the global exploration ability, and a stop criteria is given to avoid over-repeated iteration. Moreover, the annealing scheduling is updated and a storage place is built in EQA. At last, an example of port dispatching is taken, and the effectiveness of the integrated model and superiority of the EQA are verified.

Keywords: container terminals, berth allocation, quay crane assignment, quay crane scheduling, enhanced quantum annealing algorithm

1 Introduction

Due to the rapid development of containerization and globalization, maritime transportation has become an important part of global supply chains. According to the statistics, maritime transport occupies 80% of global trade. With the request of higher work efficiency and less labor costs, many countries and areas are constructing automated container terminals, and the promotion on work efficiency has been a crucial issue over the past decades.

A seaport can be divided into two parts: the water-side part and the land-side part. Considering the water-side part of automated container terminals, there are various problems that can be discussed: berth allocation problem (BAP), quay crane assignment problem (CAP), quay crane scheduling problem (CSP), and the combination of them such as berth allocation and quay crane assignment problem (BACAP). Considering the land-side part, we can discuss VRP and CVRP problem, yard crane (YC) scheduling problem, and other relevant problems.

Diabat and Theodorou in [1] used genetic algorithm (GA) to solve CAP and CSP. Xiang et al. in [2] proposed a bi-objective robust model for berth allocation scheduling problem under uncertainty. Hu in [3] designed a multi-objective genetic algorithm (MOGA) to solve BAP considering daytime preference. Correcher and Ramon in [4] put forward a biased random-key genetic algorithm for BACAP and BACASP problem. Sun et al. in [5] discussed the CSP with vessel stability constraints. Malekahmadi et al. in [6] presented an integer programming model for integrated continuous BACAP in container terminals, and then put forward a random topology particle swarm optimization algorithm (RTPSO) to solve a large-size instance. Correcher et al. in [7] proposed a new mixed integer linear model to solve BACAP and extended it to solve BACASP. Tasoglu and Yildiz in [8] proposed a simulation optimization based solution approach for the integrated BAP and CSP considering simultaneously multi-quay hybrid berth layout, dynamic arrival of vessels, stochastic

*Corresponding author: lishurong@bupt.edu.cn.

handling time and non-crossing constraints of quay cranes. Liang et al. in [9] developed a coordination scheduling model which is composed of a storage subsystem model, a YC scheduling subsystem and a coordinate controller model, and then developed a coupling algorithm based on a genetic mechanism to solve it. He et al. in [10] developed a YC scheduling problem considering risk caused by uncertainty, and proposed a GA-based framework combined with three-stage algorithm to solve the problem.

Compared with the above existing research, the main contributions of this paper include the following three aspects:

(i) A description of an integrated two-stage model of berth allocation, quay crane assignment and scheduling problem (BACASP) is given.
(ii) An enhanced quantum algorithm (EQA) is put forward to improve traditional quantum annealing algorithm (QA).
(iii) Experiment simulations, including the comparison with EQA and other algorithms, and real-life experiment of BACASP are taken.

The rest of this paper is organized as follows: in section 2,we put forward a two-stage model of BACASP including BACAP and CSP, and the solution of the former sub-model is the initial solution of the latter one. In Section 3, we propose the EQA to further improve the global searching ability on the basis of the traditional QA algorithm. In Section 4, the relevant simulations are completed.

2 Model formulation

To solve berth allocation, quay crane assignment and scheduling problem (BACASP), we put forward a two-stage model which is a combination of two subproblems: berth allocation and quay crane assignment problem (BACAP), and quay crane scheduling problem (CSP), where the optimal solution of the former is reasonably considered as the input of the latter. The final scheme of BACASP can be obtained according to the integrated solution of these two sub-models.

2.1 Assumptions and nomenclatures

Before establishing BACASP mathematical model, several common assumptions (see [11]) are proposed first:

(1) The berth area is divided into berth segments in equal size.
(2) The passing channel time of vessels is negligible so that when a vessel finishes its service, it will depart immediately and the next vessel can berth immediately if it satisfies the berth condition.
(3) A quay crane can serve at most one vessel at any moment.
(4) A berth section can berth at most one vessel at any moment.
(5) Each vessel has a minimum and maximum number of quay cranes that can be assigned to.
(6) The start and relocation time of quay cranes is negligible, which means cranes can start working immediately after a vessel's berthing.
(7) Quay cranes cannot cross over each other, which means that cranes in the yard have a specific order.
(8) The berth position of each vessel is fixed before its departure.
(9) The number of cranes available in the quay is fixed and all cranes have the same work capacity.

