The Model to Determine the Location and the Date by the Length of Shadow of Objects

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Abstract. Sun-shadow positioning technology is a new positioning method, namely, by providing the changes of objects' sun-shadow, to determine the location and date of shooting. Based on the analysis of changes about the sun-shadow. This paper, by using the solar azimuth, elevation, declination angle and solar hour angle, has established the mathematical model to determine the position and date of some objects.Firstly, through the analysis of forming principle of the sun-shadow, combining with the relevant parameters with sun-shadow and geographical coordinates and the moments provided, we have constructed two different models about the sun-shadow length of straight, which are without the time differences and with the time difference. By meanings of solving these models, we get a amount of the changes about shadow length value of the straight in Tiananmen Square, and then draw the curve of shadow through MATLAB software. Above all, a simple conclusion has obtained, which is when we do not consider the factor of time difference, the shadow length curve is symmetrical; and while the time difference is considered, the curve is no longer symmetric.Secondly, this paper has used two different methods to calculate the positions.Method I： firstly, according to the relevant definitions about solar elevation angle, the shadow length model is established. Then we find the latitudes and the longitudes are almost the same even if the straight length are changed, which are calculated by computers, N 2.4° E 110°. We could determine the places may appear in Malaysia judging from the latitudes and the longitudes.Method II: Let the ground plane where the shadow is in be the sundial plane, and the fixed straight bar be the gnomon, we could get the oritations and coordinate system of directions using the sun-benchmark orientation method based on Cartesian coordinate system mentioned above. Then we obtain the approximate range of longitude (111°~130.5°) from the relationship about difference between the time and and longitude. After the correction of accuracy, the range of longitude become (109.5°~129.3°). Combining the solution of latitude (N 2.6° and N 18.2°), we find the places maybe in

Hainan or Malaysia. Although the methods are different, the results are basically consistent.Again, based on the geographical location of the straight, we also need to find the date. According to the relationship between the motion pattern of sun direct point and the parameters of solar elevation angle, meanwhile, supposing that spring equinox be the reference time and the tropic of cancer be the reference latitude, we finally get the positions of N 22 ,E 75, possibly in Xinjiang, dated July 7 by calculated from Appendix II; N 33, E112, possibly in Hubei, dated July 21 from Appendix III.

At last, we could get 40 groups of information about the length of straight shadow at every one minutes from 8:55 by using CAD software for video information processing. Then we have soluted the actual length of shadow making use of similar relationship. According to the relationship between the camera coordinate system and the world coordinate system, consulting the solution of the latitude and the longitude under the sundial model, we find the places in the video maybe in Hohhot during July.

Keywords: Analemmatic sundial model; sun-bechmark orientation method; camera coordinate system; longitude correction

1 Problems Need to be Solved

1.1 Background

In the era of rapid development of Internet technology, processing technology of image and video has become an important means of extracting information, which has caused more and more international attentions in the world. How to determine the position and recording date is an important aspect of video data analysis. Meanwhile, the sun-shadow positioning technology, which is an important method to determine the video shoot location and date, has more accuracy by analyzing the change of sun-shadow in video. Also, establishing the mathematical and physical model to catch the changes about the sun-shadow of objects to get the position and the shooting date has a very wide range of applications, especially in military field. We will choose an interesting problem on the internet to build the appropriate model, then test and improve the model by Appendix data.

1.2 Problems Need to be Solved

1. Establish the mathematical model of the changes about the length of the shadow; analyze the principles of the shadow length’ s changes with various parameters; draw the curve of the shadow length of 3-meter straight at Tiananmen Square

(latitude 39 degrees 54 minutes 26 seconds, east longitude 116 degrees 23 minutes 29 seconds) from 9:00 to 15:00 (Beijing time) Oct. 22, 2015.

2. According to the data of coordinate about endpoints of straight shadow fixed on the ground, a mathematical model is built to determine the location where the straight is. Then we take the data of Appendix 1 into this model to get a number of possible locations.

3. Another mathematical model is constructed to determine the date when the straight bar is shooting. Then we put the data of Appendix 2 and 3 in this model to get a number of possible positions and moments.

4. Appendix 4 is a video about the changes of a straight’ s sun-shadow. Also we could estimate the height of the straight is 2 meters in some way. Establishing a mathematical model to determine the possible location where the video is shooting. Further, if the shooting date is unknown, could you determine the location and date from the video at the same time?

2 Analysis of Problems

2.1 Analysis of Problem I

Problem I asks us to analyze the rule about the length of sun-shadow followed with the various parameters, to draw the change curve of a 3 meters straight bar’ s shadow. Access to relevant information available, we could construct a mathematical model, based on the solar elevation angle and some relevant parameters refereed, to describe the principle about the changes of shadow.

2.2 Analysis of Problems II

Problem II asks to determine the possible position of the straight bar from the data of vertex coordinate of sun-shadow on the ground. Since the straight bar’ s height is unknown, under some reasonable assumptions on the height of the straight bar, we could get the solution by the definition of solar elevation angle and related parameters.

Secondly, we have found that the model could in analogy with sundial model when the related parameters are unknown. By solving a number of other parameters, we can also obtain the physical location and time of objects.

2.3 Analysis of Problems III

On the basis of Problem II, besides to get the latitude and longitude, we also want to determine the shooting date. According to the movement of the sun direct point and the relation between time difference and latitude difference of the right position and referent position, we could obtain the results about the position and date.

2.4 Analysis of Problems IV

Problem IV asks the shooting position in the video judging from the changes of straight’ s shadow. This problem can be solved from several models established in the front. Considering that the coordinate system in the video is different from the practical level, we choose the professional software to process the data, then predict the actual shadow length from the similar relationship. We can transform the coordination into the solution of length of shadow on the basis of relationship between the camera coordinate and the world coordinate system.

