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Hilbert Mathematics Versus Gödel Mathematics III. Hilbert Mathematics by Itself, and Gödel Mathematics Versus the Physical World Within It: Both as Its Particular Cases

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Hilbert mathematics versus Gödel mathematics
III. Hilbert mathematics by itself, and Gödel mathematics versus the physical world within it: both as its particular cases

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Abstract. The paper discusses Hilbert mathematics, a kind of Pythagorean mathematics, to which the physical world is a particular case. The parameter of the "distance between finiteness and infinity" is crucial. Any nonzero finite value of it features the particular case in the frameworks of Hilbert mathematics where the physical world appears "ex nihilo" by virtue of an only mathematical necessity or quantum information conservation physically. One does not need the mythical Big Bang which serves to concentrate all the violations of energy conservation in a "safe", maximally remote point in the alleged "beginning of the universe". On the contrary, an omnipresent and omnitemporal medium obeying quantum information conservation rather than energy conservation permanently generates action and thus the physical world. The utilization of that creation "ex nihilo" is accessible to humankind, at least theoretically, as long as one observes the physical laws, which admit it in their new and wider generalization. One can oppose Hilbert mathematics to Gödel mathematics, which can be identified as all the standard mathematics until now featureable by the Gödel dichotomy of arithmetic to set theory: and then, "dialectic", "intuitionistic", and "Gödelian" mathematics within the former, according to a negative, positive, or zero value of the distance between finiteness and infinity. A mapping of Hilbert mathematics into pseudo-Riemannian space corresponds, therefore allowing for gravitation to be interpreted purely mathematically and ontologically in a Pythagorean sense. Information and quantum information can be involved in the foundations of mathematics and linked to the axiom of choice or alternatively, to the field of all rational numbers, from which the pair of both dual and anti-isometric Peano arithmetics featuring Hilbert arithmetic are immediately inferable. Noether's theorems (1918) imply quantum information conservation as the maximally possible generalization of the pair of the conservation of a physical quantity and the corresponding Lie group of its conjugate. Hilbert mathematics can be interpreted from their viewpoint also after an algebraic generalization of them. Following the ideas of Noether's theorem (1918), locality and nonlocality can be realized both physically and mathematically. The "light phase of the universe" can be linked to the gap of mathematics and physics in the Cartesian organization of cognition in Modernity and then opposed to its "dark phase", in which physics and mathematics are merged. All physical quantities can be deduced from only mathematical premises by the mediation of the most fundamental physical constants such as the speed of light in a vacuum, the Planck and gravitational constants once they have been interpreted by the relation of locality and nonlocality.

Keywords: energy conservation, Gödel mathematics, Hilbert mathematics, light and dark phases of the universe, locality and nonlocality, Noether theorems of conservation, Pythagoreanism, quantum information conservation

I THE PARAMETER OF DISTANCE BETWEEN FINITENESS AND INFINITY

The essential analogy of the relation of Hilbert mathematics and Gödel mathematics to that of non-Euclidean geometry and Euclidean geometry is introduced and widely utilized in the previous, first and second parts of the present paper. Another paper (Penchev 2023 March 13) elucidates why the analogy at issue is really essential: by the mediation of pseudo-Riemannian space of Einstein's general relativity and the mapping of entangled qubit Hilbert spaces into it. Then, one can represent the variable of the degree of entanglement as that of superluminal velocity. It will be now interpreted as the parameter of the "distance between finiteness and infinity".

One can notice that the locality of subluminal speeds, furthermore inherent for physics at all until now since it has obeyed the postulate of relativity for not exceeding the speed of light in a vacuum[1], can be reinterpreted to be infinitesimally small to the superluminal velocities. Then, one has and may use two descriptions of the same: however, the one inherent for physics, where the bound of the speed of light in a vacuum divides more or less artificially and conventionally *two intervals of a single finite variable*, but the other one is meant by mathematics since Newton and Leibniz's age so that the physical locality of the former subluminal is granted to be *infinitesimally small and only its derivatives are able to be finite*.

One can utilizes at least two different "Rosetta stones" for translating the two languages between each other, each of which very instructive, but in a proper way: the one absolutely unknown until now, the other one known as Robinson's nonstandard analysis (but not as a "Rosetta stone") for the translation at issue. Indeed, the relativistic rule of how one adds the velocities (e.g. notated as v_1 and v_2) of reference frames moving to each other by the relative speed v_0 (i.e.: $v_2 = \frac{v_1+v_0}{1+v_1v_0/c^2}$) implies for them to be non-Archimedean quantities in relation to the constant c (the speed of light in a vacuum) just as the actually infinitesimally small differentials in Leibniz's or rather in Robinson's manner to any finite quantity[2].

Actually, if the axiom is equivalently reformulated to the operation of addition rather than by its original formulation by multiplication (that: $\forall a, b \in R \wedge a < b, \exists n \in N: na > b$), the above rule implies for $\forall v_1$ and c to be non-Archimedean quantities since: $\forall v_1 \wedge v_1 < c, \nexists n \in N: nv_1 > c$. Then, all subluminal speeds obey inherently the limit of locality, i.e., the speed of light in a vacuum dividing them from all superluminal speeds being Archimedean ones. A still more precise formulation is that all subluminal speeds are non-Archimedean quantities *only to* all superluminal velocities rather than to each other, by the way, just as all differentials after Robinson (and more or less, after Leibniz) to all finite quantities.

So, the bound of light in a vacuum can be reinterpreted purely mathematically as distinguishing all Archimedean from all non-Archimedean quantities (meaning that the latter are non-

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[1] It can be (a little) ironically called "light physics" opposed to "dark physics" forthcoming in the near future.

[2] Really, if one well-orders the set of all finite quantities by virtue of the axiom of choice so that the set at issue possesses a single least element, that element would correspond to the constant "c" of the speed of light in a vacuum, and all superluminal velocities could map all finite quantities unambiguously, both being well-ordered.

Archimedean only to the former, but not to each other), or even more loosely: as a physical constant able to distinguish unambiguously both dual or complementary members of any pair of axiomatic systems differing from each other by a single postulate so that the one includes it, and the other one means idempotently its logical negation. Thus (and particularly), it is able to embody the bound of all local Leibnizian differentials to all "normal" finite quantities since the former ones confess the non-Archimedean "belief" unlike the latter. So, the "Rosetta stone" of Robinson's nonstandard analysis translates between subluminal and superluminal velocities by means of the Archimedes axiom being valid to all the latter, and invalid to all the former in relation to latter (but not to each other just as in the case of differentials in a proper sense).

The other Rosetta stone, both equivalent and alternative to nonstandard analysis, utilizes the Dirac δ-function being consistent to the standard infinitesimal quantities, i.e., after Cauchy's "standard analysis" rather than after Robinson's "nonstandard" one. So, one can mean it as a usual probability density distribution however defined on an infinitesimally small area as to all subliminal velocities being just the measure of that fussiness, though an infinitesimally small fussiness measured by a corresponding infinitesimally small reference unit so that the result of that measurement can be finite just as after Cauchy not involving any non-Archimedean quantities being redundant if that is the case.

So, one means the Dirac δ-function to be reinterpreted as a "normal" (that is "non-generalized") function or a probability density distribution however defined on an infinitesimally small area and thus differing from any "normal" probability density distribution (the characteristic function of which is a certain wave function in the mathematical formalism of quantum mechanics) only by the fact that the scope of its function argument variable is infinitesimally small (so that that the scope of its values is infinitesimally great[3]).