Parameters involved in the mathematical model are expressed in Table 1, and decision variables are listed in Table 2.

Table 1. Parameter nomenclatures of the model

Parameter	Description
i	index of vessels
j	index of berth sections
q	index of quay cranes
t	index of time periods
N	number of vessels
B	number of berth segments
K	number of quay cranes
T	maximum time
e_i	expected arrival time of vessel i
d_i	departure time of vessel i
l_i	length of vessel i
k_{imin}	lower bound on the number of cranes that can be assigned to vessel i
k_{imax}	upper bound on the number of cranes that can be assigned to vessel i
p_i^k	processing time of vessel i if k cranes are assigned to it
w_i	workload of vessel i

Table 2. Decision variables of the model

Decision variable	Description
x_{ijt}^k	one if vessel i start berthing at section j in period t are assigned to k cranes, zero otherwise
y_{it}^g	one if crane group g is assigned to vessel i in period t, zero otherwise
z_{qt}	position of crane q in period t

2.2 Stage 1: BACAP

BACAP is the combination of two subproblems: berth allocation problem (BAP) and quay crane assignment problem (CAP). The BAP aims to find the optimal berth time and berth segments of each vessel, and the purpose of the CAP is to find the optimal number of cranes assigned to each vessel to minimize the total time.

The objective function and constraints of BACAP are shown as follows:

$$\sum_{i=1}^{n} (d_i - e_i + 1) \ . \tag{1}$$

Objective function (1) minimizes the total time of vessels in berth section.

$$\sum_{j=1}^{B-l_i+1} \sum_{k=k_{i\,\min}}^{k_{i\,\max}} \sum_{t=e_i}^{d_i} x_{ijt}^k = 1 \quad \text{for} \ \ i = 1, ..., V \ . \tag{2}$$

$$\sum_{i=1}^{N} \sum_{j=max(1,\,\hat{j}-l_i+1)}^{min(B-l_i+1,\,\hat{j})} \sum_{k=\,k_{imin}}^{k_{imax}} \sum_{t=s_i}^{d_i} x_{ijt}^k \leq 1 \quad \text{for} \ \ \hat{j} = 1, ..., B \ . \tag{3}$$

Constraints (2) and (3) represent the constraints of BAP. Constraint (2) guarantees that each vessel is served once. Constraints (3) ensures that there is at most one vessel in a berth section at any moment (assumption 4).

$$w_i \leq \sum_{t=e_i}^{e_i+p_i-1} \sum_{q=0}^{k_i} q y_{it}^g \quad \text{for} \ \ \hat{j} = 1, ..., B \ . \tag{4}$$

$$\sum_{i=1}^{N}\sum_{k=k_{imin}}^{k_{imax}}\sum_{j=1}^{B-l_i+1}\sum_{t=max(e_i,\,\hat{t}-p_i^k+1)}^{max(T-p_i^k+1,\,\hat{t})} kx_{ijt}^k \le K \quad \text{for } \hat{t}=1,...,T\ . \tag{5}$$

Constraints (4) and (5) are the constraints of CAP. Constraint (4) means that the service must satisfy the vessel's workload. Constraints (5) shows that the number of working cranes is no more than the number of available cranes. The above is the BACAP mathematical model, which is the first-stage model of BACASP as well.

2.3 Stage 2: CSP

Since BACAP model cannot determine the specific crane assigned to a vessel, the second stage problem named quay crane scheduling problem (CSP) is proposed. The constraints of CSP are shown as follows:

$$z_{qt} \le z_{(q+1)t} \quad \text{for } q=1,...,K-1 \text{ and } t=1,...,T\ . \tag{6}$$

$$z_{it} \ge 1 \quad \text{for } t=1,...,T\ . \tag{7}$$

$$z_{it} \le B \quad \text{for } t=1,...,T\ . \tag{8}$$

Constraint (6) determines the order of cranes. Constraints (7) and (8) ensure that all cranes are located in the berth area.

To sum up, the mathematical model of BACASP consists of the objective function (1) and constraints (2)-(8).