3 Model Assumptions and Symbol Description

3.1 Model Assumptions

1. Assuming the plane which video objects are in is the horizon;
2. Assuming the Earth is the standard sphere;
3. Ignoring atmosphere refractive index;
4. Suppose the sunlight on the earth are parallel, and the horizontal ground in somewhere on the Earth is its the tangent plane;

3.2 Contact Author Information

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3.3 Symbol Description

α	Solar elevation angle	H	Straight height
δ	Solar declination angle	L	Central meridian
φ	Local longitude	$L_{meridian}$	Object longitude values
β	Solar hour angle	t_{lc}	Longitude calibration value
l	The length of shadow	h	Time angle
l_n	The distance from point to straight	σ	Direct sunlight latitude

4 The establishment and solution of model

4.1 What Will Be Done with Your Paper

4.1.1 Preparation for Model of Problem I

Problem I asks to draw the graph of trend from the solution of the length of shadow. From the definition of the solar elevation angle, we will discuss the difference of graphs when time difference is considered and not to be considered.

4.1.2 Establishment the model of Problem I

Method I: Calculation of the length of shadow without time difference

(1)Solution of length of Straight rop’s sun-shadow [1]
Making the use of geometric relation between solar elevation angle [1] and straight bar`s shadow length, the shadow lengths of each time between 9:00-15:00 are obtained, which can draw a curve of straight bar’s shadow length.

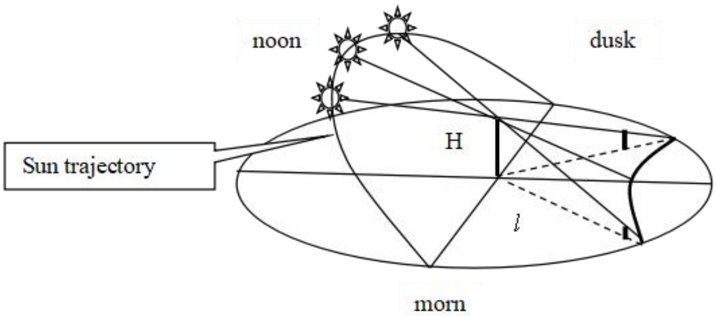


Fig.1. Relationship between shadow length and sun elevation angle

From Fig.1, a formula to calculate the length of the shadow is obtained, as follows:

$$l = \frac{H}{\tan\alpha} \qquad (1)$$

H represent the length of the straight bar, α is the sun elevation angle, from the relevant references, we could get a formula for the solar elevation angle as follows:

$$\sin\alpha = \sin\varphi \cdot \sin\delta + \cos\varphi \cdot \cos\delta \cdot \cos\beta \qquad (2)$$

And φ is the local latitude, δ is the solar declination [3], β is the the sun angle [2].

In order to determine the relevant parameters,we need to calculate the length of the shadow:

a. Establishment of solar declination angle [3]

From the references, we could find the precise formula for calculation of solar declination [3] as follows:

$$\delta = \frac{180^\circ}{\pi} \cdot (0.006918 - 0.399912\cos\gamma + 0.070257\sin\gamma - 0.006758\cos 2\gamma + 0.000907\sin 2\gamma - 0.002697\cos 3\gamma + 0.00149\sin 3\gamma) \qquad (3)$$

$$\gamma = \frac{2\pi}{365}(N-1) \qquad (4)$$

δ is measured in degrees, N is counted from Jan.1st.

b.Calculation formula for the solar hour angle is [2]

$$\beta = (T-12) \times 15^\circ \qquad (5)$$

c. Calculation formula for the solar azimuth is[4]

$$\cos A = \frac{\sin\alpha \sin\varphi - \sin\delta}{\cos\alpha \cos\varphi} \tag{6}$$

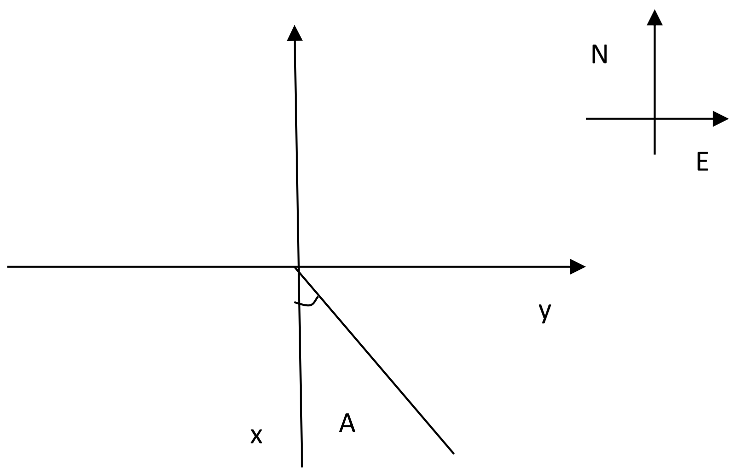
$$\sin A = \frac{\cos\delta \sin\beta}{\cos\alpha} \tag{7}$$

(2) Establishment of the trajectory of the shadow

• From the references, the solar azimuth [4] stand for the angle between the sunlight projected on the ground and the local meridian, which can be approximately regarded as the angle between the erected rop`s shadow and the right south. In order to describe the trajectory of the shadow length, and further to determine solar azimuth, we could construct the Cartesian coordinate system, in which the right East direction is X-axis and the right North direction is Y-axis.

• Therefore, the sun-shadow of the objects should change from the west to east. That is to say, before the noon the shadow should be on the west side; while after the noon the shadow should be on the east, and the length of the shadow should be the shortest at noon.

