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Super Edge-magic Total Labeling of Combination Graphs

Jingwen Li* Bimei Wang, Yanbo Gu, Shuhong Shao †

Abstract—**A (p,q) graph G has edge-magic total labeling if there exists a bijective function f: $V(G) \cup E(G) \to \{1, 2, ..., p+q\}$, such that $f(u)+f(v)+f(uv) = k$ is a constant for any edge uv of G. Moreover, G is said to be super edge-magic total labeling graph if $f(V(G)) = \{1, 2, ..., p\}$. In this paper, we introduce a new operation $\triangle$ called generalized coalescence, then we investigate super edge-magic total labeling of composite graph $F_m \triangle F_n \triangle C_i \triangle S_j$ which has four components. Finally, by giving some specific labels, we prove that for any i and j with $3 \leq i \leq 7$ and $j \geq 2$ both $F_3 \triangle F_2 \triangle C_i \triangle S_j$ and $F_3 \triangle F_3 \triangle C_i \triangle S_j$ are super edge-magic total labeling graphs.**

Keywords: super edge-magic total labeling; generalized coalescence; composite graph

1 Introduction

The graph labelling has attracted wide attention because its rich practical application background. Labeled graphs serve as useful models for a broad range of applications such as: circuit design[1], communication network addressing[2] and so on. In recent years, many scholars have done a lot of researches on super edge-magic total labeling, and also received lots of results on it.

H. Enomoto, A. S. Llado proved in 1998: every caterpillars has a super edge-magic total labeling(see also [3],[4],[5]). In [6], it has proved that the linear forest $F \cong P_3 \cup nP_2$ is super edge-magic total labeling for every integer n, $F \cong P_2 \cup P_n$ is super edge-magic total labeling for every integer $n \geq 3$; In [7] Figueroa-Centeno, Ichishima, Muntaner-Batle and Oshima investigated super edge-magic total labeling of graphs with two components. Among their results, we can see $C_3 \cup C_n$ is super edge-magic total if and only if $n \geq 6$ and n is even; $C_4 \cup C_n$ is super edge-magic total if and only if $n \geq 5$ and n is odd; $C_5 \cup C_n$ is super edge-magic total labeling if and only if $n \geq 4$ and n is even; if m is even with $m \geq 4$ and n is odd with $n \geq m/2+2$, then $C_m \cup C_n$ is super edge-magic total labeling; for $m = 6, 8$, or 10, $C_m \cup C_n$ is super edge-magic total labeling graph if and only if $n \geq 3$ and n is odd. J. Gallian sumed up the current research results in [8].

Inspired by above, we introduce a new operation generalized coalescence based on vertices merging. The number of edges and vertices of new graphs connected by this operation is invariable and decreased, respectively. In this paper, we investigate several families of graphs are super edge-magic total labeling graphs. Formulating the labels of vertices and edges is a fundamental and meaningful way to prove some theorems in graph theory. So by given a characteristic labels of vertices and edges, theorems about super edge-magic total labeling are proved.

All graphs considered in this paper are undirected simple connected graphs. We denote by $|V(G)|$ and $|E(G)|$ the number of vertices and edges of graph G. Let F_n, C_n and S_n denote the fan graph, circle graph, star graph, respectively. The star graph S_n is made up with $n-1$ leaf nodes and one centre node. We give some useful definitions which are required in the proof of the main results.

Definition 1[3] Let $G(p,q)$ be a finite simple connected graph, a bijection f from $V(G) \cup E(G)$ to $\{1, 2, ..., p+q\}$ is called an edge-magic total labeling of G if there exists a constant k (called the magic number of f) such that $f(u)+f(v)+f(uv) = k$ for any edge of G. An edge-magic total labeling f is called super edge-magic total labeling if $f(V(G)) = \{1, 2, ..., p\}$ and $f(E(G)) = \{p+1, p+2, ..., p+q\}$.

Magic number k always belongs to $[p+q+3, 2(p+q)]$.

Definition 2 Suppose that $V(F_n) = \{u_0, u_1, ..., u_n\}$, $V(F_m) = \{v_0, v_1, ..., v_m\}$, $V(C_i) = \{x_1, x_2, ..., x_i\}$, $V(S_j) = \{y_1, y_2, ..., y_j\}$, u_0, v_0, x_1 and y_1 are centre vertices. let u_0, v_0, x_1 and y_1 conduct generalized coalescence operation and forming a new vertex $u_0/v_0/x_1/y_1$, then called this composite graph $F_n \triangle F_m \triangle C_i \triangle S_j$.