3 Enhanced quantum annealing algorithm (EQA)

3.1 Traditional quantum annealing algorithm (QA)

Recently, different kinds of quantum heuristic algorithms have received increasing attention as important realization means of quantum computing in the optimization field (see [12] , [13] and [14]). Among these quantum heuristic algorithms, QA is an efficient algorithm specializing in solving integer optimization problems. It is an updated heuristic technique based on simulated annealing (SA), which refers to the quantum tunneling effect that particles with low energy can jump out of energy barriers without giving them external energy. Crispin and Syrichas in [15] used QA to solve TSP problem. Liu and Li in [16] employed QA to solve mixed-integer optimization (MIO), and extended the original QA to solve multi-objective MIO in complex industrial applications (see [17] and [18]).

With Path-Integral Monte Carlo (PIMC), a quantum system is mapped onto a classical model to approach the lowest energy state of Ising model. In QA, the fitness function can be described by Hamiltonian energy as

$$H = H_p - J_{\Gamma}\Delta H_k\ . \tag{9}$$

In equation (9), H_p is the average potential energy of all replicas, and ΔH_k is the change of kinetic energy which provides a disturbance during iteration. ΔH_k should tend to zero as H approaches the average potential energy. J_{Γ} is the transverse ferromagnetic coupling coefficient defined as follows:

$$J_{\Gamma} = -\frac{T}{2}\ln\tanh(\frac{\Gamma}{ZT})\ . \tag{10}$$

In equation (10), T is the temperature, Γ is the tunneling field strength parameter, and Z is the number of replicas (expressed as symmetric spin matrices in QA).

The average potential energy H_p is defined as follows:

$$H_p = (\sum_{n=1}^{Z}\sum_{i=1}^{N}\sum_{j=1}^{N} d_{nij}S_{nij})/Z \ . \tag{11}$$

In equation (11), d is the distance between row i and column j, and Z is the number of replicas. And we define S_{ij} equals to 1 if the order of i is connected to the order of j, and 0 otherwise. Meanwhile, the kinetic energy is calculated as follows:

$$H_k = \sum_{n=1}^{Z}\sum_{i=1}^{N}\sum_{j=1}^{N} S_{(n-1)ij}S_{nij} + \sum_{n=1}^{Z}\sum_{i=1}^{N}\sum_{j=1}^{N} S_{(n+1)ij}S_{nij} \ . \tag{12}$$

In equation (12), the former term is the sum of products of the current replica with its previous replica, while the second term is the sum of products of the current replica with its next replica of all Z replicas.

3.2 Enhanced QA algorithm (EQA)

For large-scale problems with multiple extremums, traditional QA algorithm may fall into local optimal solution. To improve traditional QA algorithm, an enhanced QA algorithm (EQA) is designed.

Compared with QA, the improvements of the EQA can be summarized as follows:

(i) A compensation mechanism is added.
(ii) The annealing strategy is updated.
(iii) A storage place is set.

3.2.1 Compensation mechanism

QA is composed by two loops. The inner one is to execute the Monte Carlo step and update the current optimal solution, and the outer is to execute the annealing process until the terminal condition is reached. The number of steps in the inner loop is the number of Monte Carlo steps in each iteration step, and the number of steps in outer loop is the number of final iteration step.

To increase the global searching ability of QA, a compensation mechanism is added to the inner loop and a parameter M is set as a threshold. A characteristic of QA is that the probability of avoiding local optimal extremum is decreasing as the iteration increases. That is to say, in the front part of iteration steps with high temperature, it has strong global search ability. Therefore, we make compensation in the front part of the iteration steps (for example, the front 1/2 iteration steps) and set a parameter K as a threshold in the outer loop to avoid excessive iteration. Proper parameters should be set to enhance the ability to avoid local extremum and premature convergence.

3.2.2 Annealing scheduling

The annealing scheduling has influenced the efficiency of the algorithm as well. If the annealing rate is too fast, the algorithm may stop iteration before reaching global optimal solution, while if the annealing rate is too small, the efficiency of algorithm will be influenced. To increase the accuracy and efficiency of EQA, a new annealing strategy is defined as follows:

$$T_n = \begin{cases} W * \alpha^n * T, n \leq K \\ \alpha^n * T, n \geq K \end{cases} \ . \tag{13}$$

Here T is the initial temperature, n is the n^{th} annealing process, α is a coefficient, and W is the weight less than 1. In the former part of iteration, the global search ability is needed so that the annealing rate is slower. In the latter part of iteration, the global search ability decreases and a faster rate of annealing strategy is set to reduce the number of iteration steps and improves the efficiency of algorithm.