Then one can immediately notice that the corresponding wave function of the probability density distribution whether being infinitesimal or finite (i.e. that mean usually) is the same, but the two cases can be anyway distinguished from each other, granting to belong to the two dual, but identical Hilbert spaces whether the separable complex one of quantum mechanics or the qubit one of quantum information. In other words, that Hilbert space corresponding to subliminal speeds (thus in terms of relativity) means the quantity of position, and its dual, but identical twin refers to the quantity of speed, and properly to superluminal speeds, eventually renormalized to be equivalent to subliminal ones for being consistent with relativity.

As a conclusion, the utilization of the above reinterpretation of the Dirac function as a relevant "Rosetta stone" for the translation at issue implies the formalism of quantum mechanics and information able to ignore the distinction of superluminal and subliminal velocities therefore suggests a uniform description for both, unifying them by means of a more general principle of "quantum relativity"[4] introduced in other papers (e.g. Penchev 2023 March 13; 2021 June 8).

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[3] So that the integral of the Dirac function is finite, or conventionally in definition, a unit.

[4] In other and rather awkward words, the principle of quantum relativity is more general than the principle of general relativity, and thus it is that of a kind of "more general relativity".

This means: if one should describe the transition through the light barrier, the language of energy conservation (e.g., as in Pauli's particle paradigm or the Standard model) generates nonsense thus being irrelevant. Nonetheless, the language of quantum information conservation generalizing the former as a particular case among the latter, relevant also to all "dark", entangled and more or less coherent states, allows for a quite reasonable description of all physical processes transcending the light barrier in either direction, being an inherent subject of quantum information meaning entanglement. Even more, it may conjecture that general relativity describes the same, but in still one, already third language of locality by substituting the superluminal dimension by another second subluminal dimension resulting in the curvature of pseudo-Riemannian space (in detail in: *Penchev 2023 March 13*).

So, if one has two "Rosetta stones" able to translate the same text into two different translations into two divergent languages, a derivative third "Rosetta stone" is available, although being implicit initially: and just it is the subject of the present section tending to introduce the parameter of the distance between finiteness and infinity in various contexts in order to facilitate its use further. One can notice that each of both "Rosetta stones" above translates a certain physical quantity such as velocity into two different mathematical languages though the one of them, that of Hilbert spaces, is utilized by a fundamental experimental (thus being physical) science such as quantum mechanics.

Then, one may bracket (in a Husserlian manner) physical reality meant in the input text referring to the quantity of speed, whether "revolutionarily" and nonlocally superluminal or "classically" and locally subluminal, in order to build the derivative Rosetta stone able to supply a relevant translation between two languages both mathematical: the one being that of nonstandard analysis, and the other of the qubit Hilbert space. One can immediately reveal that the third and derivative Rosetta stone already exists, suggested by the paper of Alain Connes (1995). However, the objective of the present paper is rather philosophical or both philosophical and mathematical (in fact, linking it to the foundations of mathematics), relating the newly introduced third Rosetta stone to the initial input text of physical reality and generalizing it into reality at all, i.e., the reality meant by philosophy, for example to be "ontological".

So, the one output mathematical text "translating" the same input physical text describes Leibnizian differentials as non-Archimedean quantities (to the usual finite and Arcimedean quantities, but not to each other), on the one hand, and usual finite Archimedean quantities, on the other hand, thus also representable as two dual axiomatics differing from each other by the two idempotent logical alternatives of the same axiom, that of Archimedes (in the case, but also generalizable as above).

The other output, also mathematical is another translation of the same input texts referring to the unification of locality and nonlocality both distinguished and linked by the limit of the light speed in a vacuum, now in the mathematical language of quantum mechanics and especially, into its "dialect" of quantum information, i.e. in terms of the qubit Hilbert space. Then, both local and nonlocal quantities are represented uniformly as the same kind of qubit values (i.e., as quantum

information), however incommensurable simultaneously being belonging to the dual or complementary identical branches of the same qubit Hilbert space.

In other words, Bohr's complementarity as well as wave-particle duality both being inherent for quantum mechanics are now reinterpreted by the opposition of locality and nonlocality in a way also relevant to relativity (whether special or general). Then, measurement (as it is defined in classical quantum mechanics) being only local determines which of both "identical twins" of the same separable complex Hilbert space is "alleged" to be local by the preliminary preparation for a certain quantum measurement; that is: which of both conjugate quantum quantities is meant to be measured.

However, the third "Rosetta stone" targeted to translate between two thoroughly mathematical texts should "bracket" the above physical interpretation (however, only initially intending then to restore it even philosophically and ontologically following the talweg of the present paper). Though, the language of Leibnizian differentials means literally the pair of infinitesimally *small* and finite quantities, Robinson's nonstandard analysis (1966) demonstrates that it is isomorphic to the pair of finite and infinitesimally *great* quantities by virtue of the lemma of ultrafilters and the axiom of choice in the final analysis, since the former is a weaker version of the latter.

As the "distance between finiteness and infinity" in the present paper is meant just by the pair of finite and infinitesimally great quantities (as to the terms of nonstandard analysis), though coinciding mathematically with the description of the pair of infinitesimally small and finite quantities. So, the distance at issue is an infinitesimally great quantity of nonstandard analysis, however represented by a qubit value of quantum information after utilizing the third "Rosetta stone" or as a certain wave function corresponding unambiguously to a certain probability (density or not) distribution as to the terms of the separable complex Hilbert space or classical quantum mechanics.

Then and following the essential analogy or reinterpretation in the concepts of pseudo-Riemannian space (justified in detail in another paper: *Penchev 2022 February 4*), one can speak of "concave", "convex", or "flat" mathematical ontologies according to the "positive", negative or zero distance of finiteness to infinity: accordingly. They correspond to three different schools for the foundations of mathematics: intuitionistic, "dialectical" or paraconsistent, and "classical" or "Gödelian" (i.e., accepting his dichotomy about the relation of arithmetic to set theory: "either incompleteness or inconsistency").

In other words, they can be distinguished between each other by the relation of the law of excluded middle (or that of noncontradiction sometimes called also sometimes the "law of contradiction") to the opposition of finiteness and infinity. Both laws are valid in propositional logic. It may be anyway shared by the three schools though they historically generated quite different kinds of attitude to classical propositional logic granted to be thoroughly valid only by the "classical" (or "Gödelian") school.

Intuitionism suspends it only in relation to the special case of the opposition of finiteness to infinity, or more exactly, even only the law of excluded middle, not touching directly that of noncontradiction since no mathematical entity can be finite and infinite simultaneously. On the

contrary, the dialectical or paraconsistent approach to the foundations of mathematics tends to replace propositional logic following the Hegelian philosophical tradition though the opinions about which exactly the new "zero-order" logic might be and are various, furthermore exchanging directly the law of noncontradiction and admitting consistent propositions for which it is not valid: and thus "dialectical entities" to which they are relevant.