According to the change principle of sundial shadow and the concept of the solar azimuth, we could construct the following coordinate system.



• Put the coordinate of the endpoint of the rop`s shadow are $\left(x_i, y_i\right)\left(i = 9,10,\cdots,15\right)$, then

$$\begin{cases} x_i = -l \cdot \cos(180^\circ - A) \\ y_i = l \cdot \sin(180^\circ - A) \end{cases} \quad (8)$$

4.1.3. Solution of the model for Problem I

(1) Solution of the solar declination angle [3]
If Jan.1st is the first day in a year, then Oct. 22nd is the 295th day. That is

$$N = 295$$

Then take N into Equation (3), we could get

$$\delta = -11.0740^\circ$$

(2) Acording to Equation (5), the solar hour angle could be obtained as below:

Tab 1. Solar hour angle

Time (unit: hour)	9	10	11	12	13	14	15
Solar hourAngle(unit:degree)	−45	−30	−15	0	15	30	45

(3) According to Equation (2), the solar elevation angle could be calculated as below in Table 2:
It is known that the Beijing Tiananmen Square is located in North latitude 39 degrees 54 minutes 26 seconds, East longitude 116 degrees 23 minutes 29 seconds. After the transformation of the units, we get

$$\varphi = 39.907^\circ, \mu = 116.391^\circ$$

Taking the values of δ, β, φ into Equation (2), we obtain the solar elevation angle at each times as follows:

Tab 2. Solar elevation angle

Time (unit: hour)	9	10	11	12	13	14	15
Solar elevation Angle (unit:degree)	24.15	31.92	37.15	39.02	37.15	31.92	24.15

Data analysis:
The data from above shows that the solar elevation angle from 9 a.m. to 15 p.m. is symmetric with respect to 12:00, and the angle reaches the maximum at noon.

(4) Solution of the length of rop`s shadow
The length of straight is 3 meters, that is $H=3$. Taking all of the values of α at every moments into Equation (1), we get the length of the straight’ s shadow is like this:

Tab 3. the length of straight shadow

Time (unit: hour)	9	10	11	12	13	14	15
Shadow length (unit： meter)	6.6917	4.8161	3.9593	3.7022	3.9593	4.8161	6.6 917

Fitting the data above by MATLAB software, we could find the function expression of the length of shadow as follows:

$$y = 0.3375x^2 - 8.101x + 52.2 \qquad (R^2 = 0.99)$$

(5) Curve of change of shadow length

The curve of change of shadow length with 3 meters straight is drwan according to the data in Table 3 as Figure 2

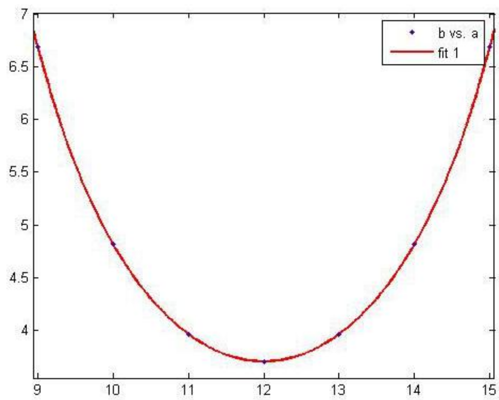


Figure. 2. Change curve of shadow length of 3-meter straight

Data analysis:

The shadow length from 9 a.m. to 15 p.m. is symmetric with respect to 12:00, and the shadow length reaches the minimum at noon. The range of the shadow length is the closed interval $[3.7, 6.69]$

Solution of solar direction angle

Taking the values of δ , β and α into the Equations (6) and (7), we will find the solar azimuth as follows:

Tab 4. Solar azimuth

Time (unit: hour)	9	10	11	12	13	14	15
Solar azimuth(unit:degree)	-50.4°	-35.5°	18.6°	0°	18.6°	35.5°	50.4°

(6) Solution of coordinates of straight shadow endpoints.

Time (unit: hour)	9	10	11	12	13	14	15
x -coordinate	4.2626129	3.9203054	3.75222861	3.7022	3.752229	3.920787	3.894569
y -coordinate	−5.152609	−2.793338	1.2630167	0	1.263017	−2.79334	−5.42028

Method II: Calculation the shadow length with time difference

Since the Beijing time widely used is not the local time (true solar time), then the solar hour angle could become more accurate if we add the time difference. Therefore, we could optimize the model in Method I by taking the time difference, between the local time and Beijing time (standard time) into account.

Optimizing the model in the establishment of solar hour angle, and other steps remain unchanged.

(1) Establishment formula of the solar hour angle is as follows:

$$\beta_0 = (TT - 12) \times 15^{\circ} \tag{9}$$

$$TT = T_0 + E_q \tag{10}$$

$$E_q = \left(120^\circ - \mu\right)/15^\circ \quad (11)$$

TT stands for true solar time, T_0 stands for Beijing time, E_q stands for time difference, μ stands for the region longitude.

Solar declination angle

$$N = 295,\ \delta = -11.0740^\circ$$

Solution of the solar hour angle

Tab 4. Solar hour angle

Time (unit: hour)	9	10	11	12	13	14	15
Solar hour angle (unit:degree)	−41.391	−26.391	−11.391	3.609	18.609	33.609	48.609

4) Solution of solar elevation angle

It is known that the Beijing Tiananmen Square is located in North latitude 39 degrees 54 minutes 26 seconds, East longitude 116 degrees 23 minutes 29 seconds. After the transformation of the units, we get

$$\varphi = 39.907^\circ, \mu = 116.391^\circ$$

Taking the values of δ, β, φ into Equation (2), we obtain the solar elevation angle at each times as follows:

Tab 5. Solar elevation angle

Time (unit: hour)	9	10	11	12	13	14	15
Solar elevation angle (unit:degree)	26.23	33.45	37.93	39.04	36.19	30.27	21.88

Data analysis:
The data from above shows that the solar elevation angle from 9 a.m. to 15 p.m. is symmetic with respect to 12:00, and the angle reaches the maximum at noon.

