Modelling and Control of Brushless DC Motor

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Abstract—*Brushless direct current (BLDC) motors, also known as permanent magnet motors perceives vast applications in many industries due to their high efficiency, reliability, durability and ease of control. The paper introduces a model of three phase brushless dc motor and its different control mechanism. The major target of the paper is to present three different control techniques to stabilize each state of the plant with the aim to achieve high performance of BLDC motor in MATLAB/SIMULINK. The paper also describes motive to check the ability of each control technique to force the rotor to follow a preselected speed/position track. The first control approach designed sis conventional PID controller which reduces the steady state error of the system and gives a stable response at each state. The other two advance control techniques used are SMC which has given more efficient response by tracking the reference effectually in presence of disturbance and MPC also has given best performance at each state by effectively predicting the values in the presence of constraints. By the comparison of results it has been found that the proposed control approaches stabilizes each state of the system and have achieved better results with disturbance.*

Keywords - **BLDC, PID, MATLAB/SIMULINK, SMC, MPC, State-feedback**

I - INTRODUCTION

Brushless Direct Current motors are also well known synchronous motors having permanent magnet rotor and armature windings on the stator .The most apparent advantage of BLDC motor is that it does not have brushes which will damage because of friction. So it eliminates the brush maintenance problem, the erosion of commutator and elimination of sparking problem associated with the motor. As rotor is made up of a permanent magnet and have no winding on it so experience minimal electrical losses [1]. Brushless Dc motor offers a variety of advantages including better torque to weight ratio, high efficiency producing more torque per watt, highly reliable, noiseless operation, long lasting, high speed ranges, and rapid response and there is no risk of explosion or possibility of RF radiation due to arcing. [2]. A BLDC motor employs electrical commutation instead of mechanically commutation system. In BLDC motor armature is made of permanent magnet which gives rotation and stator composed of electromagnet whose purpose is to create field. As in BLDC there is electronic commutation so for this purpose mainly hall sensor or rotary encoders are used. The position of the armature is detected by these sensors feedback then according to this feedback, sensor will decide when to switch the armature current. Then according to principle of BLDC motor that when current carrying conductor is placed in the magnetic field it will experience force so due to this force rotor will rotate. Both the rotor and stator rotates at the same frequency, so BLDC motor also does not experience 'slip' concept [3]. Brushless Direct Current motors are one of the motors types that are increasingly being used in a wide spectrum of applications such as belt driven systems, CNC machine tools power tools rolling and paper mills computer hard drives and DVD/CD players. Also used in aviation, robotics, defense equipment, automobiles, transportation, Instrumentation, medical and in domestic equipment [4]. There are different control approaches which are used to control the speed of BLDC motor. Here, for the proposed system a comparison has been done among conventional and advance control, strategies. First approach implemented is state feedback control. The controller is designed by the pole placement method in which poles of a closed loop system are placed in pre-determined locations in s-plane. By setting controller gain the closed loop system poles locations can be determined in s plane and this location corresponds directly eigenvalues of the system [5].

Then second conventional control approach used is Proportional derivative integral controller (PID). PID controller is adapted in many fields in engineering due to its feasibility, ease of implementation, easily tuning of parameters, provides good stability and robustness as proposed in paper. Major hurdles in the way of PID control technique are sudden change in parameter variation, can be unstable unless tuned properly and derivative noise amplification makes PID control technique to give unstable results [4]. This problem can be eliminated by implementing advanced control approaches such as SMC and MPC control. One of the Variable structure controller types is sliding mode control which looks simple but it is difficult to implement. This is because there is possibility of the abrupt change in the control signal in SMC designing, which will then affect the operation of the plant. But with addition to this it is one of best robust controller to deal with higher order complex non-linear systems [5]. Then another advance control technique is used that is Model Predictive Control and it is used to control a process while satisfying a set of constraints. MPC is constructed using control and optimization tools. It's flexural, simple control policy for complex systems, generic consideration of constraints and it will fully exploit the plant dynamics, by using an optimization algorithm with the aim to find optimal control actions to predict future values. So MPC is considered as the best control approach to control speed of Brushless DC motor as verified in the paper.

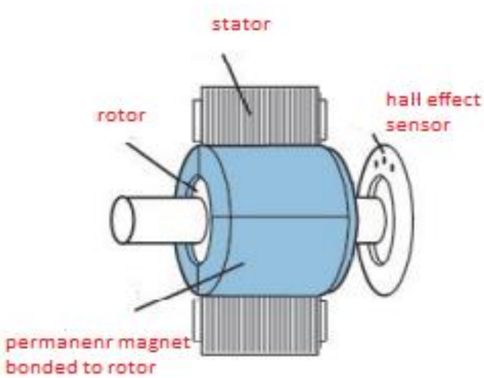

Figure -1.1 BLDC MOTOR

The arrangement of the rest of the paper is as follows, section II describes mathematical modelling of the BLDC motor which includes both the transfer function model that is in frequency domain and the state space model which is in time domain. In Section III proposed control strategies (Methodology) of BLDC motor is described. Then section IV presents the graphs of simulation. In Section V there is discussion on results on the basis of simulation results. Ultimately in Section VI the conclusion has been elucidated.