The present paper by introducing the distance between finiteness and infinity is closest to the intuitionist idea but generalizing it: propositional logic is conserved to be universally valid as the relevant zero-order logics for all mathematics, however the parameter at issue is involved so that its positive values are to be related to intuitionistic mathematics, the zero value to Gödelianism, and negative values as to "dialectical school" granting an area of overlapping finiteness and infinity, in which the law of noncontradiction is not valid, but only to them remaining true in all other cases. In other words, the innovation touches only a special numerical variable introduced to describe quantitatively the relation of finiteness (arithmetic) and infinity (set theory) rather than the "throne" of propositional logic.

Furthermore, one can immediately notice that the distinction of the class of ontologies of positive distances to that of negative distances is rather conventional by virtue of the duality or equivalency of the laws of contradiction and excluded middle in propositional logic. There is a nonzero area of "no man's land" in the former cases which can be interpreted not worse to be "shared" as to the latter cases, to which only the case of a zero area therefore excluding any "territorial disputes between the countries of finiteness and infinity" can be opposed.

The main idea of the present paper consists in the conjecture that the eventually disputable area of "no man's land" supplied by the statute of "shared management"[5] should be identified with all the physical world, i.e the universe as a particular case of Hilbert mathematics generalized to the usual understanding of it, reducing it to the "zero" or "flat" case of Gödelian mathematics in turn isomorphic to Gödel mathematics. The physical world can be absolutely opposed only in relation to Gödel mathematics and following the episteme of Cartesianism. On the contrary, it may be interpreted to be a particular case in the framework of Hilbert mathematics, therefore arising "ex nihilo" by itself, i.e., by virtue of mathematical laws[6].

Only if one tends to project Hilbert mathematics on the screen of the "wall of Plato's cave", as a relevant metaphor of Gödel mathematics, the usual (in fact, mythical) picture of the "Bing Bang" would appear. Our worldview is crucially incomplete for granting the dogma of Gödel mathematics, therefore remaining the "dark phase" of the universe fundamentally incognizable by

[5] For example, though rather as a joke, as Andorra being the condominium of the French president and the bishop of Urgell.

[6] The problem of the "creation ex nihilo", including the universe" is favorite for the theology (e.g. Anderson 2018; Tanner 2013; Quilter 2010; Clavier 2012; Laraudogoitia 1998; Senor 1993; Morris 1983) ostensibly demonstrating God's existence. Furthermore, it is utilized as a more or less metaphorical synonym of the Big Bang therefore hinting at a pejorative attitude to it (e.g., Cruz, da Silva 2020; or not (e.g. Lincoln, Wasser 2013). If the latter is the case, the expression "creation from nothing" is often preferred (e.g., He, Gao, Cai 2014; Khosravi, Sepangi 2009; Barvinsky, Kamenshchik 2006; Minn 1990; Carroll 1988; Vilenkin 1982) as well as in philosophical papers (e.g. Neville 1980) However, the "creation from nothing" may be sometimes used in theological papers as well (e.g. McFarland 2019; 2014; Selman, 2002).

the identification of all the universe with its "light phase" being only visible on the "wall" at issue. Mathematics and physics are the same in the dark phase of the universe, at the same time being fundamentally inaccessible to both Cartesianism and Gödel mathematics in their unity.

II THREE KINDS OF HILBERT MATHEMATICS ACCORDING TO THE DISTANCE OF FINITENESS AND INFINITY

Speaking quite loosely, one might state that the physical world is the particular case of any nonzero "curvature" of mathematics or the mathematical world; or that the universe appears if the general (and thus comprising it) mathematical world is divided into overlapping ("entangled") parts such as "finiteness" studied by arithmetic and "infinity" for set theory.

A necessary condition is the reinterpretation of the Gödel incompleteness statement (1931) from a theorem deducible from the axioms of arithmetic, set theory, and propositional logic into an independent axiom as this is demonstrated in detail in the first part of the paper. The analogy to the Fifth postulate of Euclid is essential rather than only superficial following the same first part or another paper (Penchev 2022 October 21; 2023 March 13) considering an entanglement theory of quantum gravitation[7] by the mapping between Einstein's general relativity[8] and the superluminal domain as what entanglement is interpreted by the mediation of quantum information.

That reinterpretation of the Gödel incompleteness statement (1931) can be visualized by realizing a bit of information as two oppositions rather than as only one following the usual prejudice to it. Those two oppositions are: (1) that of the two equally probable alternatives of a bit of information, to which the prejudice at issue restricts its understanding; (2) a preliminary choice between the state before choice and that after choice meant by (1). So, (2) is a necessary condition of (1).

Then and analogically, the Gödel incompleteness statement (1931) can be reinterpreted as an axiom since the opposition of finiteness by arithmetic and infinity by set theory is presupposed by their opposition in advance versus the state before the same opposition, or in "Eden" and before the "original sin to be consumed" (i.e., before the "apple" of the opposition of finiteness and infinity to have been "eaten" by Cantor's set theory). The second part of the paper (Penchev 2023

[7] The "matter-gravity entanglement hypothesis" suggested by Kay (2018) can be interpreted analogically as well as Kwon, Jang, Kim, and Tolla's approach (2018) for "gravity for entanglement" or Lee, Kim, and Lee's "gravity as a quantum entanglement force" (2015) linkable by themselves (2007) to dark energy. The paper of Moffat (2004) interprets Einstein's gravity as the local mapping of entanglement. Neuman (2013) links gravity and field theory by "action and entanglement". Following the same direction, one can mention the papers of Simonov, Capolupo, Giampaolo (2019); Svesko (2019); Ruiz, Giacomini, Brukner (2017); Varadarajan (2016); Obregón (2015); or Peres (2004)

[8] For example, papers (e.g., Pitts 2022; Duerr 2019; Maudlin, Okon, Sudarsky 2019; Hynecek 2018; Schäfer 2018; Wu 2016; Epp, McGrath, Mann 2013; Petrov, Lompay 2013; Yarman 2006; Brading 2005; Mensky 2004; Yoon 2004; 2001; Bondy 1990; Nissani, Leibowitz 1989; 1988; Kyrala 1981; Rylov 1963) consider conservation and nonconservation in general relativity, after which action conservation versus energy non-conservation can be meant as local counterparts of quantum information conservation. The paper of Fischer (1982) discusses conservation laws in general relativity linked to those in gauge theories and elementary particle physics, and that of Haucking (1970): "The conservation of matter in general relativity".

January 3) demonstrates that Russell's logicism to the foundation of mathematics, following the ancient precedent of Aristotle's ontological revisionism to Plato's doubling of all "things" by their "ideas", can be identified as that "Eden" before Cantor's "actual infinity".

The subject of the present, third part of the paper means, figuratively speaking, to describe "Paradise" as it has been "lost" and after the "Fall", as if from the "viewpoint of Adam and Eve already living on Earth", no more "in Eden". Abandoning the Bible metaphor, this means the description of the naive or native state in mathematics before Cantor's actual infinity from the contemporary worldview of mathematics by the explicit opposition of finiteness (arithmetic) and infinity (set theory). Then, the "zero intersection of finiteness and infinity", in fact presupposed in advance as an axiom and only ostensibly proved after that in the Gödel incompleteness paper (1931), is only an option along with that of intuitionistic or dialectical mathematics as they are introduced and distinguished above.

The case of the nonzero intersection of the two alternatives of a bit of information (as the Planck constant needs physically, and this will be justified in detail in the next section) is equivalent, which can be granted as the two dual counterparts of a qubit of quantum information. Following the same approach, quantum information is analogically interpreted as originating from classical information. In other words, the initial (primary) choice or opposition is that between quantum information and classical information after and only after which the secondary choice or opposition between the two alternatives of a bit of information is already possible.

