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Improving Disturbance-Rejection Performance Based on Combination of Equivalent-Input-Disturbance Approach and Proportional-Integral Observer

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***Abstract*—This paper presents a method that combines an improved equivalent-input-disturbance (IEID) approach and a proportional-integral observer (PIO). A PI loop in the PIO is used to reduce the estimation error of an EID. The conventional EID estimator is improved by inserting a low-pass filter to further reduce the estimation error of the EID. The gains of the PIO and the IEID estimator are designed using the linear matrix inequality (LMI) technology. Two matrices are used In the design to find suitable parameters of the filters. The effectiveness of the method is shown by numerical simulations.**

***Index Terms*—disturbance rejection, equivalent input disturbance (EID), linear matrix inequality (LMI), proportional-integral observer (PIO)**

I. Introduction

Actively compensate for disturbances plays an important role in disturbance rejection. Many active methods have been proposed to improve disturbance rejection performance [1]–[3], such as, the disturbance observer (DO) [4], the active-disturbance-rejection control (ADRC) [5], and the equivalent-input-disturbance (EID) approach [7].

DO is mainly in the form of frequency domain for disturbance suppression [4]. It is widely used to observe the state of disturbances because of its simplicity. It only needs to design a filter to ensure the stability and performance of the system. However, for a non-minimum-phase plant, it is hard to guarantee the stability.

ADRC is a control method which consists of the tracking differentiator, set jump point, the nonlinear feedback, and extend state observer (ESO) [6]. Among them, ESO is aimed at disturbance rejection. The ESO linearizes the plant into an integral chained system to simplify the design. However, the linearization process may increase the error between the observer and the plant, and make the whole system unstable.

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Recently, the EID approach was presented for disturbance rejection. It create a signal on the control input channel which affects the output the same as disturbances does. The EID approach does not require an inverse model of a plant. Thus, it avoid the unstable poles and zeros. Moreover, the main goal of EID approach is to estimate the disturbance, not to construct the integral chained system. Therefore, the error of disturbance estimation will be smaller.

A Luenberger observer uses the proportional loop of the system output error and produces an estimate of the EID. However, even if the high gain method is adopted, there must be errors in the proportional loop. The simplest way to deal with errors is to introduce an integral loop. Therefore, the PI observer is more accurate than the Luenberger observer to reflect the state of the plant.

An EID-based control system by incorporating a PI observer has been presented to improve disturbance-rejection performance [8]. The estimation of state is more accuracy and the design of the control gains are more flexible.

To further improve the performance of EID-PIO method, an estimation of error of EID-PIO method is compensated to the system to reduce the effect of the error on the output. A stability condition of closed-loop system is derived by using linear matrix inequality (LMI) technology. The gains of controller are designed based on LMIs. The effectiveness and superiority are demonstrated by numerical simulation. The rest of paper is organized as follows: a configuration of improved EID-PIO-based system is presented in Section II; The design of controller gains and a stability condition are presented in Section III. Comparisons in numerical simulations show the validity of improved method in IV. Some conclusions are given in V. In this paper, $\mathbb{R}^n$ is an n dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; I and 0 stand for an identity matrix and a zero matrix with a suitable dimension, respectively; and a symmetric matrix $\begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix}$ is

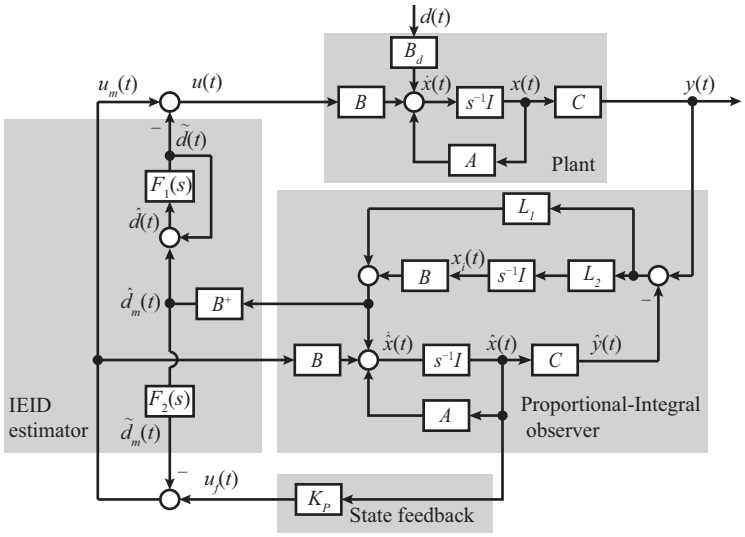


Fig. 1. Configuration of IEID-based control system.

denoted by $\begin{bmatrix} A & B \\ \star & C \end{bmatrix}$.