It easy to see that $V(F_n \triangle F_m \triangle C_i \triangle S_j) = V(F_n) \cup V(F_m) \cup (C_i) \cup (S_j) \setminus 3v$, $|V(F_n \triangle F_m \triangle C_i \triangle S_j)| = |V(F_n)| + |V(F_m)| + |V(C_i)| + |V(S_j)| - 3$, $E(F_n \triangle F_m \triangle C_i \triangle S_j) = E(F_n) \cup E(F_m) \cup E(C_i) \cup E(S_j)$. Also we see that $F_n \triangle F_m \triangle C_i \triangle S_j$ is $F_n \triangle F_m$(see[12]) if C_i and S_j

*Corresponding author. E-mail addresses: lijingwen28@163.com (J. LI).

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are null graph and G , H are F_m and F_n, respectively.

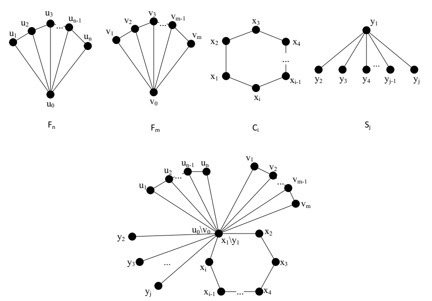

Figure 1: $F_n \triangle F_m \triangle C_i \triangle S_j$.

In section 2, we give main results about super edge-magic total labeling.

2 Main Results

Theorem 1 *Composite graphs $F_3 \triangle F_2 \triangle C_m \triangle S_n$ have super edge-magic total labeling if $3 \leq m \leq 7$, $n \geq 2$.*

Proof In order to complete the proof of graph G(p,q) has super edge-magic total labeling, we only need to prove $f(V(G)) = \{1, 2, ..., p\}$ and $f(E(G)) = \{p+1, p+2, ..., p+q\}$. We now prove the theorem by considering five cases according the value of index m.

Case 1 when $m = 3$, $n \geq 2$, graph $F_3 \triangle F_2 \triangle C_3 \triangle S_n$ is shown in figure 2(1). Super edge-magic total labeling of $F_3 \triangle F_2 \triangle C_3 \triangle S_2$ is shown in figure 2(2).

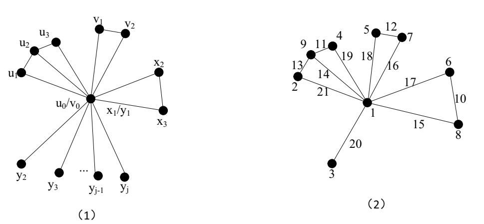

Figure 2: (1) $F_3 \triangle F_2 \triangle C_3 \triangle S_n$ (2) $F_3 \triangle F_2 \triangle C_3 \triangle S_2$.

If $n \geq 3$, magic number $k \in [2n+20, 4n+34]$, when $k = 2n+20$, we have

$$|V(F_3 \triangle F_2 \triangle C_3 \triangle S_n)| = 4+3+3+n-3 = n+7$$

$$|E(F_3 \triangle F_2 \triangle C_3 \triangle S_n)| = 5+3+3+(n-1) = n+10$$

$$|V(F_3 \triangle F_2 \triangle C_3 \triangle S_n)| + |E(F_3 \triangle F_2 \triangle C_3 \triangle S_n)| = 2n+17$$

Vertex labels of $F_3 \triangle F_2 \triangle C_3 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i = 0 \\ 2, & i = 1 \\ n+7, & i = 2 \\ 3, & i = 3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 0 \\ 5, & i = 1 \\ n+6, & i = 2 \end{cases}$$

$$f(x_i) = \begin{cases} 1, & i = 1 \\ 7, & i = 2 \\ n+5, & i = 3 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i = 1 \\ 4, & i = 2 \\ 6, & i = 3 \\ i+4, & 4 \leq i \leq n \end{cases}$$

Let S be a set of the sum of labels of two adjacent vertices. $S = \{3, 4, 5, 6, 7, 8\} \cup \{9, 10, \cdots, n+12\}$. Vertex labels are $f(V) = \{1, 2, 3, 4, 5, 6, 7\} \cup \{8, 9, \cdots, n+4, n+5, n+6, n+7\}$. By definition 1, $f(uv) = k - (f(u) + f(v))$. So the edge labels are $f(E) = \{2n+17, 2n+16, \cdots, 2n+12\} \cup \{2n+11, 2n+10, \cdots, n+8\}$. We can see $f(V) \to [1, n+7]$, $f(E) \to [n+8, 2n+17]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_2 \triangle C_3 \triangle S_n$ have super edge-magic total labeling if $n \geq 2$.