Speaking again figuratively, "information in Eden" is quantum information, and "information on Earth and after the Fall" is classical information. Furthermore, quantum information (i.e. "heavenly", "divine" information) is immediately the physical quantity of action by virtue of the Planck constant unlike classical information which can be transformed into physical action only by the mediation of humans. This is an interpretation of the Bible "Word" being in the beginning by Goethe's Faust stating for the "dead in the beginning" in a way merging both so that "In the beginning was the Word and the Word was the Deed" as if after John's Gospel and Goethe's Faust simultaneously or only after the former if "God" is substituted by the "Deed": "In the beginning was the Word, and the Word was with the Deed, and the Word was the Deed" instead of the original: "In the beginning was the Word, and the Word was with God, and the Word was God".

The relation of Hilbert mathematics to Gödel mathematics can be interpreted in two ways. In the one of them, they are opposed to each other according to the logical status of the Gödel incompleteness statement (1931): either an axiom in the former or a theorem in the latter. In the other approach, Gödel mathematics is a particular case of Hilbert mathematics according to the zero value of the parameter of the distance between finiteness and infinity. One can compare with the relation of Euclidean and non-Euclidean geometries. Logically, they can be opposed to each other by the Fifth postulate where the former grants it, and the latter accepts its negation. However, Euclidean geometry can be also considered to be a special, zero particular case among all non-Euclidean geometries after Riemann's parameter of space curvature has been introduced.

If one follows literally the analogy with the relation of Euclidean geometry to its non-Euclidean counterpart(s), Hilbert mathematics is to be opposed to Gödel mathematics according to one

metamathematical axiom (or "meta-axiom") about whether the Gödel incompleteness statement (1931) is an axiom or a theorem correspondingly. In fact, that meta-axiom is a certain interpretation of the class of axioms (or eventually, meta-axioms as in the case at issue) among which can be also included that about a bit of information (either one opposition or two oppositions) or that of the nonstandard bijection (either a bijection or not).

The three kinds of Hilbert mathematics (namely, the "hyperbolic" intuitionist mathematics, the "spherical" "dialectical" mathematics, and the "straight" "Gödelian" mathematics) can be distinguished only by the parameter of the distance of finiteness and infinity. Furthermore, the former two ones can be unified as "curved" cases versus the third one of zero curvature. The essential analogy to pseudo-Riemannian space as the fundamental mathematical formalism of general relativity is mentioned above and justified in the first and second part of the paper[9]. However, it can be also continued further following the approach of the unification of locality and nonlocality by pseudo-Riemannian space and thus by the gravitational theory of general relativity (Penchev 2023 March 13).

Then, the "straight" "Gödelian" mathematics (being isomorphic to Gödel mathematics) can be situated "on the light cone" (i.e. to be associated with the newly introduced "light ether" of special or general relativity) now reinterpreted to be a new absolute "reference frame" (though neither special nor general relativity allow for any reference frame to be linked to it, besides in some generalized sense) as the exact bound between locality and nonlocality and between finiteness and infinity correspondingly. The special local mathematics and the nonlocal hyperbolic or spherical mathematics can be unified following the approach of general relativity by pseudo-Riemannian space and thus both being interpreted only locally though in a rather ontological or metamathematical sense.

Then, gravity can be reinterpreted ontologically or meta-mathematically as well. It is already fundamental by generating all the physical world only in the framework of mathematics, i.e. as a certain class of mathematical structures not belonging to the "flat" Gödelian mathematics (or respectively, to Gödel mathematics), and consequently within a kind of neo-Pythagoreanism[10]. An essential analogy to pseudo-Riemannian space utilized by general relativity to describe gravity is relevant.

III DIALECTICAL MATHEMATICS, OR HOW THE DIALECTICAL CONTRADICTION OF FINITENESS AND INFINITY IS THE PHYSICAL QUANTITY OF ACTION

So, Hilbert mathematics (unlike Gödel mathematics) postulates that the physical world originates from mathematics being a particular mathematical structure determining a class of possible physical worlds, among which our universe is featured (and thus chosen) by certain values of the three fundamental physical constants: namely, the speed of light in a vacuum, the Planck

[9] One can relate the "flatness problem" of the universe to quantum gravity (Padmanabhan 1983).

[10] For example, papers (e.g., Henderikus 2006; Zhmud' 1989; Stapleton et al. (1958) consider *Pytha*goreanism in a contemporary context as well as Kemp (2017): to Quine; Martin (1997): to the properties. The original Pythagoreanism in the context of Greek philosophy is investigated e.g., in the collection of papers, edited by Schofield (2013).

constant, and the gravitational constant. Their exact values as to our universe are not a subject of Hilbert mathematics. Anyway, the three constants determine unambiguously the relation of locality and nonlocality regardless of their certain values as the only mathematical justification (and thus synonymically, "source", "reason", or "cause") of the physical world.

The speed of light in a vacuum is the exact bound of locality and nonlocality. Thus, it can be linked to the absolute reference frame or the immobility attached to the classical concept of the "ether", now modified to be that of electromagnetic field. Indeed, the classical "ether" meant an imaginary "zero" speed which cannot be defined in any way after special or general relativity since it refers differently to various reference frames in relative motions to each other. There does not exist any sharable property of them able to serve for the definition of the classical "ether".

Speaking rather loosely, one might say that the classical ether cannot be rigorously defined to be immovable: any zero speed cannot be zero to all reference frames. On the contrary, the boundary between locality and nonlocality, as the speed of light in a vacuum is interpreted, is the same to all reference frames and thus allows for the newly introduced "light ether" to be the necessary universal property shareable by all reference frames without any exception: by all inertial or non-inertial reference frames of special or general relativity as by all discrete or external reference frames necessary for quantum relativity (e.g., defined in Penchev 2021 June 8) and the generalized relativity able to describe the area of nonlocality consistently to that of locality.

Once any certain value is attached to the boundary of locality and nonlocality, any finite curvature unifying both cases of positive, "hyperbolic" or "intuitionistic" and negative, "spherical" or "dialectical" curvature features the particular case of the physical world generated as if "ex nihilo" (at least in a physical sense[11]) only by virtue of mathematical laws and necessity. That finite curvature implies quantum information, entanglement, all the "dark phase of the universe" (in which mathematics and physics are also "entangled", speaking rather figuratively), on the one hand, and a finite boundary of the minimal possible physical action (which is specified by the Planck constant in our universe), on the other hand, furthermore being shared by both "dark and light phases of the universe".

The light phase of the universe can be distinguished from its dark counterpart by energy conservation, locality, unitarity, etc.[12], and by all featuring physics until now in the final analysis since physics has always been identified with the physics of the light phase of the universe. Then, what is nonlocal is anyway represented locally by means of Einstein's general relativity. Then, the gravitational constant determines how both reference units are to be related as a ratio to each other for nonlocality to be able to be mapped locally as the corresponding variable curvature in any point of pseudo-Riemannian space furthermore according to the certain value of gravitational field acting on it.