II. Structure elucidation

The configuration of an IEID-based system is shown in 1. It has four parts: a plant with disturbance; a proportional-integral observer, an IEID estimator, and a set of state-feedback gains.

For the sake of simplicity, the following linear single-input-single-output (SISO) system with unknown disturbances is considered:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_d d(t) \\ y = Cx(t) \\ y_o(t) = C_o x(t) \end{cases} \tag{1}$$

where $x(t)$ $(\in \mathbb{R}^n)$ is the state vector, $y(t)$ $(\in \mathbb{R})$ is the measurable output, $y_o(t)$ $(\in \mathbb{R})$ is the controlled output, $d(t)$ $(\in \mathbb{R}^{n_d})$ is the external disturbance, $u(t)$ $(\in \mathbb{R})$ is the control input. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $B_d \in \mathbb{R}^{n\times n_d}$, and $C \in \mathbb{R}^{1\times n}$, respectively. The following assumption are made about the follows:

Assumption 1. *(A, B) is controllable, and (A, C) is observable. Moreover,*

$$\operatorname{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + 1 \tag{2}$$

Assumption 2. *$P(s)$ has no zeros on the imaginary axis.*

Assumption 3. *B has a full column rank.*

Assumptions 1 and 2 are made for convenience and are standard. To remove some redundancy in the input, B always a full column rank.

Note that, B and B_d may have different dimension. $d(t)$ may contain matched disturbance and mismatched disturbance. So, using the concept of EID, a control input signal is produced on the control input and the plant is described as [7],

$$\begin{cases} \dot{x}(t) = Ax(t) + B[u(t) + d_e(t)] \\ y(t) = Cx(t) \\ y_o(t) = C_o x(t) \end{cases} \tag{3}$$

where $d_e(t)$ is the EID.

A PIO of the plant is

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu_m(t) + L_1[y(t) - \hat{y}(t)] + Bx_i(t) \\ \hat{y}(t) = C\hat{x}(t) \\ \dot{x}_i(t) = L_2[y(t) - \hat{y}(t)] \end{cases} \tag{4}$$

where $\hat{x}(t)$ is the estimate of the state $x(t)$, $\hat{y}(t)$ is the estimate of the output $y(t)$, $u_m(t)$ is the input of the PIO, $L_1 \in \mathbb{R}^n$ and $L_2 \in \mathbb{R}$ are the gains of PIO, respectively.

Defining the state-estimation error as

$$\Delta x(t) = x(t) - \hat{x}(t) \tag{5}$$

According to [8], we obtain an estimate of the EID

$$\hat{d} = B^+ L_1 C \Delta x(t) + u_m(t) - u(t) + x_i(t) \tag{6}$$

where B^+ is a pseudo inverse of B.

A low-pass filter, $F_1(s)$, which satisfies

$$|F_1(j\omega) \approx 1|, \ \omega \in [0, \omega_c) \tag{7}$$

where ω_c is the highest angular frequency required for the EID estimate. When the highest frequency of disturbance is ω_d, it is suitable to select ω_c to be

$$\omega_c \approx (5 - 10)\omega_d. \tag{8}$$

The state-space of $F_1(s)$ is

$$\begin{cases} \dot{x}_{f1}(t) = A_{f1} x_{f1}(t) + B_{f1} \hat{d}(t) \\ \tilde{d}(t) = C_{f1} x_{f1}(t) \end{cases} \tag{9}$$

where $\widetilde{d}(t)$ is the filtered disturbance. The control law after compensation is

$$u(t) = u_m(t) - \widetilde{d}(t). \tag{10}$$

On the other hand, a combination of (3), (4), and (5) gives

$$\begin{bmatrix} \Delta\dot{x}(t) \\ \dot{x}_i(t) \end{bmatrix} = \begin{bmatrix} A - L_1C & -B \\ L_2C & 0 \end{bmatrix} \begin{bmatrix} \Delta x(t) \\ x_i(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} [d_e(t) - \widetilde{d}(t)] \tag{11}$$

which is the error between the plant and the observer. Note that, when the disturbance in the system changes continuously and persists, $d_e(t) - \widetilde{d}(t)$ always not equal to zero. Thus, $\Delta x(t)$ and $x_i(t)$ also not equal to zero. Define an error of estimation to be

$$\Delta d(t) = d_e(t) - \widetilde{d}(t). \tag{12}$$

(3), (10), and (12) yield

$$\begin{cases} \dot{x}(t) = Ax(t) + B[u_m(t) + \Delta d(t)] \\ y(t) = Cx(t). \end{cases} \tag{13}$$