3.2.3 Storage Strategy

Moreover, a storage space is established to collect the optimal solutions of each iteration step and the final solution. By comparing all solutions saved in the storage space, the final solution is ensured to be the optimal solution.

Finally, the specific step of EQA can be summarized as follows:

Step 1: Randomly initialize a matrix that only one value of each row and column is 1 while other elements are 0.

Step 2: Randomly select step dimension d and steps a and b (for $a \neq b$). Elements of each row a and b is one, and the corresponding columns are p and q, so that the change of energy after spins flipped at the intersection of a, b, p and q is the criteria to judge whether the flip is needed or not. And a probability is set to execute the tunneling effect.

Step 3: Reduce the annealing parameter after repeating Step 2 in Monte Carlo step.

Step 4: Execute the flipping if there are no flips in MC steps.

Step 5: Repeat the above steps until reaching the termination condition.

4 Experiments

4.1 Part 1: comparison of EQA and other algorithms

To verify the feasibility of EQA, a TSP problem of 52 cities is used and the optimal solution is shown in Fig 1. Meanwhile, to verify the superiority of EQA, we conduct 50 repeated experiments for each algorithm and the results are shown in Table 3. The operating environment of all experiments is set in MATLAB R2019a with a Core i7-2.6 GHz CPU/ Windows 10 in 64 bit.

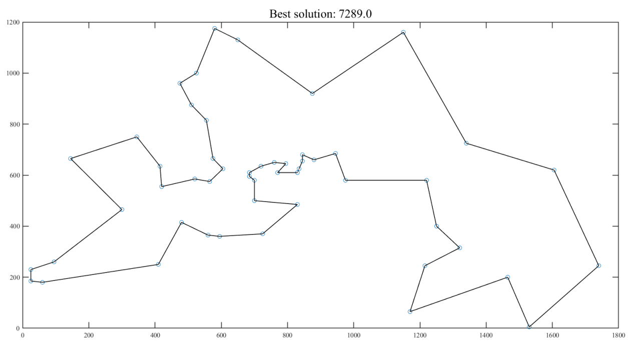

Fig. 1. Solution of TSP by EQA

Some evaluation indicators are defined as follows:

(i) Steps: to reflect the ability of convergence of an algorithm, equivalent to the average number of steps in reaching the optimal solution of the repeated experiments.
(ii) Accuracy: to reflect the global search ability of an algorithm, equivalent to ratio of times reaching global optimal solution and total times.

Table 3. Comparison of SA, QA, and EQA

	SA	QA	EQA
Parameter M	0	0	100
Parameter K	0	0	700
Steps	954	677	643
Accuracy	0.88	0.92	0.98

It can be seen in Table 3 that EQA shows a superior global searching ability and a slight better searching efficiency compared with SA and QA.

4.2 Part 2: experiments of BACASP

In this part, two experiments of BACASP are set. Experiment 1 is a small-scale BACASP problem with 8 vessels arriving in the same time, 10 berth segments, and 8 quay cranes, and Experiment 2 is a large-scale BACASP problem with 30 vessels arriving in different time, 20 berth segments and 12 quay cranes.

The necessary parameter settings of the BACASP model are set in Table 4. Parameters and the optimal scheme of Experiment 1 are shown in Table 5 and Fig 2. Furthermore, a comparison is performed and each algorithm is operated for 50 times respectively. The result of comparison is shown in Table 6. Parameters and the result of Experiment 2 are shown in Table 7.

Based on the calculation results below, the proposed EQA can be regarded as a competitive algorithm to solve the BACASP with ideal efficiency and searching capability.

