

A New Sequence of Prime Numbers

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Abstract. In this paper, we discovered a new sequence of prime numbers, every term of this sequence is either a prime number or equal to 1.

Keywords. Prime numbers, sequence.

1. Introduction

A number is said to be a prime number if the number is divisible by 1 and itself; otherwise it's composite. In this paper, we present two new sequences related with the continued fraction.

2. The sequence of b(n)

The sequence b(n) satisfy the following recursive formula

$$b(n) = (n-1)b(n-1) - nb(n-2)$$

With the starting conditions b(3) = 1, and b(4) = 7

Table 1. The first few values of b(n)

| n | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|-----------------------|---|---|----|----|-----|------|------|-------|--------|
| b (n) | 1 | 7 | 23 | 73 | 277 | 1355 | 8347 | 61573 | 523913 |

Theorem 2.1 For $n \ge 3$.

i)
$$\frac{b(n)}{n^2 - n - 1} = \frac{1}{2 - \frac{3}{3 - \frac{4}{\frac{4 - 5}{\ddots}}}}}_{(n-1) - \frac{n}{n - (n+1)}}$$

For $n \ge 5$.

ii)
$$b(n) = (2n^2 - 6n + 3) \cdot A051403(n - 5) - (2n^2 - 5n + 2) \cdot A051403(n - 6)$$

Proof. By using some simplification of the denominator of the continued fraction.

3. The sequence of a(n)

In this section, we present our sequence of prime numbers defined in the conjecture as follows

Conjecture 3.1. The sequence a(n) of the prime numbers satisfy the following formula

$$a(n) = \frac{n^2 - n - 1}{gcd(b(n), n^2 - n - 1)}$$

Table 2. The first few values of a(n)

| n | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|-----------------------|---|----|----|----|----|----|----|----|-----|
| a (n) | 5 | 11 | 19 | 29 | 41 | 11 | 71 | 89 | 109 |

Also we have

$$a(37) = a(43) = a(48) = a(53) = 1$$

Conjecture 3.2. every term of this sequence is either a prime number or equal to 1.

References

[1] Richard Guy, Unsolved Problems in Number Theory, Springer science (2004).

[2] N. J. A. Sloane et al., The On-line Encyclopedia of integers sequences, https://oeis.org

(Concerned with the sequence *A*051403)