5) Solution of the length of rop's shadow

The length of straight is 3 meters, that is $H=3$. Taking all of the values of α at every moments into Equation (2), we get the function expression of the length of the straight's shadow is like this:

$$y=0.0009792x^5-0.04713x^4+0.8553x^3-6.758x^2+17.04x+27.97$$

Obtained the shadow length of the straight at each moments as Table 6:

Tab 6. The shadow length of the straight

Time (unit: hour)	9	10	11	12	13	14	15
Shadow length(unit：meter)	6.09	4.54	3.849	3.7	4.1	5.14	7.47

6) Curve of change of shadow length of straight

The curve of change of shadow length with 3 meters straight is drwan according to the data in Table 6 as Figure 3:

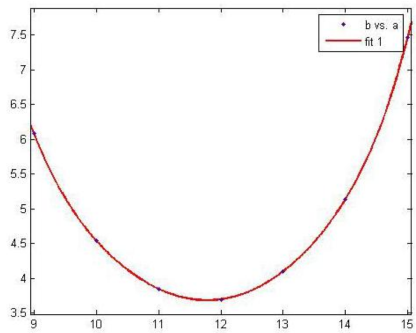


Fig. 3. Curve of change of shadow length of 3 meters straight

Data analysis:

From the data in Table 6 and Figure 3, we find that the shadow length from 9 a.m. to 15 p.m. is no longer symmetric with respect to 12:00, and the shadow length reaches the minimum after the noon. This difference from Fig 2 explains the existence of the time difference. Therefore, the moment when the minimum shadow length appears could shift a little, and the range of the shadow length is the closed interval $[3.7, 7.47]$.

4.2 The model establishment and solution for Problem II

Method I: To solve the longitude and latitude by using the given club length1.

The establishment and solution to the positioning system based on the length of shadow

(1)The solving ideas

In order to determine the possible sites of the given club, i.e., the longitude and latitude of the

club, we can first be sure the specific length of the club. Then, the possible sites can be obtained by using the coordinate data of the shadow vertex

(2)The conduction of the information

By using the date of the shadow vertex, we can calculate the shadow length at various points which are shown in the table 7.

Table 7. The shadow length at various points

Chinese standard Time	14:42	14:45	14:48	14:51	……	15:30	15:33	15:36	15:39	15:42
Shadow length	1.149626	1.182199	1.215297	1.249051	……	1.746206	1.790051	1.835014	1.880875	1.927918

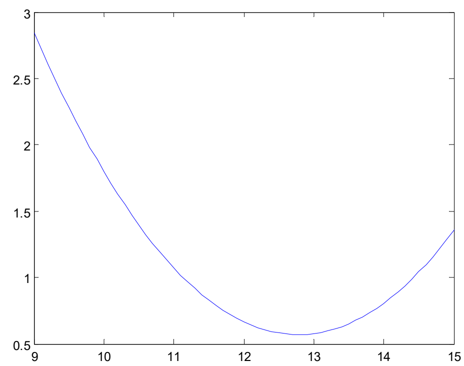


Figure. 4. The variation of the shadow length

The discussion of the relevant parameters

a. To determine the declination angle of the sun

According to the calculation formula of the declination angle of the sun, we have

$$\delta = \frac{180^\circ}{\pi} \cdot (0.006918 - 0.399912\cos\gamma + 0.070257\sin\gamma - 0.006758\cos 2\gamma$$
$$+ 0.000907\sin 2\gamma - 0.002697\cos 3\gamma + 0.00149\sin 3\gamma) \quad (12)$$

$$\gamma = \frac{2\pi}{365}(N-1),$$

Where the unit of δ is degree and N begins with January 1.

b. To determine the solar hour angle

Using the calculation formula of the solar hour angle, we get

$$\beta_0 = (TT - 12) \times 15^\circ$$
$$TT = T_0 + E_q \quad (13)$$
$$E_q = (120^\circ - \mu)/15^\circ,$$

where TT is apparent solar time, T_0 is Chinese Standard Time, E_q is time difference and
μ is area longitude.

c. To determine the club length

It is well known that the different objects can generate the shadow with identical length, the reason may be the different sun angle and the different geographic position of the objects. So, in this part, according to the physical length of shadow, we can calculate the sun angle to different length of club. The results are shown in table 8.

Table 8. The solar elevation angle for the different club length at different time

Time（unit：hour）		9	10	11	12	13	14	15
Club length（unit：meter）	1	0.0026	0.0036	0.0043	0.004	0.004	0.0036	0.148
	2	0.545	0.0072	0.008	0.0094	0.008	0.007	0.293
	3	0.007	0.0108	0.013	0.014	0.013	0.582	0.433

Now, we further solve the problem.