II- **MATHEMATICAL MODELLING:**

TRANSFER FUNCTION MODEL:

In this paper the model of Brushless Dc motor has been designed considering transfer function model and state space model as well. Transfer function model provide us with simple power analysis and design techniques and also provide information about the behavior of the system. As there the transfer function based mathematical model are broadly used in automatic control application so that's why transfer function model is considered as one of the best model in control theory.

For the proposed system there is two phase conduction mode used for the three phase BLDC motor [6].

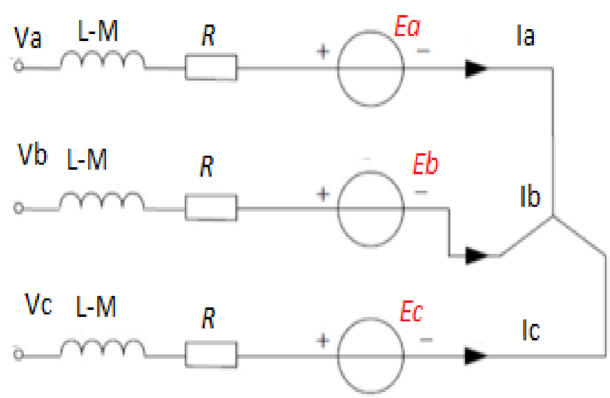

Fig. 2.1 Three Phase circuit of the BLDC motor

As it is working in two-phase conduction mode so at a time only two phases are exited AB, BC, or CA. The equivalent circuit diagram is given below in figure2.2.

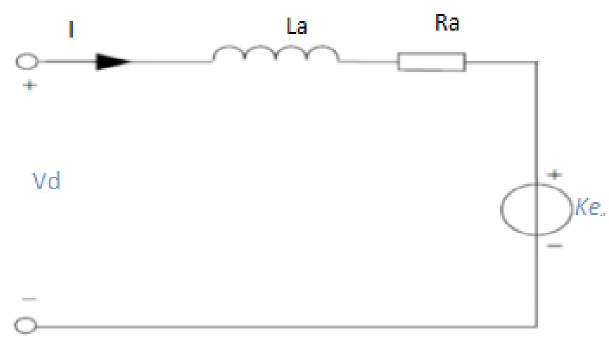

Fig 2.2.Equivalent circuit diagram of the brushless DC motor.

$I_a = - I_b = I$ (1)

$\frac{dI_a}{dt} = - \frac{dI_b}{dt} = \frac{dI}{dt}$(2)

$V_{ab} = 2Ri + 2(L\text{-}M)\frac{di}{dt} + (E_a - E_b)$ (3)

$\therefore E_b = - E_a$(4)

$V_{ab} = V_d = 2Ri + 2(L\text{-}M)\frac{dI}{dt} + 2E_a$

$= R_a I + L_a\frac{dI}{dt} + K_e\,\Omega$(5)

$K_t I - T_L = J\frac{d\Omega}{dt} + b\,\Omega$(6)

Assume Load Torque T_L=0.

$I = \frac{J}{K_t}\frac{d\Omega}{dt} + \frac{b}{K_t}\Omega$(7)

Substituting (7) in (5)

$V_d = R_a(\frac{J}{k_t}\frac{d\Omega}{dt} + \frac{b}{k_t}\Omega) + L_a\frac{d}{dt}(\frac{J}{K_t}\frac{d\Omega}{dt} + \frac{b}{K_t}\Omega) + K_e\,\Omega$

$V_d = \frac{L_aJ}{K_t}\frac{d\Omega^2}{dt^2} + \frac{R_aJ+L_ab}{K_t}\frac{d\Omega}{dt} + \frac{R_ab+K_eK_t}{Kt}\Omega$(8)

Using Laplace transform:

$C_1(s) = \frac{\Omega(s)}{V_d(s)} = \frac{K_t}{L_aJs^2+(R_aJ+Lab)s+(R_ab+K_eK_t)}$ (9)

$C_2(s) = \frac{\Omega(s)}{T_L(s)} = \frac{ra+La\,s}{L_as^2+(R_aJ+L_ab)s+(R_ab+K_eK_t)}$ (10)

Where:
V_d = Total direct current Voltage.
E_a =Overall Back emf(electromotive force).
R = Equivalent Armature Resistance
L_a = Equivalent inductance of Armature.
J= Moment of inertia of the rotor.
T_L =Torque against load.
I= Total current of equivalent circuit.
$\dot{\Omega}$= Rotor angular velocity.
B_v = Friction coefficient/ Damping ratio.
Ke = Coefficient of Back-emf (Voltage constant).
K_t = Torque constant.
$\dot{\theta}$= Rotor angular acceleration.