Case 2 when $m = 4$, $n \geq 2$, graph $F_3 \triangle F_2 \triangle C_4 \triangle S_n$ is shown in figure 3(1). Super edge-magic total labeling of $F_3 \triangle F_2 \triangle C_4 \triangle S_2$ and $F_3 \triangle F_2 \triangle C_4 \triangle S_3$ are shown in figure 3(2) and 3(3).

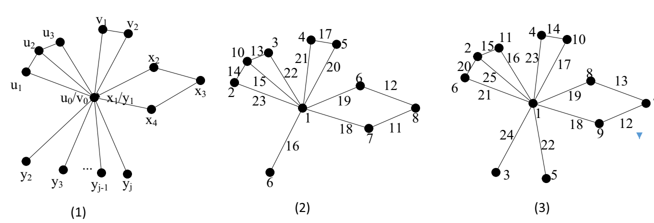

Figure 3: (1) $F_3 \triangle F_2 \triangle C_4 \triangle S_n$ (2) $F_3 \triangle F_2 \triangle C_4 \triangle S_2$, (3) $F_3 \triangle F_2 \triangle C_4 \triangle S_3$.

If $n \geq 4$, magic number $k \in [2n+22, 4n+38]$, when $k = 2n+22$, we have

$$|V(F_3 \triangle F_2 \triangle C_4 \triangle S_n)| = 4+3+4+n-3 = n+8$$

$$|E(F_3 \triangle F_2 \triangle C_4 \triangle S_n)| = 5+3+4+(n-1) = n+11$$

$$|V(F_3 \triangle F_2 \triangle C_4 \triangle S_n)| + |E(F_3 \triangle F_2 \triangle C_4 \triangle S_n)| = 2n+19$$

Vertex labels of $F_3 \triangle F_2 \triangle C_4 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i = 0 \\ 4, & i = 1 \\ 2, & i = 2 \\ n+8, & i = 3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 0 \\ 8, & i = 1 \\ n+5, & i = 2 \end{cases}$$

$$f(x_i)=\begin{cases}1, & i=1\\ n+6, & i=2\\ 5, & i=3\\ n+7, & i=4\end{cases}$$

$$f(y_i)=\begin{cases}1, & i=1\\ 3, & i=2\\ 6, & i=3\\ 7, & i=4\\ i+4, & 5\le i\le n\end{cases}$$

In this case, $S=\{3,4,5,6,7,8\}\cup\{9,10,\cdots,n+13\}$, $f(V)=\{1,2,3,4,5,6,7,8\}\cup\{9,10,\cdots,n+4,n+5,n+6,n+7,n+8\}$, $f(E)=\{2n+19,2n+18,\cdots,2n+14\}\cup\{2n+13,2n+12,\cdots,n+9\}$. We can see $f(V)\to[1,n+8]$, $f(E)\to[n+9,2n+19]$ and $f(V)\cap f(E)=\phi$, so composite graphs $F_3\triangle F_2\triangle C_4\triangle S_n$ have super edge-magic total labeling if $n\ge 2$.

Case 3 when $m=5$, $n\ge 2$, graph $F_3\triangle F_2\triangle C_5\triangle S_n$ is shown in figure 4.

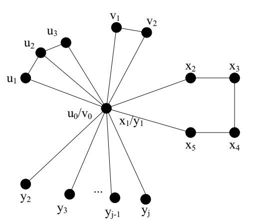

Figure 4: $F_3\triangle F_2\triangle C_5\triangle S_n$.