Substituting the data of Table 8 into (1) yields

$$\begin{cases} \sin\varphi_1 \sin\delta_1 + \cos\varphi_1 \cos\delta_1 \cos\beta_1 = \sin A_1 \\ \sin\varphi_2 \sin\delta_2 + \cos\varphi_2 \cos\delta_2 \cos\beta_2 = \sin A_2 \\ \cdots \cdots \\ \sin\varphi_{20} \sin\delta_{20} + \cos\varphi_{20} \cos\delta_{20} \cos\beta_{20} = \sin A_{20} \\ \sin\varphi_{21} \sin\delta_{21} + \cos\varphi_{21} \cos\delta_{21} \cos\beta_{21} = \sin A_{21} \end{cases}$$

The equation set above can be rewritten as a overdetermined set

$$\begin{pmatrix} \sin\varphi_1 & \cos\varphi_1 \cos\beta_1 \\ \sin\varphi_2 & \cos\varphi_2 \cos\beta_2 \\ \cdots & \cdots \\ \sin\varphi_{20} & \cos\varphi_{20} \cos\beta_{20} \\ \sin\varphi_{21} & \cos\varphi_{21} \cos\beta_{21} \end{pmatrix} \begin{pmatrix} \sin\delta_i \\ \cos\delta_i \end{pmatrix} = \begin{pmatrix} \sin A_1 \\ \sin A_2 \\ \cdots \\ \sin A_{20} \\ \sin A_{21} \end{pmatrix}$$

By using the MATLAB, we can see

1m		2m		3m	
latitude	longitude	latitude	longitude	latitude	longitude
2.401	110.8	2.400	110.5	2.402	111.0
2.401	110.8	2.400	111	2.402	110.3
2.406	111.1	2.402	111	2.562	110.6
2.401	110.7	2.401	110.7	2.552	110.1
2.406	110.7	2.402	110.8	2.442	110.9
2.401	110.08	2.402	110.2	2.442	110.4
2.403	110.7	2.403	110	2.442	110.6
2.403	111	2.127	111	2.402	110.9
2.403	110.4	2.144	111	2.403	111

Conclusion:

By calculating the longitude and latitude under the different shadow length, we can obtain that the longitude and latitude remained about the same. So, the northern latitude is 1.57° and the east longitude is 163°. Then, the position can be determined to Malaysia.

Method II: To solve the longitude and latitude by using the Analemmatic Sundial model

(1)The theoretical basis of the model

According to the relative information, we can estimate the latitude by using the projection trajectory when the size of object is unknown. This method to calculate the longitude and latitude is based on the Analemmatic Sundial theory. We can use the foot position only and the visibility of object is not to need. The Analemmatic Sundial theory has some helpful properties. For example, the projection trajectory is an ellipse. Its major axis is east-west and the minor axis is north-south. The projection on the sundial moves along the minor axis, the shadow trajectory is shown as Figure 5.

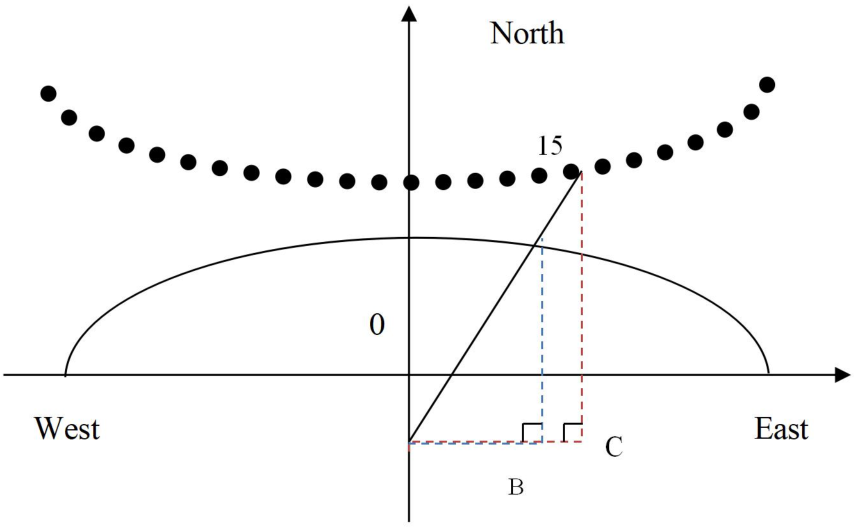


Figure. 5. The shadow trajectory

(2) The modeling of the Analemmatic Sundial

a. To determine the longitude

The information shows that the local time has a difference with the standard time. The difference is affected by the time offset, the equilibrum time and daylight saving time. When the longitude of two areas has a differential of 15°, the time differs by one hour. Then, we can see that the longitude of two areas has a differential of 1°, its time differs by four minutes from the other.

So, the longitude of the object can be expressed as follow

$$L = L_{meridian} - 0.25 \cdot t_{1c} \tag{14}$$

where $L_{meridian}$ notes the central meridian of this timezone, L notes the longitude of the object and t_{1c} is the calibration of the longitude deviation.

Because of the calibration of the longitude deviation equal to the sum of the time offset, the equilibrum time and the daylight saving time, i.e.

$$t_{1c} = t_{OFF} + t_{EOT} + t_{DST} \tag{15}$$

where t_{OFF} is the time offset, t_{EOT} is the equilibrum time and t_{DST} is the daylight saving time.

b. To determine the latitude

The Analemmatic Sundial expresses the ellipse trajectory by using the rule which the different latitude can generate different flattening. So, we choose the ground as the sundial horizontal. The projection of shadow on north-south and east-west are the minor axis and major axis of the sundial. Based on the definition of the flattening, we have

$$\frac{\left|Y_h + Y_E\right|}{X_h} = \frac{|b|}{|a|} \tag{16}$$

where $X_h = \sin h$ notes the sundial on horizontal.

The time angle is h

$$h = \left(T_{24} - 12\right) \times 15^{\circ} \tag{17}$$

where T_{24} is Chinese Standard Time.

$Y_h = \sin\phi\cos h$ is the calculation formula of the sundial on vertical direction. The position the foot point of object $E = \left[0, Y_E\right)^T$ which is relative the ellipse center is

$$Y_E = \tan\sigma\cos\phi$$

where ϕ is local latitude, σ is direct sunlight latitude and

$$\sigma = \arcsin\left(\sin 23^{\circ}26^{'} \cdot \sin\left(\omega t\right)\right) \tag{18}$$

where t is the day after vernal equinox, i.e., March 21 denotes as 1.