B. STATE SPACE MODEL:

State-space based mathematical models are gaining popularity as it is considered as one of best analysis methods in modern control theory.
As the transfer function model has certain drawbacks such as is it is defined under zero initial conditions, can be applied only to LTI systems and major limitation is that it provides no information regarding states of the system. So state-space models always gain importance in control theory [7].
For the proposed system the state space model is given below:

$I_a + I_b + I_c = 0$ (11)

$V_{ab} = R_a(I_a - I_b) + L_a\frac{d}{dt}(I_a - I_b) + E_{ab}$ (12)

$V_{bc} = R_a(I_a + 2I_b) + L_a\frac{d}{dt}(I_a + 2I_b) + E_{bc}$ (13)

Subtract equation (12) from equation (13)

$V_{ab} - V_{bc} = -3R_a - 3L_a\frac{d}{dt}I_b + E_{ab} - E_{bc}$(14)

$I_b' = \frac{R_a}{L_a}I_b - \frac{1}{3L_a}(V_{ab} - E_{ab}) + \frac{1}{3L_a}(V_{bc} - E_{bc})$ (15)

By the same method:

$V_{ab} = R_a(2I_a + I_b) + L_a\frac{d}{dt}(2I_a + I_b) + E_{ab}$(16)

$V_{ca} = R_a(I_c - I_a) + L_a\frac{d}{dt}(I_c - I_a) + E_{ca}$(17)

Subtracting equation (16) from equation (17)

$(V_{ab} - E_{ab}) - (V_{ca} - E_{ca}) = 3R_a I_a + 3L_a\frac{d}{dt}I_a$(18)

$I_a' = -\frac{R_a}{L_a}I_a + \frac{1}{3L_a}(V_{ab} - E_{ab}) - \frac{1}{3L_a}(V_{ca} - E_{ca})$ (19)

$V_{ab}= V_{bc}$(20)
$E_{ab}= E_{bc}$(21)

$V_{ca}= -(V_{ab}+ V_{bc}) = 2 V_{ab}$(22)

$E_{ca}= -(E_{ab}+ E_{bc}) = -2 E_{ab}$(23)

By substituting (20) , (21) , (22) , (23) in (19) will be:

$$I_a' = -\frac{R_a}{L_a} I_a + \frac{1}{3L_a}(V_{bc}-E_{bc}) + \frac{2}{3L_a}(V_{ab}- E_{ab}) \quad (24)$$

From equation (6):

$$\Omega' = \frac{b}{J}\ \Omega + = \frac{1}{J}(T_e - T_L) \quad (25)$$

Where:

$$T_e = K_t I \quad (26)$$

$$\begin{bmatrix} \dot{I}a \\ \dot{I}b \\ \dot{\Omega} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{-R_a}{L_a} & 0 & 0 & 0 \\ 0 & -\frac{R_a}{L_a} & 0 & 0 \\ 0 & 0 & \frac{-b}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ \Omega \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{2}{3L_a} & \frac{2}{3L_a} & 0 \\ \frac{-1}{3L_a} & \frac{1}{3L_a} & 0 \\ 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ab} - E_{ab} \\ V_{bc} - E_{bc} \\ T_e - T_L \end{bmatrix} \quad (27)$$

Where θ is rotor position.

C: Parameters for BLDC

Rating	Symbol	Value	units
Resistance	R	0.25	Ω
Inductance	L	0.32	mH
Number of poles	P	8	
Rated DC voltage	V	15	V
Armature Inertia	J	0.0042	$Kg.m^{-2}$
Damping ratio	b	0.0096	N.M.S
Rated current	I	43.5	A
Peak torque	T_P	2.83	NM

III METHODOLOGY:

The proposed methodology targets various control strategies that are as follow:

1- STATE-FEEDBACK CONTROLLER:

For the BLDC motor state-feedback controller designing is always employed in feedback control systems and the major technique used for that is pole placement method. The system should be in controllable form in order to design state-feedback control for it which will stabilize it. The control input for sate-feedback control is U=-KX. Where k is the feedback vector, so by placing closed loop eigenvalues at that vector it will stabilize each state of the system to some extent [8]. The design of a feedback control system for BLDC motor is:

$\dot{x} = Ax + Bu$
$Y = Cx + du$

$$A = \begin{bmatrix} \frac{-R_a}{L_a} & 0 & 0 & 0 \\ 0 & -\frac{R_a}{L_a} & 0 & 0 \\ 0 & 0 & \frac{-b}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{2}{3L_a} & \frac{2}{3L_a} & 0 \\ \frac{-1}{3L_a} & \frac{1}{3L_a} & 0 \\ 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix}$$

Where gain matrix K is :

K= [k1 k2 k3 k4]

So,

$U = -k1x1 - k2x2 - k3x3 - k4x4$
$U = -KX$
State variables are:

$\dot{I}_a$= Current in Phase A

$\dot{I}_b$= Current in Phase B
$\dot{\Omega}$= Rotor angular velocity
$\dot{\theta}$= angular acceleration

B- PID CONTROLLER:

A proportional integral & derivative controller (PID) is a generic control loop approach involving feedback concept/mechanism. The main objective behind a PID controller is that each part must be individually

"tuned". For that purpose there are various tuning techniques like auto tuning of parameters, genetic algorithm tuning method and Ziegler Nichols tuning method. [9].Basically PID looks the set point and then compares the desired and actual output value and on the basis of it eliminates error from the proposed system. In PID controller the proportional part reduces the disturbance from the plant; derivative mode improves the stability of the system so that each state will get stabilize quickly and also increases the gain K which increases speed of controller response whereas integral part reduces steady state error in the system.
The generic form of transfer function of PID controller is:

$$K(s) = K_p + \frac{K_i}{s} + K_d\, s$$

The proportional part is proportional to error:
$U = (K_p * e(t))$
Integral part will be proportional to area under error curve:
$U = K_i * \int_0^t e(t)d(t)$
The derivative part is proportional to derivative of the error signal :
$U = K_d * \frac{d}{dt}e(t)$
Now PID control function is:
$U = (K_p * e(t)) + (K_i * \int_0^t e(t)d(t)) + K_d * \frac{d}{dt}e(t)$