To compensate the error of estimation, $\Delta d(t)$, $\hat{d}_m(t)$ is defined to estimate $\Delta d(t)$. In Fig. 2, the state-space representation of $\hat{d}_m(t)$ is given by

$$\begin{cases} \begin{bmatrix} \Delta\dot{x}(t) \\ \dot{x}_i(t) \end{bmatrix} = \begin{bmatrix} A - L_1C & -B \\ L_2C & \end{bmatrix} \begin{bmatrix} \Delta x(t) \\ x_i(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta d(t) \\ \hat{d}_m = \begin{bmatrix} B^+L_1C & 1 \end{bmatrix} \begin{bmatrix} \Delta x(t) \\ x_i(t) \end{bmatrix} \end{cases} \tag{14}$$

The transfer function from $\Delta d(t)$ to $\hat{d}_m(t)$ is

$$\hat{G}_m(s) = \begin{bmatrix} B^+L_1C & 1 \end{bmatrix} \left(sI - \begin{bmatrix} A - L_1C & -B \\ L_2C & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{15}$$

which is approximately one in the low frequency range. A low-pass filter, $F_2(s)$, is needed to limited the compensated bandwidth which satisfies

$$|F_2(j\omega) \approx 1|, \ \omega \in [0, \omega_m) \tag{16}$$

where ω_m is the highest angular frequency required for the error of EID estimate. In general,

$$\omega_c \approx \omega_m. \tag{17}$$

The state-space of $F_2(s)$ is

$$\begin{cases} \dot{x}_{f2}(t) = A_{f2}x_{f2}(t) + B_{f2}\hat{d}_m(t) \\ \widetilde{d}_m(t) = C_{f2}x_{f2}(t) \end{cases} \tag{18}$$

where $\widetilde{d}_m(t)$ is the filtered error for EID estimate. A new control law is given as The control law after compensation is

$$u_f(t) = u_m(t) + \widetilde{d}_m(t) \tag{19}$$

where

$$u_f(t) = K_P\hat{x}(t). \tag{20}$$

III. Design of Improved Equivalent-Input-Disturbance Estimator and Proportional-Integral-Observer

The design of the controller is based on the stability of the system. Therefore, a stability condition is given first, and then the controller parameters are designed according to the stability condition.

Since the control system in Fig. 1 is linear, the stability of the system is independent of the external signal. Letting $d(t) = 0$ ($d_e(t) = 0$), a combination of (3), (4), (5), (6), (9), (10), (18), (19), and (20) gives the closed loop system in Fig. 1 as

$$\dot{\varphi}(t) = \bar{A}\varphi(t) \tag{21}$$

where

$$\varphi(t) = \begin{bmatrix} \hat{x}^{\mathrm{T}}(t) & \Delta x^{\mathrm{T}}(t) & x_i^{\mathrm{T}}(t) & x_{f1}^{\mathrm{T}}(t) & x_{f2}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$$

$$\bar{A} = \begin{bmatrix} A + BK_P & L_1C & B & 0 & -BC_{f2} \\ 0 & A - L_1C & -B & -BC_f & 0 \\ 0 & L_2C & 0 & 0 & 0 \\ 0 & B_{f1}B^+L_1C & B_{f1} & A_{f1} + B_{f1}C_{f1} & 0 \\ 0 & B_{f2}B^+L_1C & B_{f2} & 0 & A_{f2} + B_{f2}C_{f2} \end{bmatrix} \tag{22}$$

A matrix M can be decomposed to be

$$M = \bar{U} \begin{bmatrix} \bar{\Sigma} & 0 \end{bmatrix} \bar{V}^{\mathrm{T}} \tag{23}$$

by singular-value, where $\bar{\Sigma}$ is a diagonal matrix with positive diagonal elements in decreasing order, and $\bar{U}$ and $\bar{V}$ are unitary matrices.

The following lemma [9] is used to derive a stability condition for the design of improved EID-PIO-based system.