One can summarize more or less loosely that the physical world can be generated from Hilbert mathematics by virtue of any finite non-zero curvature, regardless of being negative or positive, from the relation of locality and nonlocality as to the physical viewpoint or by the underlain it

[11] For example by means of "infinity machines" (e.g., as in*: Laraudogoitia 1998*).
[12] For example as in the classical paper of Wigner (1954).

mathematical relation of finiteness and infinity by the mediation of that of infinitesimal quantities (whether infinitesimally small or infinitesimally great) to finite ones. On the contrary, the pure case of Gödelian mathematics in the framework of Hilbert mathematics is "flat" or featured by "zero curvature". It is also isomorphic to Gödel mathematics as contemporary mathematics until now or in the framework of the organization of cognition in Modernity.

The concept of curvature is rather mathematical and related to the class of vector spaces, to which pseudo-Riemannian space utilized by general relativity or the separable complex Hilbert space of quantum mechanics belong in particular. Any finite nonzero curvature corresponds to a certain mismatch of the two dual spaces[13] versus the case of their exact coincidence if the curvature at issue is zero. Since gravity correlates with pseudo-Riemannian space after Einstein's general relativity one can speak of it in any case of curvature defined on a certain vector space as originating from the mismatch of its two dual counterparts and always tending to remove their discrepancy, to reset the corresponding curvature into zero, or in terms of Hilbert mathematics: to reduce any of both general cases of non-Gödelian mathematics to Gödelian one, i.e., to cancel the physical world in the final analysis, transforming it to be mathematical in the narrow sense of Gödel mathematics.

If one utilizes the metaphor of "love" for "gravity" since both can be thought as "forces of attraction" (at least figuratively), the above generalized and mathematical understanding of gravity can be likened to Freud's "death drive", i.e., destroying the physical existence by annihilating its inherent and definitive contradiction. Nonetheless, gravity or "love" can be figuratively interpreted as the opposite force of creation in relation to the physical world generating it "ex nihilo" in virtue of mathematical laws and necessity (after the present consideration), realizing both as if in the manner of Newton's "third principle": as the identification of a pair of forces directed oppositely, surely rather as a figure of speech than in a rigorous mathematical meaning.

Anyway the case of "hyperbolic" intuitionistic mathematics can be opposed to that of "spheroidal" dialectical mathematics in the framework of Hilbert mathematics in a sense. For example, the Planck constant can be interpreted in a rather intuitionistic way as "no man's area" compromising the entire light cone and separating locality and nonlocality so that direct conflict can be avoided by the limit of the minimal, physically possible action. Nonetheless, the quantity of action itself, therefore exceeding the threshold at issue can be relaxed as a still unresolved contradiction once it has overcome that crucial "doorstep" in the manner of Hegel's ontological "dialectical logic" generating also a physical or "procedural" period of time, during which the "litigation" of both sides can be decided by an ultimate sentence in favor of the "plaintiff" or the

[13] The same mismatch though being actual (locally) can be no less interpreted non-locally as information and a relation of (probability or not) density distributions. In fact, papers (e.g. Jacobson 2012; Carol 2005) links gravity and information (or entropy; or Einstein's gravity and thermodynamics as *Mäkelä, Peltola 2009*). One may mention Eric Verlinde's theory as well as a forthcoming intention in a future paper to be reinterpreted in terms of the entanglement theory of gravity.

"defendant" in the contradiction at issue or by any agreement of them[14]. The juridical metaphor can be continued as to energy conservation as the immutability of the "claim" in the course of the "judicial process" as a necessary condition for its consideration and decision in the final analysis in any of the enumerated ways. Then, the Planck constant is the natural low limit of the "interest in the litigation", after which one can "lay claim".

IV INFORMATION AND QUANTUM INFORMATION INTRODUCED ON THE GROUND OF THE FIELD OF ALL RATIONAL NUMBERS AND SET THEORY WITHOUT THE AXIOM OF CHOICE

Before continuing to the unification of the dimensionless quantity of quantum information and the dimensionful quantity of physical action by the fundamental Planck constant, information and its kinds of classical (or "finite") information and quantum (or "infinite") information are to be defined set-theoretically, i.e. in the foundations of mathematics rather than usually, as a rather technical property of messages for communication. though eventually interpreted as widely as Norbert Wiener's cybernetics does. In other words, a more fundamental understanding of information than that already available in cybernetics is necessary for the objective and objectivity of investigating the unity of physics and mathematics featuring the "dark phase of the universe".

That fundamental and mathematical introduction of information and quantum information is relevant to the intention of Hilbert mathematics to overcome the restriction of Gödel mathematics and particularly the Gödel dichotomy about the relation of arithmetic and set theory ("either incompleteness or contradiction") by an information bijection embedded in the definition of a bit of information (called also "nonstandard bijection" because of its special property for the straight and inverse mapping to be dual or complementary to each other).

The property at issue can be traced back even to the general idea about the conservative generalization of any proposition to its logical negation avoiding any contradiction, but at the cost of some relevant generalization of the law of noncontradiction or that of excluded middle, however now restricted exceptionally only to the relation of finiteness and infinity (particularly relating to the Gödel dichotomy) or involving the parameter of the distance between them a way recollecting Riemann's space curvature to the class of all non-Euclidean geometries in order to comprise Euclidean geometry as a special case of zero curvature.

The key idea for avoiding contradiction between arithmetic and set theory consists in the derivative inference of Peano arithmetic from the field of all rational numbers (in fact as an sufficient exemplification of any field after the theorem corresponding to the axiom of choice in the usual approach) rather than vice versa. The field of all rational numbers can be exhaustively defined by two linked Abelian groups obeying a single distributive law and transformed immediately into Boolean algebra (i.e. homomorphic to propositional logic as the universally shared zero-order logic of mathematics at all) only by adding the second distributive law. In other words, The field of all rational numbers can be simply defined by means of removing the one

[14] The conflict of locality and nonlocality can be also illustrated by the "interaction-free measurement" (Rohrlich, Aharonov, Landsberger 2018), by the "pure-state identification problems" (Ishida, Hashimoto, Horibe, Hayashi 2008), or a two-proton EPR experiment (McWeeny, Amovilli 1995).

distributive law from Boolean algebra or respectively the dual counterpart of the field of all rational numbers complementing it to propositional logic.

Due to their divergent historical origin, propositional logic and arithmetic are standardly alleged to be quite different and independent in default. In fact, the present approach elucidates that they are "close relatives" algebraically, ever siblings, together with set theory allowing for the following less or more metaphorical "genealogical motto": "arithmetic is the half of propositional logic", which in turn is "the zero-order twin of set theory", being the homomorphic class of all possible consistent first-order logics or mathematical theories. Arithmetic, logic and set theory is the "ruling dynasty of all mathematical theories": their "blood ties" are demonstrated by their "common origin" from information and quantum information if one approaches them algebraically, by the mediation of the field of all rational numbers. In turn, information and quantum information can be interpreted to be "siblings" of the quantity of action and the physical world at all[15]. Then, the dark phase of the universe is the common "parent", from which both mathematics and physics originate.

The present section will represent the above idea only sketched by the "genealogical" metaphor in a more rigorous way relevant to the foundation of mathematics so that the next section will be able to reinterpret Noether's theorems (1918) in the new context of Hilbert mathematics.