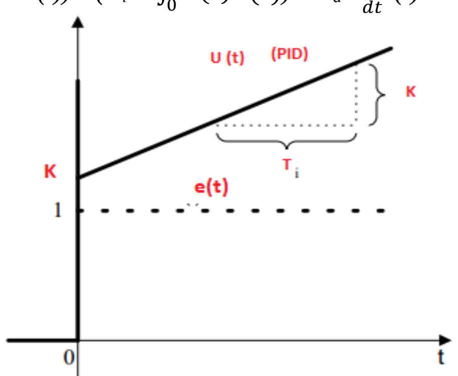

Fig- 3.1 Transfer function of PID

After testing various approaches for tuning parameters of PID controller the proposed system has given best results at auto tuning and stabilizes each state of the plant. The suitable values of the gains K_p, K_i and K_d for the PID controller are given below:

Gains	value
K_p	0.481868
K_i	0.7457686
K_d	0.069449
Filter coefficient	30.15132

C- SLIDING MODE CONTROL (SMC):

The sliding mode controller (SMC) is mainly a nonlinear control strategy that alters the dynamics of a nonlinear plant [10]. The major idea behind SMC was to define a sliding surface that force the system trajectory to "slide" on the sliding surface to reach its desired final value keeping its entire derivative to zero.

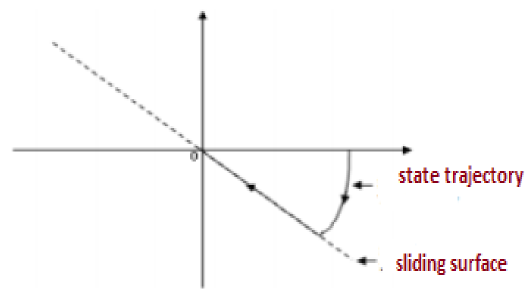

Figure-3.2 Phase portrait of a sliding motion

Then the proposed system is at first defined by equation of the sliding surface so that it becomes independent of the system parameters and external disturbance.
Designing a sliding mode control two main points are kept under consideration:

1- The selection of stable sliding on which system will slide to follow its pre-described trajectory.
2- Then control law should be designed so that system will converge to the sliding surface in finite time.

The generic form of state variables for SMC design is defined as:

$x_1 = x$, $x_2 = \dot{x}_1$

$$\begin{cases} \dot{x}_1 = x_2 x_1(0) = x_{10} \\ \dot{x}_2 = u + f\,(x_1, x_2, t) \qquad x_2\,(0) = x_{20} \end{cases}$$

Where u is control force $f\,(x1, x2, t)$ is disturbance function.
A linear state-feedback control law which provides asymptotic stability of the origin only for $f\,(x1, x2, t) = 0$.

$u = -k_1x_1 - k_2x_2$, $k_1 > 0$, $k2 > 0_1$

Then after parameterizing the sliding variables in order to ensure the desired compensated dynamics we get the equation as follow:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 - cx_2 - psign(x_2 + cx_1, t + f(x_1, x_2, t) \end{cases}$$

At first as system is of 4rth order so by applying matrix reduction technique it is converted into second order system using some commands in MATLAB.
For the proposed system, firstly a sliding surface is selected which is like designing the objective function. By definition is s(x)=0. Sliding surface for a conventional mode sliding control is designed by choosing sliding variable **'c'.** By using sliding surface equals to zero the sliding variable becomes C=1.5.
Then for this sliding surface equation is:

Function s= fcn (x_1,x_2)

$s = x_2 + 1.5 * x_1$

Then control law is defined by creating array in the workspace for variables x_1 ,x_2 as described below:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u + sin(t)$$

For the reference we have used unit step function:

r(t) = u(t)

Disturbance used is a sinusoidal signal:
d(t) = sin(t)

The parameters used in SMC are:

Parameters	value
controller gain (P)	6
Sliding variable (c)	1.5

D- MODEL PREDICTIVE CONTROL:

It is an advance control technique that uses proposed model and in response it forecast future output response of a system. MPC aims to improve the upcoming plant behavior by computing chain of prospective control variable. The main objective is to minimize cost function.

s

$$£ = \sum_{k=0}^{Np} (y - s)^T Q (y - s) + \sum_{k=0}^{Np} (\Delta u^T E \, \Delta u)$$

Np: Forecast horizon

s: settle-point

y: Output

Δu: Change in control signal (predicted).

Q: Weight Matrix for error on O/P.

E: Weight matrix for control I/P.

For the proposed system to achieve the better results the advance control technique implemented is Model predictive Control (MPC). The proposed system will operate closer to constraints which were set for the system (max & min). MPC can handle the upcoming or external disturbances which can affect the system, and it can also predict the future control action on the basis of current inputs and past states ofthe system.

For the proposed plant controller parameters are selected as: control horizon N_c =3, predictionhorizon Np =15 which means system can predict future output till 15 steps and the sampling interval is Ts=0.00001.