Magic number $k\in[2n+24,4n+42]$, when $k=2n+24$, we have

$$|V(F_3\triangle F_2\triangle C_5\triangle S_n)|=4+3+5+n-3=n+9$$

$$|E(F_3\triangle F_2\triangle C_5\triangle S_n)|=5+3+5+(n-1)=n+12$$

$$|V(F_3\triangle F_2\triangle C_5\triangle S_n)|+|E(F_3\triangle F_2\triangle C_5\triangle S_n)|=2n+21$$

Vertex labels of $F_3\triangle F_2\triangle C_5\triangle S_n$ are

$$f(u_i)=\begin{cases}1, & i=0\\ 3, & i=1\\ 2, & i=2\\ n+6, & i=3\end{cases}$$

$$f(v_i)=\begin{cases}1, & i=0\\ 5, & i=1\\ n+9, & i=2\end{cases}$$

$$f(x_i)=\begin{cases}1, & i=1\\ 6, & i=2\\ n+7, & i=3\\ 4, & i=4\\ n+8, & i=5\end{cases}$$

$$f(y_i)=\begin{cases}1, & i=1\\ n+5, & 2\le i\le n\end{cases}$$

In this case, $S=\{3,4,5,6,7\}\cup\{8,9,\cdots,n+13\}$, $f(V)=\{1,2,3,4,5,6\}\cup\{7,8,\cdots,n+5,n+6,n+7,n+8,n+9\}$, $f(E)=\{2n+21,2n+20,\cdots,2n+17\}\cup\{2n+16,2n+15,\cdots,n+10\}$. We can see $f(V)\to[1,n+9]$, $f(E)\to[n+10,2n+21]$ and $f(V)\cap f(E)=\phi$, so composite graphs $F_3\triangle F_2\triangle C_5\triangle S_n$ have super edge-magic total labeling if $n\ge 2$.

Case 4 when $m=6$, $n\ge 2$, graph $F_3\triangle F_2\triangle C_6\triangle S_n$ is shown in figure 5(1). Super edge-magic total labeling of $F_3\triangle F_2\triangle C_6\triangle S_n$ are shown from figure 5(2) to figure 5(6) when n is 2,3,4,5 or 6.

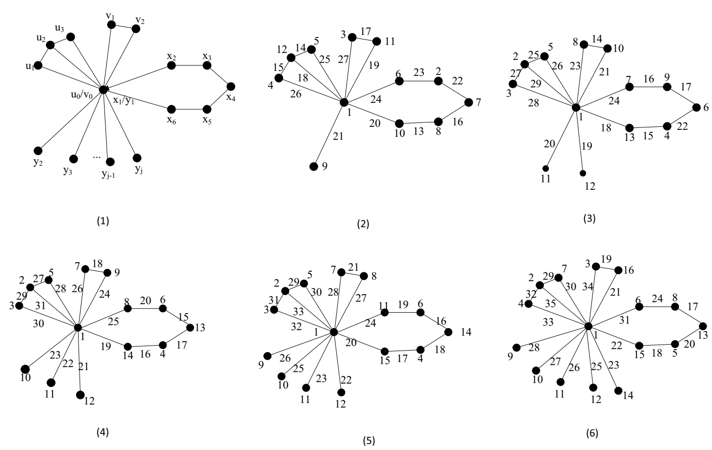

Figure 5: $F_3\triangle F_2\triangle C_6\triangle S_n$ and super edge-magic total labeling when n is 2,3,4,5 or 6.

If $n\ge 7$, magic number $k\in[2n+26,4n+46]$, when $k=2n+26$, we have

$$|V(F_3\triangle F_2\triangle C_6\triangle S_n)|=4+3+6+n-3=n+10$$

$$|E(F_3\triangle F_2\triangle C_6\triangle S_n)|=5+3+6+(n-1)=n+13$$

$$|V(F_3\triangle F_2\triangle C_6\triangle S_n)|+|E(F_3\triangle F_2\triangle C_6\triangle S_n)|=2n+23$$

Vertex labels of $F_3\triangle F_2\triangle C_6\triangle S_n$ are

$$f(u_i)=\begin{cases}1, & i=0\\ 4, & i=1\\ 2, & i=2\\ n+9, & i=3\end{cases}$$

$$f(v_i)=\begin{cases}1, & i=0\\ 6, & i=1\\ n+2, & i=2\end{cases}$$

$$f(x_i) = \begin{cases} 1, & i = 1 \\ 3, & i = 2 \\ n+10, & i = 3 \\ 5, & i = 4 \\ n+7, & i = 5 \\ 7, & i = 6 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i = 1 \\ n+8, & i = 2 \\ n+9-i, & 3 \leq i \leq 6 \\ n+8-i, & 7 \leq i \leq n \end{cases}$$

In this case, $S = \{3,4,5,6,7,8\} \cup \{9,10,\cdots,n+15\}$, $f(V) = \{1,2,3,4,5,6,7\} \cup \{8,9,\cdots,n+7,n+8,n+9,n+10\}$, $f(E) = \{2n+23,2n+22,\cdots,2n+18\} \cup \{2n+17,2n+16,\cdots,n+11\}$. We can see $f(V) \to [1,n+10]$, $f(E) \to [n+11,2n+23]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_2 \triangle C_6 \triangle S_n$ have super edge-magic total labeling if $n \geq 2$.