One can consider the field of all rational numbers consisting of two linked commutative groups: additive and multiplicative, each of which possesses its own unit element, denoted correspondingly by "0" and "1". Then, one can construct Peano arithmetic of all natural numbers N on the ground of the field of rational numbers by introducing the unary function successor "$next\ N = N + 1$" therefore combining the operation of the additive group "+" with the unit element element of the multiplicative group, "1": thus essentially involving both groups of the rational field for the definition of arithmetic. Then, any bijection of the set of all natural numbers and any other infinite set can be interpreted as a well-ordering of the latter set without the axiom of choice to be included and thus valid also in any axiomatic system in which neither the axiom of choice neither the well-ordering theorem participate.

Furthermore, one can consider the additive group of all integers, all of which belong to the rational field. All integers are an infinite set and it can be well-ordered utilizing any bijection with the set of all natural numbers as above (i.e. without needing the axiom of choice). Both additive semigroups of the two subsets of all positive integers and all negative integers are naturally well-ordered in two ways anti-isometric to each other. One can examine the natural bijection "א" of the set of all natural numbers "N" and the set of all pairs, the two members of which are "namesake" or rather "numbersake" and belong to the set of all positive integers and the set of all negative integers correspondingly:

$$\forall n \in N, \forall(-n) \in (-1), \forall(+n) \in (+1), א(def) = n \leftrightarrow (-n, +n)$$

[15] For example, Lie and Hyunseok (2020) link quantum information and information conservation laws. Roncaglia (2019) considers the "conservation of information in quantum physics". Luo (2010) means information conservation in quantum measurements.

Any element of "א", such as a "$n \leftrightarrow (-n, +n)$", defines a "bit of information", and the mapping of any finite set (or particularly, finite strings, i.e. finite well-ordered sets or respectively vectors if those sets can be identified as elements of any vector space) onto some finite subset of "א" is the quantity of information unambiguously assignable to the former set (respectively, "string", or "vector").

Then, one can extend the concept of information to infinite sets, which is notated as "quantum information"[16] only by the substitution of all natural numbers (i.e. in Peano arithmetic, here inferred as a structure, derivative from the field of all rational numbers to which the axiom of induction is added) with the set of all natural numbers (i.e. in set theory, in which the axiom of induction is replaced by its logical negation, what the axiom of infinity is: e.g. in *Penchev 2022 October 21*[17]); that of all positive integers by the *set* of all positive integers; that of all negative integers by the *set* of all negative integers. Those substitutions can be accordingly notated as:

$$N \to \{N\};\ (-I) \to \{(-I)\};\ (+I) \to \{(+I)\}.$$

That is: $\forall n \in \{N\}, \forall(-n) \in \{(-I)\}, \forall(+n) \in \{(+I)\}, \Im(def) = n \leftrightarrow (-n, +n)$.

The introduction of the concepts of information and quantum information allows for preventing the Gödel dichotomy of the relation of arithmetic to set theory in the following rigorous meaning. Though it is still valid to the three substitutions above, "$N \to \{N\};\ (-I) \to \{(-I)\};\ (+I) \to \{(+I)\}$", the derivative substitution "$\text{א} \to \{N\}$" overcomes that "either incompleteness or contradiction", which can be notated just as "$\text{א} \leftrightarrow \{N\}$. In other words, classical information being inherently finite is able to be neither incomplete nor contradictory to the set of all natural numbers, even nor to quantum information, i.e. to the quantity of classical information in relation to the set of all natural numbers,

This means that the concept of information is able to bridge finiteness and infinity as well as physics and mathematics, surprisingly even the Cartesian "body" and "mind" after a more philosophical reflection. Speaking loosely, information and quantum information are the same though the former is inherently finite, and the latter is fundamentally infinite. As it happens, one can conjecture the converse statement, namely that information can be defined by the unification of finiteness and infinity, for example, after the fact that the unification of finiteness and infinity represents only a bit of information so that the one alternative of it is "finiteness", and the other is "infinity".

The way of information not to be incomplete to the set of all natural numbers can be sketched briefly so. One can assign either "true" or "false" to all resolvable statements featured by a corresponding finite Gödel number. Nonetheless, one may attach the finite ordinal number of a bit of information being both "true" and "false" to any Gödel insoluble statement. So, the Gödel

[16] The equivalence of the standard definition of quantum information as the counterpart of "wave function" in the qubit Hilbert space and its definition as the conservative generalization of information as to infinite sets and series is suggested for the first time in another paper (Penchev 2020 July 10).
[17] For the first time in: Penchev 2016.

number is a natural number as to all soluble statements, but the finite ordinal number (i.e., a natural number as well) of a bit of information as to all insoluble statements.

Furthermore, one can elucidate the mutual equivalence of the introduction of whether the nonstandard bijection or information where the Cartesian product of all natural numbers with themselves is represented explicitly in the former case, and implicitly, by the "diagonal of the Cartesian product" in the latter case. However, the Carteseian product of all natural numbers with themselves is equated to all natural numbers in both cases. One can essentially illustrate the fact that information and quantum information are the same by identifying the two diagonals of the Cartesian product, where quantum information is anti-isometric to information, or notated more or less loosely, by the symbol of infinity "∞" as to arithmetic or to all natural numbers, but without involving the set of all natural numbers, "{N}":

$$\forall n \in N, \forall(-n) \in (-I), \forall(+n) \in (+I), \Im(def) = n \leftrightarrow (\infty - n, +n)$$

So, both tools of the nonstandard bijection and information involved already in the foundation of mathematics are able to build the conceptual ground for the intended unification of finiteness and infinity or respectively overcoming the Gödel dichotomy about the relation of arithmetic and set theory, furthermore relevant to the opposition of Gödel mathematics versus Hilbert mathematics, which is the main subject of the present paper. One is to distinguish the solution by the unification of finiteness and infinity (meant here) from the ontological and intensional decision of the same problem traceable back to Aristotle or embodied in the modern approach of logicism to the foundations of mathematics as in Whitehead and Russell's "Principia mathematica"[18].

Speaking figuratively, Aristotle's ontological decision just as logicism's solution to the foundation of mathematics tend to return back to the state of "Paradise Lost", i.e., before "the original sin to have been consumed", before "the apple to have been eaten" whether by Plato's opposition of things versus ideas in the former case or by Cantor's set theory contrasting explicitly finiteness to infinity by the express concept of "actual infinity". In other words, logicism prevents the choice of either "finiteness" or "infinity" putting a stop to the choice itself by giving itself up.

Husserl's "epoché" to reality can be interpreted as a philosophical generalization in relation to the more special problem about the foundation of mathematics, which is meant by logicism immediately, therefore restoring in a hidden form the ancient ontological solution of Aristotle. For emphasizing the similarity of Husserlian approach, one can introduce an analogical concept of "epoché to infinity" to feature the main idea of logicism though in an unexpected way contradicting common sense's prejudice (Penchev 2023 January 3) and even to illustrate it by the "lost solution" of Fermat's last theorem claimed by himself (Penchev 2021 March 9) therefore corresponding to the state of logicism's "Paradise Lost" in turn tending to cancel the "original sin" of Cantor's actual infinity by its intensional and logical unification with finiteness.