MPC can handle constraints. So the main equationsof constraint in MPC are:

On Output: $O_{min} < O < O_{max.}$

On Input: $U_{min} < U < U_{max.}$

On States :$X_{min} < X < X_{max.}$

For our system input constraints are:

U1: -50<U1<+50.

U2:-50<U2<+50.

U3:--50<U3<+50

The Output Constraints are:

Y:-[-5;-10]<Y<[+5;+10].

IV- SIMULATION RESULTS:

A- Behavior of all the states for proposed BLDC motor is given below:

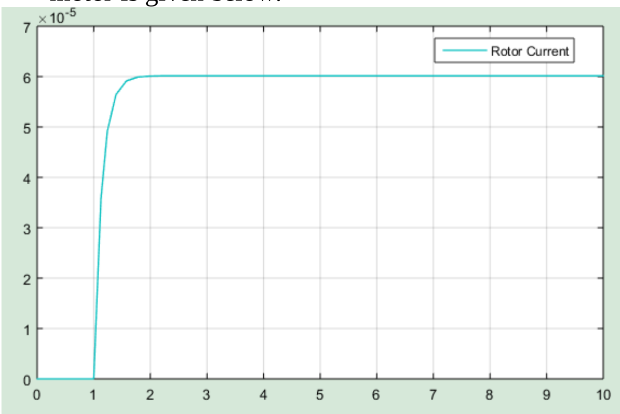

fig-4.1 Rotor current of phase A (Ia) of state-feedback control.

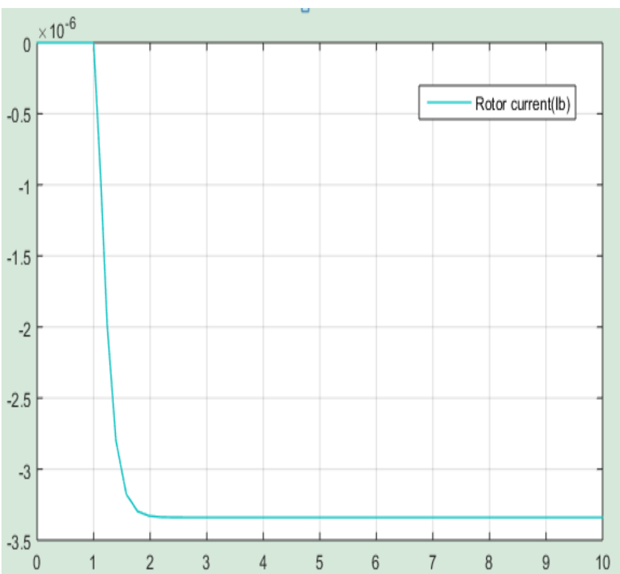

fig-4.2 Rotor current response in phase B (I_b) of State-feedback control.

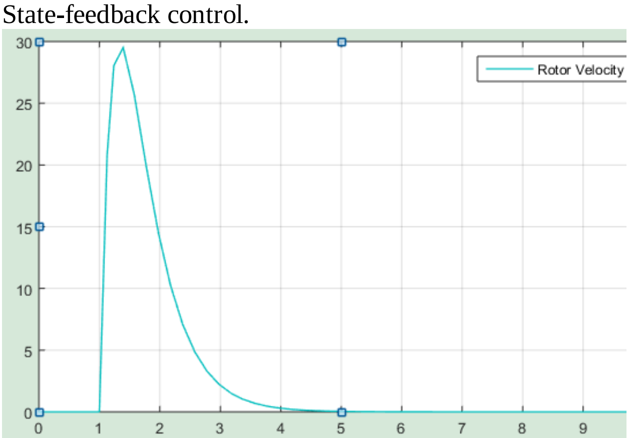

fig-4.3Rotor angular velocity response ($\dot{\Omega}$) ofState-feedback control.

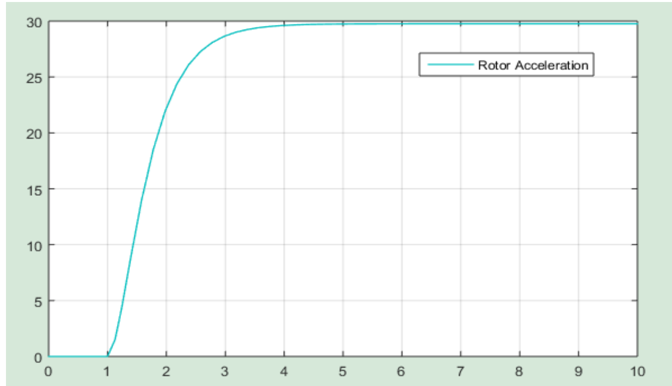

fig-4.4ResponseofRotor angular acceleration($\dot{\theta}$) for State-feedback control.

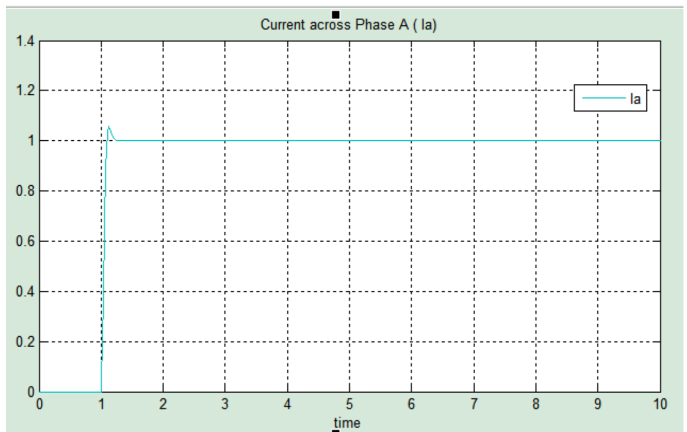

Fig-4.5 Response of Rotor current of phase A (Ia) for PID control..