Case 5 when $m = 7$, $n \geq 2$, graph $F_3 \triangle F_2 \triangle C_7 \triangle S_n$ is shown in figure 6.

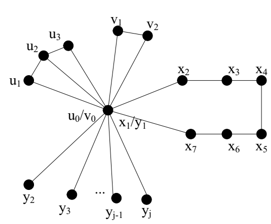

Figure 6: $F_3 \triangle F_2 \triangle C_7 \triangle S_n$.

Magic number $k \in [2n+28, 4n+50]$, when $k = 2n+29$, we have

$$|V(F_3 \triangle F_2 \triangle C_7 \triangle S_n)| = 4+3+7+n-3 = n+11$$

$$|E(F_3 \triangle F_2 \triangle C_7 \triangle S_n)| = 5+3+7+(n-1) = n+14$$

$$|V(F_3 \triangle F_2 \triangle C_7 \triangle S_n)| + |E(F_3 \triangle F_2 \triangle C_7 \triangle S_n)| = 2n+25$$

Vertex labels of $F_3 \triangle F_2 \triangle C_7 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i = 0 \\ 4, & i = 1 \\ 3, & i = 2 \\ n+7, & i = 3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 0 \\ 8, & i = 1 \\ n+8, & i = 2 \end{cases}$$

$$f(x_i) = \begin{cases} 1, & i = 1 \\ 5, & i = 2 \\ n+9, & i = 3 \\ 6, & i = 4 \\ n+11, & i = 5 \\ 2, & i = 6 \\ n+10, & i = 7 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i = 1 \\ 7, & i = 2 \\ i+6, & 3 \leq i \leq n \end{cases}$$

In this case, $S = \{4,5,6,7,8,9\} \cup \{10,11,\cdots,n+17\}$, $f(V) = \{1,2,3,4,5,6,7,8\} \cup \{9,10,\cdots,n+6,n+7,n+8,n+9,n+10,n+11\}$, $f(E) = \{2n+25,2n+24,\cdots,2n+20\} \cup \{2n+19,2n+18,\cdots,n+12\}$. We can see $f(V) \to [1,n+11]$, $f(E) \to [n+12,2n+25]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_2 \triangle C_7 \triangle S_n$ have super edge-magic total labeling if $n \geq 2$.

Thus, all cases imply that $3 \leq m \leq 7$, $n \geq 2$ and so the proof of the theorem 1 is complete.

Theorem 2 *Composite graphs $F_3 \triangle F_3 \triangle C_m \triangle S_n$ have super edge-magic total labeling if $3 \leq m \leq 7$, $n \geq 2$.*

Proof The proof way of theorem 2 is as same as the first one. We now prove the theorem by considering five cases according the value of index m.

case 1 when $m = 3$, $n \geq 2$, graph $F_3 \triangle F_3 \triangle C_3 \triangle S_n$ is shown in figure 7(1). Super edge-magic total labeling of $F_3 \triangle F_3 \triangle C_3 \triangle S_2$ is shown in figure 7(2).

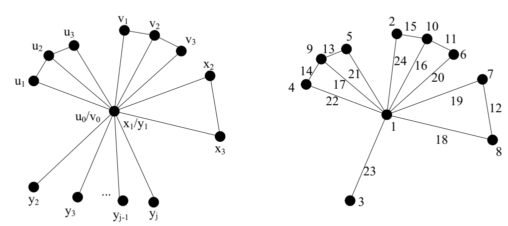

Figure 7: (1) $F_3 \triangle F_3 \triangle C_3 \triangle S_n$ (2) $F_3 \triangle F_3 \triangle C_3 \triangle S_2$.