The distinction of the approach of the present paper from the idea of ontology or logicism is necessary for its realization to be an alternative to the foundations of mathematics, on the one hand,

[18] The opposition of the intensional and extensional solutions at issue is the main subject of investigation in the previous second part of the paper.

and the unification of information and physical action, on the other hand. It is also embedded in the project of Hilbert arithmetic (e.g., Penchev 2021 August 21), after which infinity is interpreted by doubling finiteness (or philosophically and theologically, "finitude"): that is as a second finiteness gapped from the former to be independent of it. The same doubling of finiteness or Peano arithmetic can be traced in both information introduced in set-theoretical manner as above or in the nonstandard bijection.

Adopting the nonstandard bijection in set theory, the fundamental philosophical idea of duality (relative to complementarity in quantum mechanics) underlies the foundation of mathematics. It can be expressed in the new context as generalizing the bijection as to quality in a way consistent to propositional logic unlike the approaches involving any "non-classical logic" and traceable philosophically still even to Hegel's dialectical logic. The nonstandard bijection can be also interpreted to introduce infinity implicitly, e.g., after Dedekind's definition of infinity by the bijection of a set (claimed to be infinite) with its true subset. Then the nonzero complement of the true subset to the set at issue can be defined to be dual to the former subset.

In other words, the non-excluded middle of the dual subset as above is a special and sufficient property to define the concept of "actual infinity", slightly paraphrasing the idea of intuitionism: not only the rule of excluded middle is irrelevant to infinite sets, but furthermore the invalidity of the excluded middle is sufficient for determining any set to be infinite, or by its nonzero dual counterpart implied if a bijection is nonstandard. The advantage of the nonstandard bijection to actual infinity consists just in the explicit reference to the difference between finiteness and infinity, on the one hand, and then identifying that complement of finite to infinity as a second finiteness, but dual to the former one.

As to physical action, it originates from that "doubling of finiteness" by itself and once again "by itself", now by its dual counterpart. In other words, the definition of the nonstandard bijection implies the quantity of physical action as equivalent to information, on the one hand, and to mathematics, on the other hand, even to arithmetic in the meaning in which the definition of the nonstandard bijection means the reducibility of mathematics to it.

Indeed, if the nonstandard bijection is defined as to Hilbert arithmetic in previous papers (e.g. Penchev 22 October 21; 2021 March 9) as "$(P^+ \otimes P^- \leftrightarrow P^0) \leftrightarrow P$" , the physical action can be associated with the product of any two elements, i.e. natural numbers, each of which belongs to the one of the two dual anti-isometric arithmetics (notated as "P^+, P^-"). Then, the Planck constant can be interpreted as the fundamental "curvature" of the Cartesian product "$P^+ \otimes P^-$" so that its corresponding elements, i.e., relevant to the definition of information, are neither orthogonal nor infinitesimally orthogonal to each other since the Planck constant is the finite minimal possible projection on the one onto the other.

As far as the curvature of the Cartesian product "$P^+ \otimes P^-$" in turn can be realized as "entanglement", the physical equivalent of physical action to non-physical mathematical structures is due to entanglement. The "dark", both physical and mathematical phase of the universe is a necessary condition for its "proper" and only physical phase identified to be all physics until now to which mathematics is correspondingly and inherently Gödel mathematics, a position of

mathematics arising naturally from the Cartesian constitution of cognition in Modernity. Noether's theorems (1918) can be deduced from the same observation, in detail in the next section.

V ACTION AFTER EMMY NOETHER'S THEOREM (1918), NOW REINTERPRETED IN DIALECTICAL MATHEMATICS

The dimension of the "distance between finiteness and infinity" (which can be also interpreted quite loosely as the "distance between physics and mathematics" and even as that between "subject" and "object" philosophically) is quantum information mathematically, and quantity of action physically (what is the dimension of the Planck constant particularly). Thus, the physical quantity of action is fundamental by virtue of the fact that the physically dimensionless "quantum information" is equated just to "action" due to the dimension of the Planck constant.

Furthermore, this turns out to be the equating link between physics and mathematics eventually regulated by the most fundamental law of conservation[19] including to the most general case of the "dark phase of the universe" where energy conservation is not more valid since it can be related (even definitively) only to its "light phase" restricted to about 4-5% of the entire mass and energy of the universe according to all contemporary astronomical measurements of them. So, the light phase of the universe where time and energy can be discernibly distinguished between each other is rather an exception, a markedly particular case, though not only all physics but even all natural science until now has been limited within its framework alone.

If one reflects the same restriction philosophically, it is due to the necessity of repeatability and empirical or experimental verifiability featuring natural science without any exception and thus physics as an essential part of it. Any empirical observation, any experiment and even any thought experiment (e.g., Einstein's "Gedankenexperiment") obey it and thus they are interpreted to be local and causal. Indeed, just Einstein noticed that quantum mechanics admits "experimental nonlocality" seeming to be a "fallacy in definition" if the adjective "experimental" is a synonym of "local" as the repeatability and empirical or experimental verifiability are imposed to be a rule without any exception as to natural science[20]. Nonlocality would imply what he denounced as "spooky actions at a distance" essentially linked to his other very picturesque metaphor: that about "God playing dice", thus rejecting the requirements of repeatability advocated by him[21].

Even much more, he together with Podolsky and Rosen (1935) suggested a thought experiment in his manner to demonstrate in a rigorous and quantitative way (rather than by metaphors) that quantum mechanics implies nonlocality, and thus if one accepts the principle of repeatability and experimental verifiability (since the latter can be only local after natural science in his age),

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[19] Chiribella and Scandolo (2015) mean conservation of information analogically in relation to the foundations of quantum mechanics as well as Deshmukh and Libby (2010) in relation to atomic and subatomic physics. One can also consider higher order conservation laws after a relevant generalization of Noether's theorem (Cheung 1987) as well as Schaft (1981); or "inverse Noether's theorem" (Rosen 1980).
[20] Pavičić (1990) considers a "relative frequency criterion for the repeatability of quantum measurements".
[21] For example, following counterfactually the new arguments of Bohr to Einstein (Cavalcanti, Wiseman 2012).

quantum mechanics is inherently incomplete because of the absence of those additional restrictions of locality as natural science needs in the three authors' opinion.

On the contrary, the 2022 Nobel Prize for entanglement and quantum information (e.g. Penchev 2023 March 13) introduces quantum correlations to be recognized as an absolutely credible scientific fact therefore admitting exceptions or rather generalizing the principle of repeatability and local verifiability in natural science. Indeed, repeatability can be conserved if it is related to probability (density or not) distributions rather than to single events or experiments. Locality cannot be saved in any way, though: at least unless one accepts that nonlocality can be equivalently mapped locally by Einstein's general relativity or by the mediation of an entanglement theory of quantum gravity[22].

The same generalization of the principle of natural science about repeatability and local verifiability can be illustrated by an eventual deduction of quantum-information conservation (thus as a generalization of energy conservation) by means of the first theorem of Emmy Noether (1918). One can assume the most general[23] case where what is conserved (or literally in Noether's formulation: the integral invariant to the Lie group of transformations) is a physically dimensionless quantity (such as quantum information), and the Lie group of all transformations (what is that structure meant literally in Noether's formulation) of action itself is the meant set of corresponding physical changes.

So formulated, the most general case corresponds also to the wide approach underlying her paper, namely the fundamental physical problem of any conservation laws to be interpreted thoroughly and absolutely as an abstract and pure mathematical problem. Which is the relation of the integral invariant after variations to those variations themselves if they constitute a Lie[24] group? Then, one can continue Noether's idea even more abstractly (and partly rather trivially) to any group (eventually even only groupoid in both algebra and category theory) and the structure invariant to all group, thus touching the well-ordering (more precisely, two anti-isometric well-orderings) implied by any group in definition and not needing the axiom of choice.