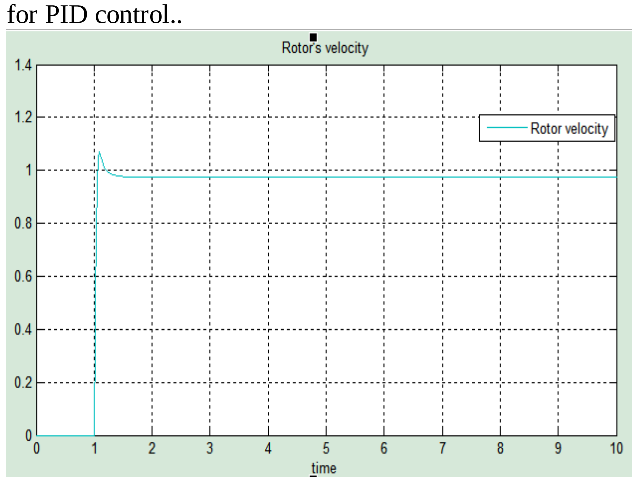

fig-4.6 Rotor angular velocity response ($\dot{\Omega}$) forPID control.

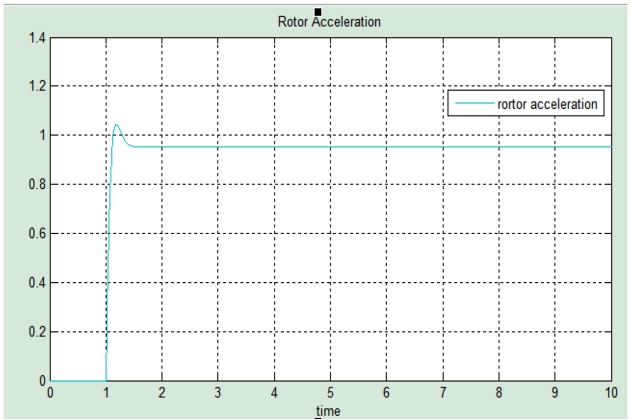

fig-4.7 Response of Rotor angular acceleration($\dot{\theta}$) for PID control.

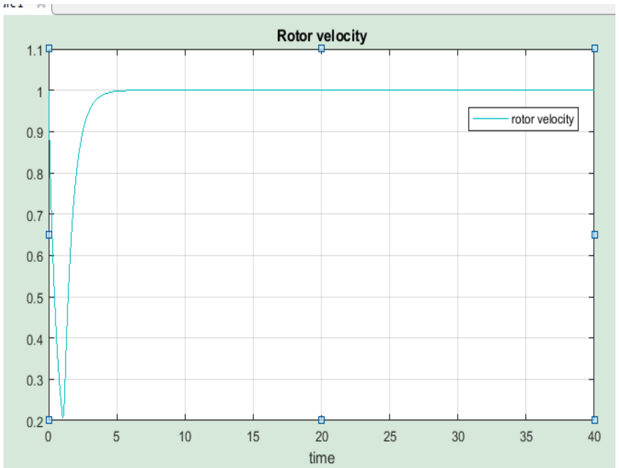

fig-4.8Rotor angular velocity ($\dot{\Omega}$) response forSMC control.

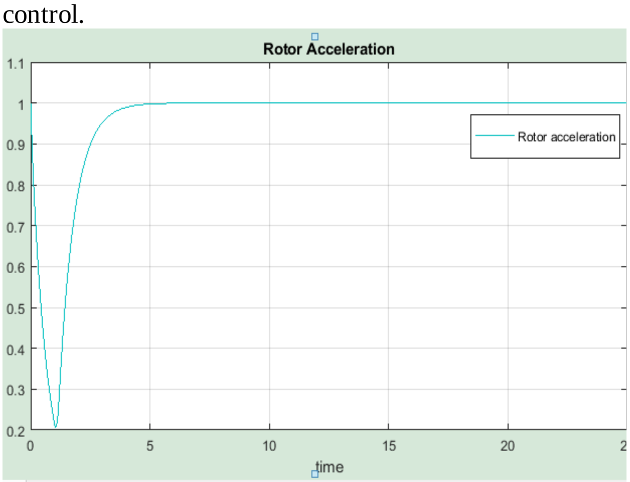

fig-4.9 Response of Rotor angular acceleration($\dot{\theta}$) for SMC control.

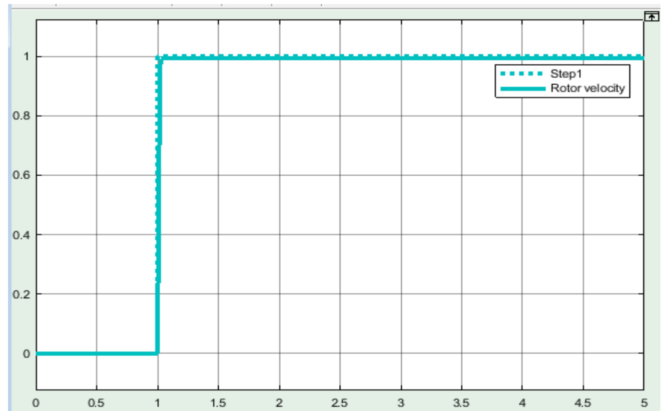

fig-4.10 Rotor angular velocity response ($\dot{\Omega}$) for MPC control.