If $n \geq 3$, magic number $k \in [2n+23, 4n+40]$, when $k = 2n+23$, we have

$$|V(F_3 \triangle F_3 \triangle C_3 \triangle S_n)| = 4+4+3+n-3 = n+8$$

$$|E(F_3 \triangle F_3 \triangle C_3 \triangle S_n)| = 5+5+3+(n-1) = n+12$$

$$|V(F_3 \triangle F_3 \triangle C_3 \triangle S_n)| + |E(F_3 \triangle F_3 \triangle C_3 \triangle S_n)| = 2n+20$$

Vertex labels of $F_3 \triangle F_3 \triangle C_3 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i = 0 \\ 5, & i = 1 \\ n+7, & i = 2 \\ 6, & i = 3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 0 \\ 2, & i = 1 \\ n+8, & i = 2 \\ 3, & i = 3 \end{cases}$$

$$f(x_i) = \begin{cases} 1, & i = 1 \\ 8, & i = 2 \\ n+6, & i = 3 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i = 1 \\ 4, & i = 2 \\ 7, & i = 3 \\ i+5, & 4 \le i \le n \end{cases}$$

In this case, $S = \{3,4,5,6,7,8,9\} \cup \{10, 11 \cdots, n+14\}$, $f(V) = \{1,2,3,4,5,6,7,8\} \cup \{9, 10, \cdots, n+5, n+6, n+7, n+8\}$, $f(E) = \{2n+20, 2n+19, \cdots, 2n+14\} \cup \{2n+13, 2n+12, \cdots, n+9\}$. We can see $f(V) \to [1, n+8] f(E) \to [n+9, 2n+20]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_3 \triangle C_3 \triangle S_n$ have super edge-magic total labeling if $n \ge 2$.

case 2 when $m = 4$, $n \ge 2$graph $F_3 \triangle F_3 \triangle C_4 \triangle S_n$ is shown in figure 8(1). Super edge-magic total labeling of $F_3 \triangle F_3 \triangle C_4 \triangle S_2$ is shown in figure 8(2).

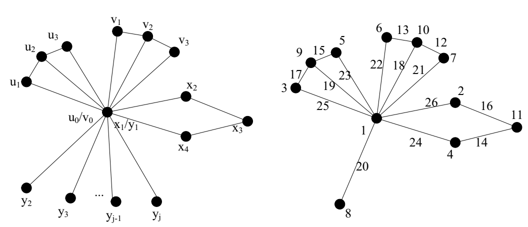

Figure 8: (1) $F_3 \triangle F_3 \triangle C_4 \triangle S_n$ (2) $F_3 \triangle F_3 \triangle C_4 \triangle S_2$.

If $n \ge 3$magic number $k \in [2n+25, 4n+44]$, when $k = 2n+25$, we have

$$|V(F_3 \triangle F_3 \triangle C_4 \triangle S_n)| = 4+4+4+n-3 = n+9$$

$$|E(F_3 \triangle F_3 \triangle C_4 \triangle S_n)| = 5+5+4+(n-1) = n+13$$

$$|V(F_3 \triangle F_3 \triangle C_4 \triangle S_n)| + |E(F_3 \triangle F_3 \triangle C_4 \triangle S_n)| = 2n+22$$

Vertex labels of $F_3 \triangle F_3 \triangle C_4 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i = 0 \\ 7, & i = 1 \\ 2, & i = 2 \\ n+9, & i = 3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 0 \\ 4, & i = 1 \\ n+8, & i = 2 \\ 5, & i = 3 \end{cases}$$

$$f(x_i) = \begin{cases} 1, & i = 1 \\ n+6, & i = 2 \\ 8, & i = 3 \\ n+7, & i = 4 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i = 1 \\ 3, & i = 2 \\ 6, & i = 3 \\ i+5, & 4 \le i \le n \end{cases}$$

In this case, $S = \{3,4,5,6,7,8,9\} \cup \{10, 11 \cdots, n+15\}$, $f(V) = \{1,2,3,4,5,6,7,8,9\} \cup \{10, 11, \cdots, n+5, n+6, n+7, n+8, n+9\}$, $f(E) = \{2n+22, 2n+21, \cdots, 2n+16\} \cup \{2n+15, 2n+14, \cdots, n+10\}$. We can see $f(V) \to [1, n+9]$, $f(E) \to [n+10, 2n+22]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_3 \triangle C_4 \triangle S_n$ have super edge-magic total labeling if $n \ge 2$.

case 3 when $m = 5n \ge 2$, graph $F_3 \triangle F_3 \triangle C_5 \triangle S_n$ is shown in figure 9(1). Super edge-magic total labeling of $F_3 \triangle F_3 \triangle C_5 \triangle S_2$ and $F_3 \triangle F_3 \triangle C_5 \triangle S_3$ are shown in figure 9(2) and 9(3).