Table 4. Parameters of vessels

Kind of vessel	Range of length	Range of cranes	Range of workload
Small	[1,3]	[1,2]	[5,14]
Medium	[4,6]	[2,4]	[15,25]
Large	[7,8]	[4,6]	[26,35]

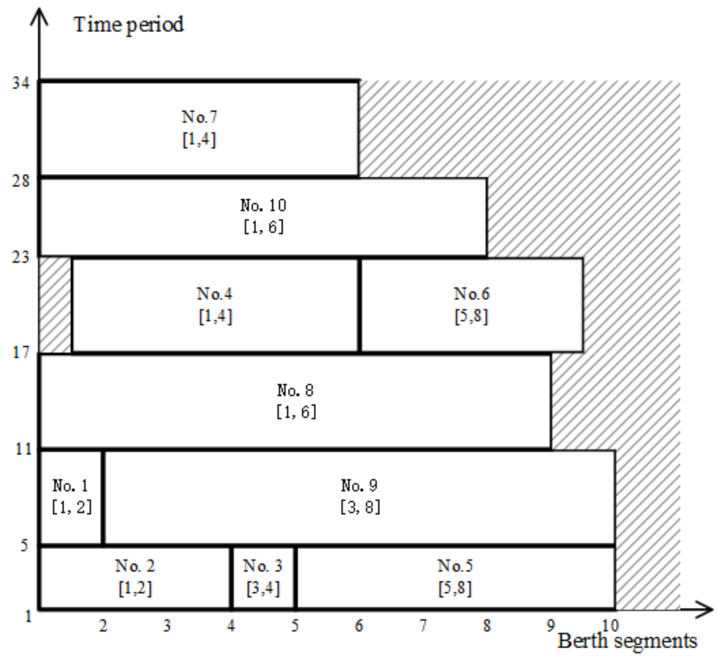

Fig. 2. Optimal solution of Experiment 1

Table 5. Parameters and optimal solution of Experiment 1

Vessel	Length	Workload	Berth time	End time	Segment	Cranes
1	1	12	5	10	1	[1,2]
2	3	9	1	4	1	[1,2]
3	1	10	1	4	4	[3,4]
4	4	21	17	22	2	[1,4]
5	5	10	1	4	5	[5,8]
6	4	22	17	22	6	[5,8]
7	5	25	28	33	1	[1,4]
8	8	26	11	16	1	[1,6]
9	8	32	5	10	2	[3,8]
10	7	30	23	27	1	[1,6]

Table 6. Comparison of SA, QA, and EQA

	SA	QA	EQA
Parameter M	0	0	100
Parameter K	0	0	700
Steps	379	284	253
Accuracy	0.84	0.90	0.98

Table 7. Parameters and optimal solution of Experiment 2

Vessel	Arrival time	Length	Workload	Segment	Berth time	End time	Cranes
1	2	3	13	6	2	8	[3,4]
2	12	2	8	7	3	5	[1,4]
3	37	3	9	1	37	40	[1,2]
4	15	3	10	5	15	19	[5,6]
5	13	3	15	9	13	20	[7,8]
6	24	2	8	10	24	27	[7,8]
7	5	2	14	2	5	11	[1,2]
8	30	2	13	18	61	66	[11,12]
9	12	3	14	14	9	16	[9,10]
10	2	3	13	17	2	8	[9,10]
11	39	5	21	14	46	51	[7,10]
12	21	5	19	15	21	25	[9,12]
13	40	5	15	1	52	55	[1,4]
14	14	5	16	14	17	20	[9,12]
15	35	5	21	7	35	40	[9,12]
16	3	5	19	9	5	9	[5,8]
17	1	5	15	11	1	4	[5,8]
18	38	6	25	1	41	47	[1,4]
19	10	4	24	1	12	17	[1,4]
20	40	6	18	1	62	66	[1,4]
21	25	7	33	6	29	34	[1,2]
22	23	7	30	7	51	55	[5,10]
23	27	8	29	11	56	60	[7,12]
24	13	8	30	1	56	61	[1,6]
25	29	7	26	1	29	33	[1,6]
26	35	8	29	11	41	45	[7,12]
27	39	8	29	2	51	56	[1,6]
28	7	8	26	6	46	50	[1,6]
29	24	8	28	1	24	28	[1,6]
30	36	8	30	12	36	40	[7,12]

5 Conclusion

In this study, a serial two-stage linear programming model of BACASP is built, which consists of BACAP responsible for berth allocation and quay crane assignment, and CSP with the purpose of quay crane scheduling. Then, considering that the BACAP is a large-scale integer optimization with multiple extremums, an enhanced quantum annealing algorithm (EQA) with strong global searching ability is developed. We propose a compensation mechanism in EQA, adding two parameters in the inner loop and outer loop to improve global searching. Also,the a updated annealing rule is set and a storage place of optimal solutions are built. Furthermore, the superiority of the EQA in global convergence is also verified compared with QA and SA. Finally, an example of port dispatching is taken. The correct optimal solution is obtained by the proposed model and algorithm.