B. COMPARISON:

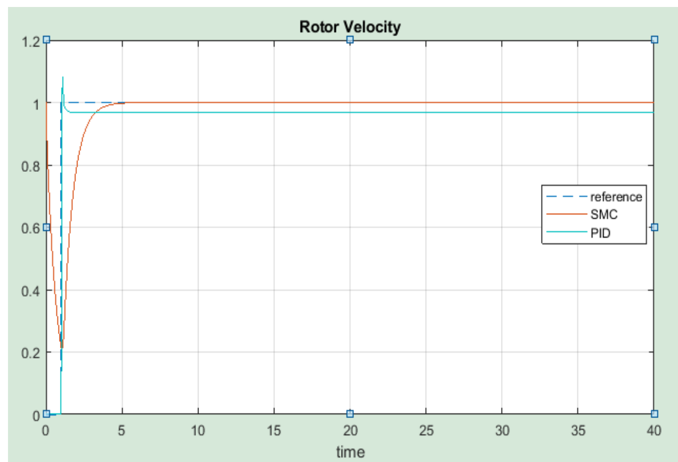

Fig- 4.11 Comparison of rotor velocity of PID and SMC control.

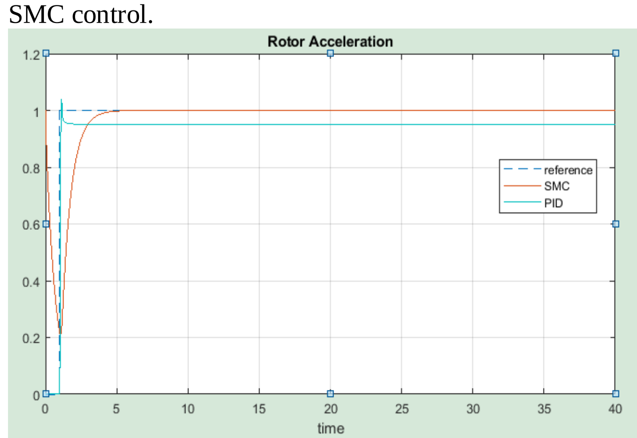

Fig- 4.12.Comparison of rotor acceleration of PID and SMC control.

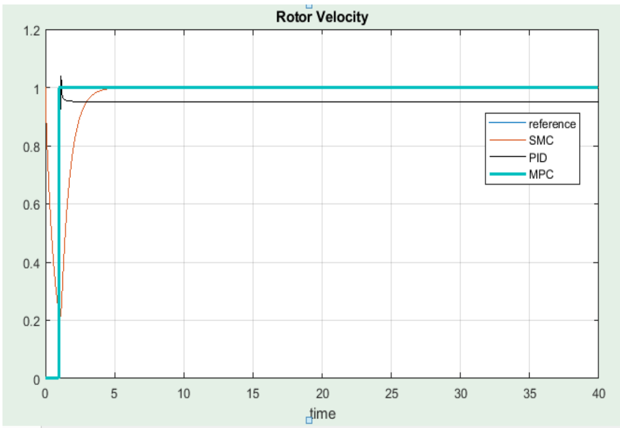

Fig- 4.13.Comparison of rotor velocity of PID, SMC and MPC control.

V- DISCUSSION ON RESULTS:

The BLDC motor is controlled with different control approaches with the aim to stabilize each state of the system.

The unit step response is used as a reference for each control technique as it includes the settling time and performance of controller. For state feedback control, fig 4.1 shows the rotor current across phase A (I_a) and by achieving steady state it exhibits a stable response. Figure 4.2 shows rotor current across phase B (I_b) it is clear that it does not showed much stable response as the curves have negative amplitude and not tracking the reference input that is with positive amplitude. Then rotor velocity ($\dot{\Omega}$) is shown in fig 4.3 at first it shows some overshoot then becomes stable. Figure-4.4 exhibits rotor acceleration ($\dot{\theta}$), which achieves steady state after a large settling time with amplitude value of 30.

The conventional control technique analyzed to achieve desired results for the proposed system is PID control, which shows better results than state-feedback control. In fig.4.5 it shows that with PID rotor current (I_a) has a rapid rise time however it's settling time is lower due to oscillatory response before it reaches to its steady state. For rotor velocity ($\dot{\Omega}$) it achieves steady state fast, as shown in fig 4.6. Then at last rotor acceleration shows almost same response like rotor velocity but stabilizes quickly, as seen in figure. 4.7.

The advance control approach analyzed is sliding mode control to deal with nonlinearities. As the major concern is to control the rotor velocity and acceleration, so for SMC only these two states are examined in the presence of disturbance function. At rotor velocity ($\dot{\Omega}$) there are no oscillations and it achieves steady state so fast in accordance with fig 4.8. The figure-4.9 shows the rotor acceleration ($\dot{\theta}$), which gives a stable response in presence of disturbance and tracking reference efficiently.