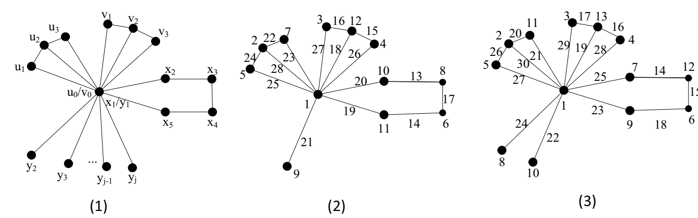

Figure 9: (1) $F_3 \triangle F_3 \triangle C_5 \triangle S_n$ (2) $F_3 \triangle F_3 \triangle C_5 \triangle S_2$ (3) $F_3 \triangle F_3 \triangle C_5 \triangle S_3$.

If $n \ge 4$, magic number $k \in [2n+27, 4n+48]$, when $k = 2n+27$, we have

$$|V(F_3 \triangle F_3 \triangle C_5 \triangle S_n)| = 4+4+5+n-3 = n+10$$

$$|E(F_3 \triangle F_3 \triangle C_5 \triangle S_n)| = 5+5+5+(n-1) = n+14$$

$$|V(F_3 \triangle F_3 \triangle C_5 \triangle S_n)| + |E(F_3 \triangle F_3 \triangle C_5 \triangle S_n)| = 2n+24$$

Vertex labels of $F_3 \triangle F_3 \triangle C_5 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i = 0 \\ 6, & i = 1 \\ 2, & i = 2 \\ n+6, & i = 3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 0 \\ n+9, & i = 1 \\ 3, & i = 2 \\ n+10, & i = 3 \end{cases}$$

$$f(x_i) = \begin{cases} 1, & i = 1 \\ 9, & i = 2 \\ n+7, & i = 3 \\ 7, & i = 4 \\ n+8, & i = 5 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i=1 \\ 4, & i=2 \\ 5, & i=3 \\ 8, & i=4 \\ i+5, & 5 \le i \le n \end{cases}$$

In this case, $S = \{3,4,5,6,7,8,9,10\} \cup \{11,12\cdots,n+16\}$, $f(V) = \{1,2,3,4,5,6,7,8,9\} \cup \{10,11,\cdots,n+5,n+6,n+7,n+8,n+9,n+10\}$, $f(E) = \{2n+24,2n+23,\cdots,2n+17\} \cup \{2n+16,2n+15,\cdots,n+11\}$. We can see $f(V) \to [1,n+10]$, $f(E) \to [n+11,2n+24]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_3 \triangle C_5 \triangle S_n$ have super edge-magic total labeling if $n \ge 2$.

case 4 when $m = 6$, $n \ge 2$, graph $F_3 \triangle F_3 \triangle C_6 \triangle S_n$ is shown in figure 10.

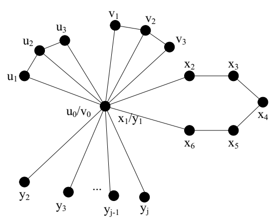

Figure 10: $F_3 \triangle F_3 \triangle C_6 \triangle S_n$.

Magic number $k \in [2n+29, 4n+52]$, when $k = 2n+29$, we have

$$|V(F_3 \triangle F_3 \triangle C_6 \triangle S_n)| = 4+4+6+n-3 = n+11$$

$$|E(F_3 \triangle F_3 \triangle C_6 \triangle S_n)| = 5+5+6+(n-1) = n+15$$

$$|V(F_3 \triangle F_3 \triangle C_6 \triangle S_n)| + |E(F_3 \triangle F_3 \triangle C_6 \triangle S_n)| = 2n+26$$

Vertex labels of $F_3 \triangle F_3 \triangle C_6 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i=0 \\ 4, & i=1 \\ 2, & i=2 \\ n+7, & i=3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i=0 \\ 6, & i=1 \\ 3, & i=2 \\ n+11, & i=3 \end{cases}$$

$$f(x_i) = \begin{cases} 1, & i=1 \\ n+9, & i=2 \\ 8, & i=3 \\ n+8, & i=4 \\ 5, & i=5 \\ n+10, & i=6 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i=1 \\ 7, & i=2 \\ i+6, & 3 \le i \le n \end{cases}$$

In this case, $S = \{3,4,5,6,7,8,9\} \cup \{10,11\cdots,n+17\}$, $f(V) = \{1,2,3,4,5,6,7,8\} \cup \{9,10,\cdots,n+6,n+7,n+8,n+9,n+10,n+11\}$, $f(E) = \{2n+26,2n+25,\cdots,2n+20\} \cup \{2n+19,2n+18,\cdots,n+12\}$. We can see $f(V) \to [1,n+11]$, $f(E) \to [n+12,2n+26]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_3 \triangle C_6 \triangle S_n$ have super edge-magic total labeling if $n \ge 2$.

case 5 when $m = 7$, $n \ge 2$, graph $F_3 \triangle F_3 \triangle C_7 \triangle S_n$ is shown in figure 11(1). Super edge-magic total labeling of $F_3 \triangle F_3 \triangle C_7 \triangle S_n$ are shown from figure 11(2) to 11(7) when n is 2,3,4,5,6 or 7.