Above all, the latitude can be expressed as

$$\frac{\left|Y_h + Y_E\right|}{X_h} = \frac{\left|\sin\phi\cos h - \tan\sigma\cos\phi\right|}{\left|\sin h\right|} = \frac{|b|}{|a|} \tag{19}$$

c. To set up the coordinate system of shadow based on the sundial model

Since the direction of the shadow coordinate system is unknown, we must deal with the coordinate system at first. By checking reference material, we find some useful method to solve this problem, like the graticules beacon, the natural characteristics to directional, the sun-benchmark orientation method and so on. In this paper, we apply the sun-benchmark orientation method.

In the first place, the secondary coordinate system on east-west must be built.

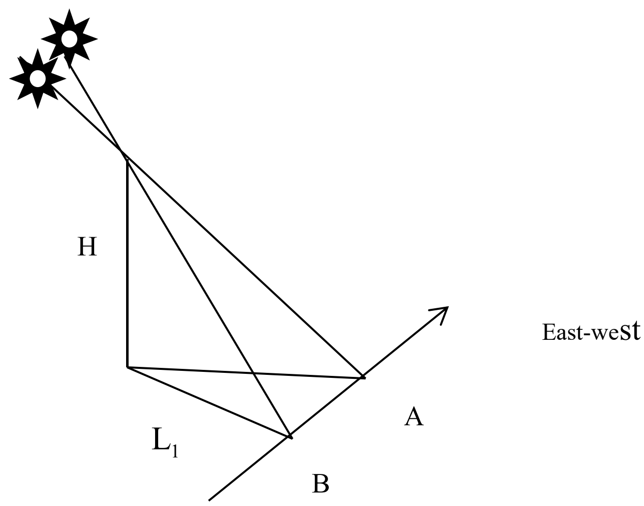


Figure. 6. The secondary coordinate system on east-west

In the second place, we can make the benchmark perpendicular to the ground and measure the shadow of this time. Then, we denote it as A . Furthermore, we use the case shown as Figure 6 . After a while, the end-points of shadow is noted as B and the direction of the wired between A and B is east-west.

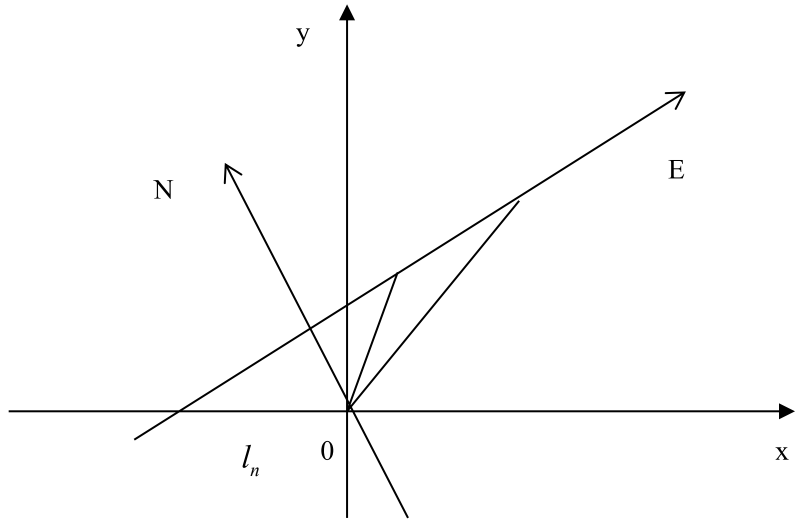


Figure. 7. The schematic diagram of the azimuth coordinate

So, we get that the wired of the shadow end-points at two moments can determine a line, and the direction of the line is east-west. Suppose the equation of the line is $y = Ax + B$.

At the Chinese standard time $12:41$, the end-point of shadow is $(-1.2352, 0.173)$, and then the shadow l is 1.2472 meters. The distance of the origin from the line is the projection of the shadow on the north-south. Then, we can obtain that the formula of the shadow on the north-south based on the geometrical relationship.

$$l_n = \frac{Ax_0 + By_0 + C}{\sqrt{A^2 + B^2}}$$

(3)The solution of the model

a. To solving the local longitude

In the model we build, we must know the central meridian of the local timezone before calculating the longitude. So, for the case that the category of longitude is known, we can use the definition to correcting the solution longitude. In order to predict the local longitude range, this paper applies the principle that the shadow is shortest at the noontime.

By calculating, we have the shadow length at different time. Then, the period of time

$12:35-12:45$ is determined while the shadow is shortest.

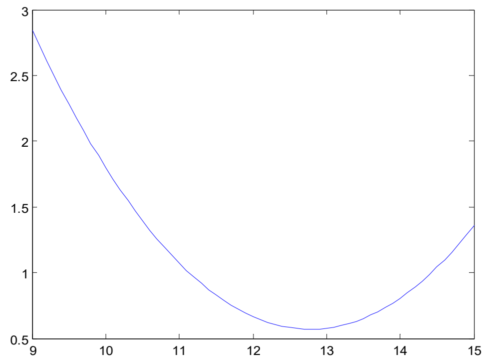


Figure. 8. The variation of the shadow length

The expression of relative reference longitude is

$$P = \frac{T - T_{\mathrm{north}}}{4} \tag{20}$$

According to the law of Earth's rotation, we have that the Earth turns 1° every four minutes. Let's take the other timezone and the longitude as a reference group, and use the longitude difference to replace the time difference. Furthermore, we deal with the longitude difference based on the principle "east plus and west substract". Then, we can obtain the longitude range.