References

1. Diabat, A., Theodorou, E.: An integrated quay crane assignment and scheduling problem.J. Comput. Ind. Eng. 73, 115-123(2014).
2. Xiang, X., Liu, C.C., Miao, L.X.: A bi-objective robust model for berth allocation scheduling under uncertainty.J. Transp. Res. E Logist. Transp. Rev. 106, 297-319(2017).
3. Hu, Z.H.: Multi-objective genetic algorithm for berth allocation problem considering daytime preference.J.. Comput. Ind. Eng. 89, 2-14(Nov.)
4. Correcher, J. F., Alvarez-Valdes, R.: A biased random-key genetic algorithm for the time-invariant berth allocation and quay crane assignment problem.J,. Expert. Syst. Appl. 89, 112-128(2017).
5. Sun, D.F., Tang, L.X., Roberto, B., Andrew, L.: An exact algorithm for the unidirectional quay crane scheduling problem with vessel stability.J. Eur. J. Oper. Res. 291, 271-283(May).
6. Malekahmadi, A., Alinaghian, M., Hejazi, S.R., Saidipour, M.A.A.: Integrated continuous berth allocation and quay crane assignment and scheduling problem with time-dependent physical constraints in container terminals.J. Comput. Ind. Eng. 147, 106672(2020).
7. Correcher, J.F., Alvarez-Valdes, R., Tamarit, J.M.: New exact methods for the time-invariant berth allocation and quay crane assignment problem.J. Eur. J. Oper. Res. 275, 271-283(2019).
8. Tasoglu, G., Yildiz, G.: Simulated annealing based simulation optimization method for solving integrated berth allocation and quay crane scheduling problems.J. Simul. Model Pract. Theory. 97, 101948(2019).
9. Liang, C.J., Gu, T.Y., Lu, B., Ding,Y.: Genetic mechanism-based coupling algorithm for solving coordinated scheduling problems of yard systems in container terminals.J. Comput. Ind. Eng. 89, 34-42(2015).
10. He, J.L., Tan, C.M., Zhang, Y.T.: Yard crane scheduling problem in a container terminal considering risk caused by uncertainty.J. Advanced Engineering Informatics, 39, 14-24(2019).
11. Türkoğulları, Y.B., Taşkın, Z.C., Aras, N., Altınel, İ.K.: Optimal berth allocation, time-variant quay crane assignment and scheduling with crane setups in container terminals.J. Eur. J. Oper. Res. 254, 985-1001(2016).
12. Liu ,M., Zhang, F., Ma, Y.: Evacuation path optimization based on quantum ant colony algorithm.J. Advanced Engineering Informatics. 30, 259-267(2016).
13. Liu, Z., Li, S.R.: A coevolutionary quantum krill herd algorithm for solving multi-objective optimisation problems. Int. J. Model Identif. Control. 34, 350-358(2020).
14. Alanis, D., Botsinis, P., Babar, Z., Nguyen H.V., Chandra, D., Ng, S.X.,Hanzo, L.: A quantum-search-aided dynamic programming framework for Pareto optimal routing in wireless multihop networks.J. IEEE Trans. Commun. 66, 3485-3500(2018).
15. Crispin, A., Syrichas, A.: Large-scale vehicle routing problems: Quantum Annealing, tunings and results.J. Comput. Oper. Res. 87, 52-62(2017).
16. Liu, Z., Li, S.R., Ge, Y.L.: Numerical computation of a mixed-integer optimal control problem based on quantum annealing.J. J. Shanghai Jiaotong Univ. Sci. 25, 623-629(2020).
17. Liu, Z., Li, S.R.: A Numerical Method for Interval Multi-objective Mixed-integer Optimal Control Problems Based on Quantum Heuristic Algorithm.J. Ann. Oper. Res. Published online (2021).
18. Liu, Z., Li, S.R., Ge, Y.L.: A quantum computing-based numerical method of mixed-integer optimal control problems under uncertainty for alkali-surfactant-polymer flooding.J. Eng. Optimiz. 53, 531-550(2021).