Model predictive control technique has been used to analyze the response of desired states. In MPC major focus is on rotor velocity ($\dot{\Omega}$) which achieves steady state quickly and has very small settling time, as compared to conventional PID and SMC control techniques.

In this paper a comparison has been done between advance and conventional control techniques for the proposed system. By the comparison of results it has been found that the advance control approach (SMC) used is better than conventional PID control technique. Figure 4.11 shows that the settling time of SMC is less than PID. In SMC the rotor velocity ($\dot{\Omega}$) achieves steady state quickly with disturbance and in PID it stabilizes later without disturbance due to large settling time. In SMC for rotor acceleration (θ) as described in figure 4.12, there are no oscillations like PID and system tracks its reference quickly and becomes stable.

Figure 4.13 shows the comparison results for both conventional and advance control approaches. In accordance with the figure 4.13 it is quite clear that MPC gives better results on rotor velocity($\dot{\Omega}$) and stabilizes rapidly as compared to conventional control techniques.

VI- CONCLUSION:

The model of BLDC motor has been designed and its performance is analyzed. Then to control its speed and to track the pre-selected reference, distinct control approaches have utilized. The proposed control strategies are being investigated for the performance, velocity and acceleration control for BLDC motor. The conventional and advance control strategies are implemented and it has been clarified through results that advance control strategies are finest. We have verified through simulations that SMC is providing high disturbance rejection and granting high stability as compared to conventional PID controller. The advance control approaches initiates the system efficiently to the preferred trajectory.

The results obtained demonstrated that MPC offers better performance than the classical PID and advance SMC controller approaches. MPC gives good trajectory tracking performance. The response of rotor velocity through MPC is superior to PID and SMC control strategies. By the comparison of results it is predicted that the advance control approach MPC grant better performance in all aspects and stabilizes system very efficiently as well.

REFERENCES:

1. Kumar, Y. Narendra, et al. "Speed Control of BIdc Motor Drive By Using PID Controller‖." *International Journal of Engineering Research and Applications* 4.4 (2014): 37-41.
2. Tayyab, Kenza, Fahad Mumtaz Malik, and Shahzad Amin Sheikh. "SAMPLED DATA CONTROL OF BLDC MOTOR USING SMC AND FUZZY CONTROLLER AND COMPARING THEIR PERFORMANCE." *Science International* 27.3 (2015).
3. Zhao, Jian, and Yangwei Yu. "Brushless DC motor fundamentals application note." *AN047, July* (2011).
4. Visconti, Paolo, and Patrizio Primiceri. "An overview on state-of-art and future application fields of BLDC motors: design and characterization of a PC-interfaced driving and motion control system." *J. Eng. Appl. Sci. ARPN* 12.17 (2017): 4913-4926.
5. Das, Sajal K., N. Mondol, and N. A. Sultana. "Design and implement of a state feedback position output controller for a Maxon S-Dc Motor with dSPACE." *Proc. of the International Conference on Mechanical Engineering*. 2011.
6. Shamseldin, Mohammed Abdelbar, and Adel A. EL-Samahy. "Speed control of BLDC motor by using PID control and self-tuning fuzzy PID controller." *15th International Workshop on Research and Education in Mechatronics (REM)*. IEEE, 2014.
7. 7- Vishnu, C. S., and Riya Mary Francis. "Speed Control of BLDC Motor using a Tuned LQR Controller." *International Journal of Computer Applications* 975: 8887.
8. 8-Muruganantham, N., and S. Palani. "State space modeling and simulation of sensorless permanent magnet BLDC motor." *International Journal of Engineering Science and Technology* 2.10 (2010): 5099-5106.
9. 9- Maung, MyoMaung, MaungMaungLatt, and Chaw MayatNwe. "DC motor angular position control using PID controller with friction compensation." *International journal of scientific and research publications* 8.11 (2018): 149.
10. 10-Galphade, Namita P., and Subhash S. Sankeshwari. "Simulation of bldc motor control using sliding mode control technique." *International Journal of Advances in Engineering & Technology* 7.6 (2015): 1857.
11. 11- Maghfiroh, Hari, AugustinusSujono, and Chico HermanuBrilliantoApribowo. "Basic Tutorial on Sliding Mode Control in Speed Control of DC-motor." *Journal of Electrical, Electronic, Information, and Communication Technology* 2.1: 1-4.
12. Fu, Guangjie, Ziming Zhao, and Haiwei Mu. "Improved Speed Predictive Control for the Brushless DC Motor Adjusting Speed Study." *Information Technology Journal* 13.14 (2014): 2298.
13. Gaya, M. S., et al. "Enhanced pid vs model predictive control applied to bldc motor." *IOP Conference Series: Materials Science and Engineering*. Vol. 303. No. 1. IOP Publishing, 2018.
14. Ali, Musayyab, Muhammad Abdullah Adnan Farooqi, and Fahad Mumtaz Malik. "H INFINITY CONTROL OF A MECHANICAL SYSTEM WITH BACKLASH."
15. El-Samahy, Adel A., and Mohamed A. Shamseldin. "Brushless DC motor tracking control using self-tuning fuzzy PID control and model reference adaptive control." *Ain Shams Engineering Journal* 9.3 (2018): 341-352.