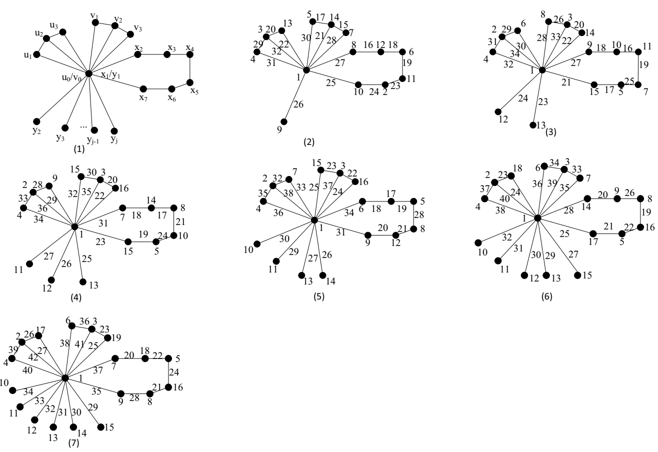

Figure 11: $F_3 \triangle F_3 \triangle C_7 \triangle S_n$ and super edge-magic total labeling when n is 2,3,4,5,6 or 7.

If $n \ge 8$, magic number $k \in [2n+31, 4n+56]$, when $k = 2n+31$, we have

$$|V(F_3 \triangle F_3 \triangle C_7 \triangle S_n)| = 4+4+7+n-3 = n+12$$

$$|E(F_3 \triangle F_3 \triangle C_7 \triangle S_n)| = 5+5+7+(n-1) = n+16$$

$$|V(F_3 \triangle F_3 \triangle C_7 \triangle S_n)| + |E(F_3 \triangle F_3 \triangle C_7 \triangle S_n)| = 2n+28$$

Vertex labels of $F_3 \triangle F_3 \triangle C_7 \triangle S_n$ are

$$f(u_i) = \begin{cases} 1, & i=0 \\ 4, & i=1 \\ 2, & i=2 \\ n+10, & i=3 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i=0 \\ 6, & i=1 \\ 3, & i=2 \\ n+12, & i=3 \end{cases}$$

$$f(x_i) = \begin{cases} 1, & i=1 \\ 7, & i=2 \\ n+11, & i=3 \\ 5, & i=4 \\ n+9, & i=5 \\ 8, & i=6 \\ n+2, & i=7 \end{cases}$$

$$f(y_i) = \begin{cases} 1, & i = 1 \\ n + 10 - i, & 2 \le i \le 7 \\ n + 9 - i, & 8 \le i \le n \end{cases}$$

In this case, $S = \{3, 4, 5, 6, 7, 8, 9\} \cup \{10, 11 \cdots, n + 18\}$, $f(V) = \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{9, 10, \cdots, n+9, n+10, n+11, n + 12\}$, $f(E) = \{2n + 28, 2n + 27, \cdots, 2n + 22\} \cup \{2n+21, 2n+20, \cdots, n+13\}$. We can see $f(V) \to [1, n+12]$, $f(E) \to [n + 13, 2n + 28]$ and $f(V) \cap f(E) = \phi$, so composite graphs $F_3 \triangle F_3 \triangle C_7 \triangle S_n$ have super edge-magic total labeling if $n \ge 2$.

Thus, all cases imply that $3 \le m \le 7$, $n \ge 2$ and so the proof of the theorem 2 is complete.

3 Conclusions and Future Work

We define the new operation $\triangle$ and $F_n \triangle F_m \triangle C_i \triangle S_j$. In the section two,we prove that $F_3 \triangle F_2 \triangle C_m \triangle S_n$ and $F_3 \triangle F_3 \triangle C_m \triangle S_n$ are super edge-magic if $3 \le m \le 7$, $n \ge 2$. Maybe when $m \ge 8$, $n \ge 2$ have the same results.

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