Lemma 1. *For a given matrix $M \in \mathbb{R}^{p\times n}$ with $\mathrm{rank}(M) = p$, there exists a matrix $\bar{X} \in \mathbb{R}^{p\times p}$ such that*

$$MX = \bar{X}M \tag{24}$$

holds for any $X \in \mathbb{R}^{n\times n}$ if and only if X can be decomposed as

$$X = \bar{V} \begin{bmatrix} \bar{X}^{(11)} & 0 \\ 0 & \bar{X}^{(22)} \end{bmatrix} \bar{V}^{\mathrm{T}} \tag{25}$$

where $\bar{V} \in \mathbb{R}^{n\times n}$ is a unitary matrix, $\bar{X}_{11} \in \mathbb{R}^{p\times p}$, and $\bar{X}_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$.

The stability of the closed-loop system is given by the following theorem.

Theorem 1. *For suitably selected filters $F_1(s)$ and $F_1(s)$; positive numbers γ_1, γ_2; positive-definite matrices X_1, X_{11},*

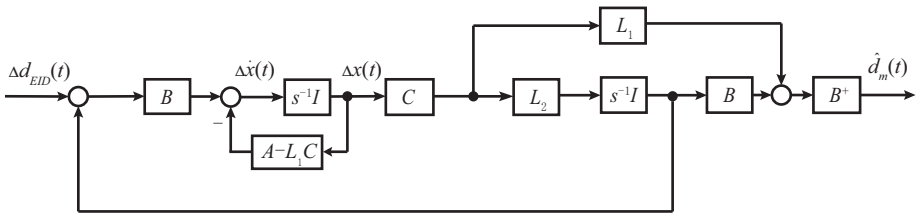


Fig. 2. Block diagram of $\hat{G}_m(s)$.

X_{22}, and X_3; and appropriate matrices M_1, M_2, W_1, W_2, and W_3; the system is stable if the following inequalities hold:

$$\begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} & \psi_{15} \\ \star & \psi_{22} & \psi_{23} & \psi_{24} & \psi_{25} \\ \star & \star & \psi_{33} & \psi_{34} & \psi_{35} \\ \star & \star & \star & \psi_{44} & \psi_{45} \\ \star & \star & \star & \star & \psi_{55} \end{bmatrix} < 0$$
$$X = \begin{bmatrix} X_1 & 0 & 0 & 0 & 0 \\ \star & X_2 & X_2W_1 & X_2W_2 & 0 \\ \star & \star & X_3 & 0 & 0 \\ \star & \star & \star & X_4 & 0 \\ \star & \star & \star & \star & X_5 \end{bmatrix} > 0 \tag{26}$$

where

$$\begin{aligned}
\psi_{11} &= X_1(A+\gamma_1 I)^{\mathrm{T}} + (A+\gamma_1 I)X_1 \\
&\quad + BW_1 + (BW_1)^{\mathrm{T}} \\
\psi_{12} &= BM_1^{\mathrm{T}}X_2 + W_2C \\
\psi_{13} &= BX_3 + W_2CM_1 \\
\psi_{14} &= W_2CM_2 \\
\psi_{15} &= -BC_{f2}X_5 \\
\psi_{22} &= X_2(A+\gamma_2 I)^{\mathrm{T}} + (A+\gamma_2 I)X_2 \\
&\quad - W_2C - (W_2C)^{\mathrm{T}} - BM_1^{\mathrm{T}}X_2 - X_2M_1B^{\mathrm{T}} \\
\psi_{23} &= (A+\gamma_2)X_2M_1 + C^{\mathrm{T}}W_3^{\mathrm{T}} - BX_3 - W_2CM_1 \\
\psi_{24} &= (A+\gamma_2)X_2M_2 + X_2M_1B_{f1}^{\mathrm{T}} \\
&\quad + X_2M_1(A_{f1}+B_{f1}C_{f1}) + (B_{f1}B^+W_2C)^{\mathrm{T}} \\
&\quad - W_2CM_2 - BC_{f1}X_4 \\
\psi_{25} &= X_2M_1B_{f2}^{\mathrm{T}} + (B_{f2}B^+W_2C)^{\mathrm{T}} \\
\psi_{33} &= W_3CM_1 + (W_3CM_1)^{\mathrm{T}} \\
\psi_{34} &= W_3CM_2 + X_3B_{f1}^{\mathrm{T}} + (B_{f1}B^+W_2CM_1)^{\mathrm{T}} \\
\psi_{35} &= X_3B_{f2}^{\mathrm{T}} + (B_{f2}B^+W_2CM_1)^{\mathrm{T}} \\
\psi_{44} &= (A_{f1}+B_{f1}C_{f1})X_4 + X_4(A_{f1}+B_{f1}C_{f1})^{\mathrm{T}} \\
&\quad + B_{f1}B^+W_2CM_2 + (B_{f1}B^+W_2CM_2)^{\mathrm{T}} \\
\psi_{45} &= (B_{f2}B^+W_2CM_2)^{\mathrm{T}} \\
\psi_{55} &= A_{f2}X_5 + X_5A_{f2}^{\mathrm{T}} \\
X_2 &= V\begin{bmatrix} X_{2(11)} & 0 \\ 0 & X_{2(22)} \end{bmatrix}V^{\mathrm{T}}.
\end{aligned}$$