In this paper, we choose Beijing as the reference group. By calculating, we get the longitude range is $(111^\circ \sim 130.5^\circ)$.

By using the longitude formulation in the sundial model, we can revise the longitude. Now, the central meridian of local timezone is 120° E and the longitude range is obtained. Suppose t_{lc} is deviation calibration value of longitude, the correction of local longitude can be expressed as $l = 120^\circ - 0.25t_{lc}$.

Then, the correction range is $(109.5^\circ \sim 129.3^\circ)$.

b. To solving the local latitude

According to the formula

$$\frac{|\sin\phi\cos h - \tan\sigma\cos\phi|}{|\sin h|} = \frac{|b|}{|a|},$$

we can calculate ϕ in equation $h = (T_{24} - 12) \times 15^\circ$, T_{24} is obtained by using the reference longitude and time difference. Then, we have $h = 49.5$ and the solar elevation angle is 12.06°.

Now, using the MATLAB, we have

$$\phi = 18.2^\circ \ or \ \phi = 2.5^\circ$$

Then, based on the longitude and latitude, we can determine that the place maybe HaiNan or Malaysia.

4.3 The model establishment and solution of the third problem III

4.3.1 The preparation of the model

Under the condition that the position is unknown, this paper needs us to determine the video data by using the vertex coordinates. According to the rules that the subsolar point moves between the tropic of Capricorn and cancer, we can calculate the moving angle of the subsolar point of every day. Then, the data can be determined when we choose the special data as the reference point.
Based on the solution of problem two, we get that the longitude range of the club location is $(72^\circ \sim 114^\circ)$.

4.3.2 The establishment and solution of the relative time model

Since the latitude difference between the North and south tropic of cancer is $47^\circ\,(46^\circ 52')$ and the sun moves twice for a tropical year , the latitude of the subsolar point changes 0.258°. Now, we choose the tropic of cancer as the relative reference latitude and the summer solstice as the relative reference time. And Suppose that the date of this area differs x days with summer solstice, we have

$$\begin{cases} \dfrac{|\sin\phi\cos h - \tan\sigma\cos\phi|}{|\sin h|} = \dfrac{|b|}{|a|} \\ h = \left(T_{24} - 12\right)\times 15^\circ \\ \sigma = \dfrac{x}{0.258^\circ} \end{cases}$$

In the case of the appendix 2, the relative time difference is $x = 72$. Furthermore, we get the local coordinates is latitude 22° and longitude 75°, possibly in Xinjiang, which the date is July 7. By using the same method, in the case of appendix 3, the relative time difference is $x = 20$ and the local coordinates is latitude 33° and longitude 112°. So the place is possibly in Hubei, dated July 21.

4.4 The model establishment and solution of problem IV

4.4.1 The camera coordinate system

In this section, we establish the coordinate $U-V$ with the origin of the left upper corner and the unit is pixel. The abscissa of pixel u and the ordinate v are the number of columns and rows in the array of images, respectively. (u is corresponding x and v is corresponding y)

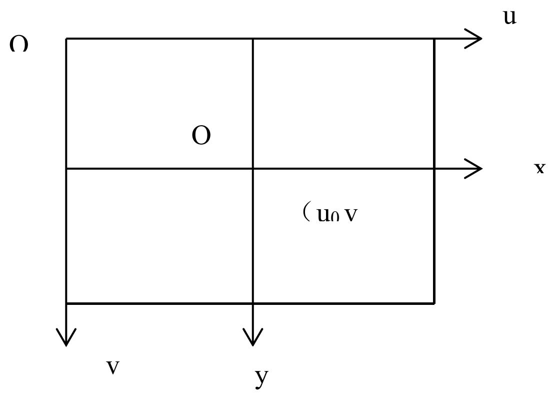


Figure. 9. The coordinate system $U-V$

The coordinates (u,v) only represents the columns and rows of the pixel. But the pixel in the location of image do not express by using the physical unit. So, we must set up the coordinate $x-y$ of the image which used the physical unit. Note the intersection of camera optical axis and image plane as the origin of the coordinate O_1 . x -axis parallels u -axis and y -axis parallels v -axis. Suppose (u_0,v_0) represents the coordinate of O_1 under the (u,v) .Note $\mathrm{d}x$ and $\mathrm{d}y$ as the physical size of every pixel which are in x -axis and y -axis, respectively. Then, we have the relationship between the point in $u-v$ coordinate and $x-y$ coordinate

$$u=\frac{x}{\mathrm{d}x}+u_0 \tag{21}$$

$$v = \frac{y}{\mathrm{d}y} + v_0 \qquad (22)$$

where the unit of the physical coordinate is millimeter ,the unit of $\mathrm{d}x$ is millimeter/pixel and the unit of $x/\mathrm{d}x$ is same to u , i.e., pixel.

For simplicity, we use the homogeneous coordinates and matrix to represent the above expression.

$$\begin{bmatrix} u \\ v \\ l \end{bmatrix} = \begin{bmatrix} \frac{1}{\mathrm{d}y} & 0 & u_0 \\ 0 & \frac{1}{\mathrm{d}y} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Then, we have the inverse relationship as follows

$$\begin{bmatrix} x \\ y \\ l \end{bmatrix} = \begin{bmatrix} \mathrm{d}x & 0 & -u_0\mathrm{d}x \\ 0 & \mathrm{d}y & -v_0\mathrm{d}y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ l \end{bmatrix} \qquad (23)$$

4.4.2 The world coordinate system

In order to describe the location of the camera, we introduce the world coordinate. It is shown as $O_w X_w Y_w Z_w$ coordinate in Fig.9. The rotation matrix R can be used to express the relationship between the world coordinate and the camera coordinate.