The singular-value decomposition of the output matrix C is

$$C = U\begin{bmatrix}\Sigma & 0\end{bmatrix}V^{\mathrm{T}} \tag{27}$$

where Σ is a positive definite matrix, and U and V are unitary matrices. Then, the gains of the observer state feedback and the PIO are

$$\begin{cases} K_P = W_1X_1^{-1} \\ L_1 = W_2U\Sigma X_{2(11)}^{-1}\Sigma^{-1}U^{\mathrm{T}}. \\ L_2 = W_3U\Sigma X_{2(11)}^{-1}\Sigma^{-1}U^{\mathrm{T}}. \end{cases} \tag{28}$$

The Theorem 1 is simple and is similar to that of Theorem 1 given in [8]. Thus, we omit the proof here.

The design algorithm is given below for the IEID-PIO-based system.

Design Algorithm:

- **Step 1)** Choose the maximum angular frequency of disturbance, ω_d, the cutoff angular frequency of the filters $F_1(s)$ and $F_2(s)$ are ω_c and ω_m, respectively.
- **Step 2)** Choose filters $F_1(s)$ and $F_2(s)$ satisfying (7), (16), respectively.
- **Step 3)** Choose positive numbers γ_1, γ_2, and suitable matrices M_1 and M_2. Find a feasible solution of the LMI (26) and calculate K_P, L_1, and L_2 from (28).

Remark 1. *There are no restrictions on the selection of filters, unlike in [8].*

IV. Numerical Simulation

In this section, the effectiveness of the IEID-PIO approach is verified through numerical simulations.

The parameters in plant 1 are

$$\begin{cases} A = \begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}} \\ B_d = \begin{bmatrix} 2 & 1 \end{bmatrix}^{\mathrm{T}}, C = \begin{bmatrix} 2 & 1 \end{bmatrix} \end{cases} \tag{29}$$

Disturbance

$$d(t) = \begin{cases} \sin\pi t + 0.25\sin 3\pi t + 0.2\tanh(t), & 5 < t \le 15 \\ 0, & t \le 5 \text{ or } t > 15 \end{cases} \tag{30}$$

was considered to add into the system.

Since the maximum frequency of disturbance,ω_d, is 3π, ω_c and ω_m are selected as 100 rad/s. The parameters of low-pass filter $F_1(s)$ and $F_2(s)$ are

$$A_{f1} = -100,\ B_f = 100,\ C_f = 1 \tag{31}$$

and

$$A_{f2} = -100,\ B_f = 100,\ C_f = 1 \tag{32}$$

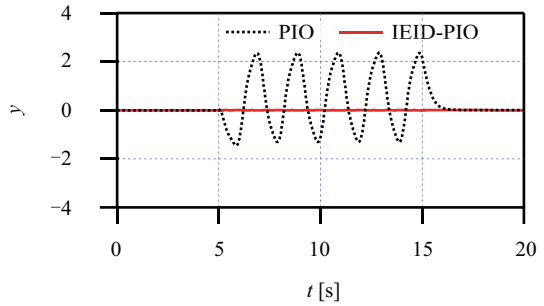


Fig. 3. Comparison with PIO-based disturbance-rejection method.