Suppose the homogeneous coordinate of point P , which is under the world coordinate is (X_w , Y_c , Z_c , 1), we get that

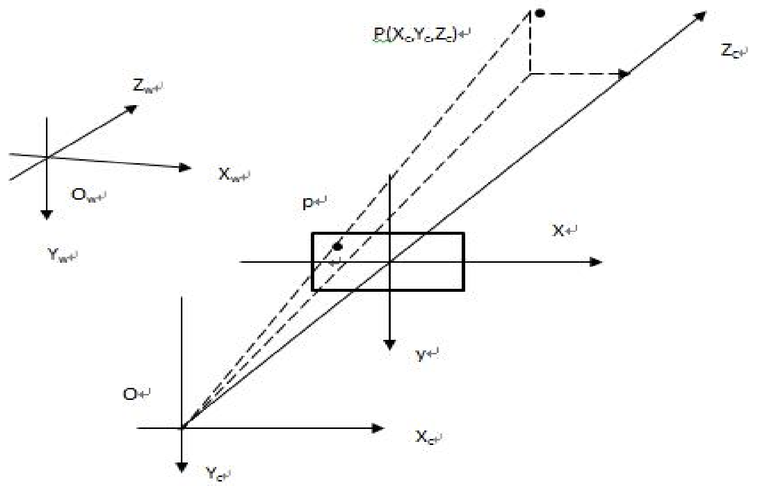


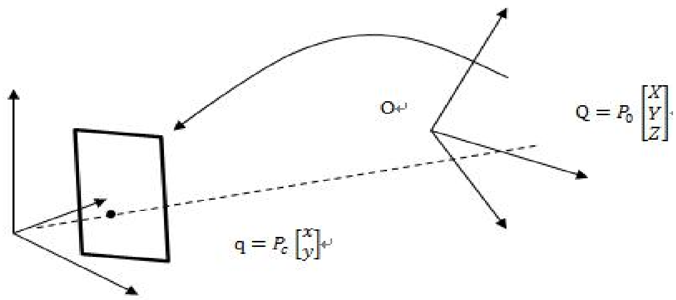
Figure. 9. The relationship between two coordinate systems

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} R & t \\ \bar{0} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = M_i \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{24}$$

where the rotation matrix R is a 3×3 orthogonal matrix and t is a 3-dimensional translation vector.

4.4.3 The relationship between the world coordinate system and the camera coordinate system

Firstly, we give the definition of the rotation matrix R.



If the rotation angle of the coordinate system is θ, we can see that the target point rotates a same angle θ with reversed. In 3-dimensional space, the rotation can be exploded into 2-dimensional space. Suppose the object rotates angles for x, y, z -axis are ψ, φ, θ, respectively, then, the general rotation matrix R is the product of $R_x(\psi), R_y(\varphi), R_z(\theta)$,i.e., $R = Rx(\psi), Ry(\varphi), Rz(\theta)$, where

$$R_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix} \qquad R_y(\varphi) = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{25}$$

4.4.4 The solution of the model

(1) On the basis of the aforementioned discussions, we can translate the points in $u-v$ coordinate system to $x-y$ coordinate system by using (20) and (21). Then, the problem becomes the solution of the actual shadow length on the horizontal plane.

(2) The solution of shadow length

Time	Shadow length in the video	Actual length	Azimuth angle in the video

08:55	37.7155	2.36832	$1^{\circ}13^{'}59^{''}$
08:56	37.4812	2.353608	$1^{\circ}12^{'}31^{''}$
08:57	37.2208	2.337256	$1^{\circ}9^{'}41^{''}$
08:58	36.9254	2.318706	$1^{\circ}5^{'}29^{''}$
08:59	36.6588	2.301965	$1^{\circ}13^{'}50^{''}$
09:00	36.5541	2.295391	$1^{\circ}8^{'}31^{''}$
09:01	36.1669	2.271077	$1^{\circ}8^{'}21^{''}$
09:02	36.0821	2.265752	$1^{\circ}9^{'}14^{''}$
09:03	35.705	2.242072	$1^{\circ}6^{'}19^{''}$
09:04	35.6412	2.238066	$0^{\circ}58^{'}33^{''}$
…	…	…	…
09:18	32.4331	2.036615	$0^{\circ}31^{'}58^{''}$
09:23	31.473	1.976327	$0^{\circ}21^{'}29^{''}$
09:24	31.3124	1.966242	$0^{\circ}23^{'}38^{''}$
09:25	31.1188	1.954085	$0^{\circ}16^{'}53^{''}$
09:26	30.8562	1.937595	$0^{\circ}24^{'}55^{''}$
09:27	30.59	1.920879	$0^{\circ}9^{'}40^{''}$
09:33	9.33	0.585871	$0^{\circ}11^{'}27^{''}$
09:34	9.24	0.58022	$0^{\circ}9^{'}57^{''}$

Then, the fitting function is $y=-12.35x+147.5$.

The solution of the longitude and latitude

According to (18) and (19), the longitude ϕ and latitude μ can be calculated

$$\begin{cases} \dfrac{\left|\sin\phi\cos h-\tan\sigma\cos\phi\right|}{\left|\sin h\right|}=\dfrac{|b|}{|a|} \\ h=\left(T_{24}-12\right)\times 15^{\circ} \\ \sigma=\dfrac{x}{0.258^{\circ}} \\ \mu=\dfrac{T_{24}-12}{4} \end{cases}$$

Then, we can obtain the solution by using the relative parameters. So, the northern latitude is 38° and the east longitude is 116°. On the basis of these, the position in the video maybe the Hohhot in July.

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