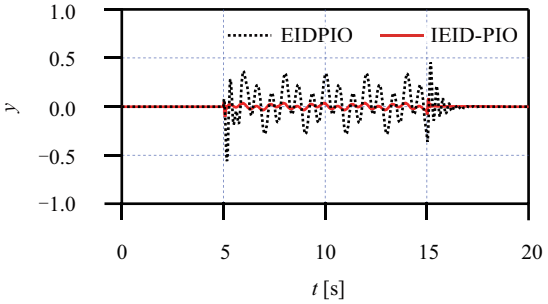


Fig. 4. Comparison with EID-PIO method.

respectively. The positive numbers γ_1 and γ_2 are

$$\gamma_1 = \gamma_2 = 1. \tag{33}$$

And matrices M_1 and M_2 are chosen as

$$M_1 = M_2 = \begin{bmatrix} -10 \\ -10 \end{bmatrix}. \tag{34}$$

Through a feasible solution of the LMI, gains of IEID-PIO-based controller are obtained, which are

$$K_P = \begin{bmatrix} -3.40 & 2.22 \end{bmatrix},\ L_1 = \begin{bmatrix} 9.43 \\ 9.80 \end{bmatrix},\ L_2 = 64.06. \tag{35}$$

For comparison, a PIO-based disturbance-rejection method and a EID-PIO method are designed. The gains of the PIO-based disturbance-rejection method and EID-PIO method were calculated from a Lyapunov function by LMI, which were given by [8]

$$K_P = \begin{bmatrix} -3.92 & 2.63 \end{bmatrix},\ L_1 = \begin{bmatrix} 2.50 \\ 2.11 \end{bmatrix},\ L_2 = 5.87 \tag{36}$$

$$K_P = \begin{bmatrix} -3.43 & 2.19 \end{bmatrix},\ L_1 = \begin{bmatrix} 4.41 \\ -1.47 \end{bmatrix},\ L_2 = 5.39 \tag{37}$$

respectively.

The simulation results are carried out for the comparisons between the PIO-based disturbance-rejection method and IEID-PIO method in Fig. 3 and between the PIO-EID method and IEID-PIO method in Fig. 4. These show that IEID-PIO is superior to other method.

V. Conclusion

This paper present a improved PIO-based EID approach to improve disturbance-rejection performance. A special structure is presented to combine with the PIO to compensate the error of EID estimation. LMIs are used to derived a stability condition for the closed-loop system. Moreover, the gains of the controller are designed based on the stability condition. Simulation shows that the improved EID-PIO method is superior to the other disturbance-rejection method.

VI. Paper Submission

References

[1] W. Chen, J. Yang, L. Guo, and S. Li. Disturbance-observer-based control and related methods—an overview. *IEEE Trans. Ind. Electron.*, vol. 63, no. 2, pp.1083–1095, Feb. 2016.

[2] J. Yang, W. Chen, S. Li, G. Lei, and Y. Yan. Disturbance/uncertainty estimation and attenuation techniques in PMSM drives - a survey. *IEEE Trans. Ind. Electron.*, vol. 64, no 4, pp. 3273–3285, Jan. 2016.

[3] W. Yu, H. Wang, H. Hong, and G. Wen. Distributed cooperative anti-disturbance control of multi-agent systems: an overview. *Sci. China-Inf. Sci.*, vol. 60, no. 11, pp.77–90, Sep. 2017.

[4] Y. Choi, K. Yang, W. K. Chung, and H. R. Kim. On the robustness and performance of disturbance observers for second-order systems. *IEEE Trans. Autom. Control*, vol. 48, no. 2, pp. 315–320, Mar. 2003.

[5] J. Han. From PID to active disturbance rejection control. *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp. 900–906, Apr. 2009.

[6] Z. Gao. Scaling and bandwidth-parameterization based controller tuning. *Proc. Amer. Control Conf.*, vol. 6, pp. 4989–4996, 2003.

[7] J. She, M. Fang, Y. Ohyama, H. Hashimoto, and M. Wu, "Improving disturbance-rejection performance based on an equivalent-input-disturbance approach", *IEEE Trans. Ind. Electron.*, vol. 55, no. 1, pp. 380–389, Jan. 2008.

[8] M. Wu, F. Gao, P. Yu, J. She, and W. Cao. Improve disturbance-rejection performance for an equivalent-input-disturbance-based control system by incorporating a proportional-integral observer. *IEEE Trans. Ind. Electron.*, vol. 67, no. 2, pp. 1254–1260, Feb. 2020.

[9] D. W. C. Ho and G. Lu. Robust stabilization for a class of discrete-time non-linear systems via output feedback: The unified LMI approach. *Int. J. Control*, vol. 76, no. 2, pp. 105–115, Nov. 2003.