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[22] Guo and Cai (2018) mean "information conservation in quantum gravity", and Hardy (2010) considers locality to quantum gravity. Ma (2018) discusses "entanglement entropy in quantum gravity". Nomura, Varela, Weinberg (2013) also connect information and quantum gravity. The paper of Bruschi, Sabin, White, Baccetti, Oi, Fuentes (2014) suggests experiments for "testing the effects of gravity and motion on quantum entanglements.

[23] It is the most general case only in a physical sense where the quantity of time is not presupposed or said otherwise, the cases relevant to the definition of time are particular to that case meant to be most general physically. Anyway, the definition of the physical quantity of time corresponds to the case where the well-ordering "theorem" (or equivalently the axiom of choice) is granted. Consequently, that most generally case in a physical sense can be interpreted also to be the most general one in a mathematical sense: the Noether theorem is formulated in mathematics grounded now in a set theory without the axiom of choice (e.g., in ZF set theory as opposed to ZFC set theory). In fact, her original proof does not rely on the axiom of choice in any particular conclusion: so it will be no less valid in that mathematics without the axiom of choice.

[24] In fact, the restriction for the group to be a Lie one and what is conserved to be accordingly an integral are not fundamentally essential: both are a consequence from the fact that classical physics considers only smooth manifolds passing in Noether's theorems by default, but arbitrarily in fact.

As to the physical interpretation of the above "most general formulation", one considers physical changes (or mathematically, their group) "without time": as a slogan, "changes without time", or as common sense would probably more like, "changes before time". As to time itself, the expression "without or before time" means the two directions of time to be available simultaneously, or in a coherent superposition, for example visualizable by "Schrödinger's cat": either certain result whether "dead" or "alive" corresponds to "changes after time". Energy as what is conserved also correlates only with "changes after time" since it needs time.

On the contrary, quantum-information conservation means what is conserved in a coherent state, to which energy conservation is inapplicable even only by virtue of the fact that energy (as well as time) cannot be yet defined as being an inseparable whole. Nonetheless, quantum information conservation continues to be valid after energy conservation or speaking figuratively, after the mythical "Big Bang" intentionally as far back in time as possible so that energy conservation is universally, omnipresently and omnitemporality valid (but only ostensibly, in humankind's belief: seemingly scientific, really religious).

Then and interpreting back, the dark phase of the universe by its light phase[25], one may say rather loosely that "after the Big Bang", the opposite direction of time (as if also available in a coherent state) has been transformed into energy, or generalizing: all pairs of dual and conjugate physical quantities can be distributed either into the one direction of time, or into the other, though idempotently, i.e., conventionally as to which member of the pair in relation to which direction to be.

One may further notice that the "destruction" or "deconstruction"[26] of the dual or dynamic quantities (to their cinematic counterparts) to the alleged origin from the neglected (unchosen or discriminate) direction of time follows even yet mathematically: from the substitution of the well-ordering due to the axiom of choice with the pair of two dual or anti-isometric well-orderings presupposed by any group structure[27]. One can further say that the idea of Noether's theorem consists of: (1) decomposing any group into two semigroups; (2) the substitution of either semigroup with an equivalent invariant (conservation law, physically); (3) confirming the same invariant to the dual semigroup if both originate from the same group; (4) linking the invariant at issue and the initial group as logically equivalent.

The invariant can be exemplified by the function successor (a concept literally borrowed from Peano arithmetic), which can be defined to any group. That is: as the unary operation originating by the additive group operation and the unit element of the multiplicative group so that two dual well-orderings appear both starting from the unit element but being idempotently anti-isometric to

[25] Another paper (Penchev 2023 March 13) introduces and coincides both light and dark phases of the universe in detail.

[26] The allusion to those philosophical ideas of Heidegger's "destruction to the origin" or Derrida's "deconstruction" is not quite serious, but maybe curious.

[27] What is meant are two anti-isometric semigroups and explained in detail a little below.

each other. Then, the algebraic sense of the idea of Noether's theorems[28] seems to be trivial and originates even from the definition of "group" in the final analysis.

In fact, Noether's paper does not mention at all, even once the physical quantity of action. However, that physical interpretation though implicit is immediate as to classical physics since the article formulates the invariants by Lagrangians in turn able to describe any motion in classical mechanics, in which "invariant variation" corresponds to the principle of least action, from which energy conservation is deducible. However, if one discusses the abstract group generalization (or an interpretation) of Noether's theorems, what corresponds to variations, or rather to the invariant inferable from those variations obeying furthermore some analogue of the principle of least action, needs some relevant elucidation in other to be corroborated the fundamentality also of that algebraic counterpart of the physical quantity of action supposedly by the mediation of information and quantum information.

If one considers the two semigroups, respectively the two anti-isometric well-orderings both starting from the zero element of the additive group (eventually being "curved" or parametrized to each other as in the second theorem), the principle of least action would correspond to the case of a "square" since its area is least among all "rectangles", the less side of which is the same, which in turn implies for the two semigroups to be exactly anti-isometric to each other, i.e. their reference units to be the same though opposite to each other. On the contrary if they are "curved" to each other, or the one is parametrized to the other, the area of the square at issue cannot be zero, but some finite value, for example, such as the Planck constant as to our universe.

Furthermore, one can distinguish the rather trivial mathematical sense of Noether's theorems from their proper and essential sense (once they have been reduced algebraically as above). The former consists in the consideration of the two anti-isometric semigroups in two alternative ways therefore avoiding the direct logical conflict due to their mutual anti-isometry. The one is conserved literally, i.e., as a semigroup, but the other is "paraphrased" by the "conservation" of its unit element since it is the same as in the other semigroup and thus no logical contradiction can appear. Summarizing that rather trivial mathematical sense, it consists in the representation of any group by either of its two well-ordered and anti-isometric semigroups, sharing only the zero element of the additive group at issue.

The relevant essential physical sense of the only algebraic consideration of Noether's theorems is due to the fact that the inherent well-ordering of time means the representation of the group in terms of either of its semigroups as only physically relevant and meaningful since time is irreversible and the dual semigroup is removed from any physical interpretation as nonsense, nonetheless being absolutely necessary for mathematical proofs and conclusions. Thus, action is

[28] Many papers (e.g. Mansfield, Rojo-Echeburúa, Hydon, Peng 2019; 2019a; Pössel 2019; Peng 2017; Dorodnitsyn, Ibragimov 2014; Gonçalves, Mansfield 2012; 2012a; Shi 2012; Fatibene, Francaviglia, Mercadante 2010; Narain, Kara 2010; Marwat, Kara, Hayat 2008; Ustinov 2007; Kosmann-Schwarzbach, Meersseman 2006; Brading 2005; 2002 Hanc, Tuleja, Hancova 2004; Sha 2004; Qiao, Yue, Dong 1994; Krivskii, Simulik 1989; 1989a; Cheung, 1987; Post 1980; Rosen 1980; Ibragimov 1969; Schröder 1968;Trautman 1967) discuss Noether's theorems in a more usual interpretation in relation to physics or its generalization.

featured to be the most fundamental physical quantity after Noether's theorems (1918) since it allows the consistent unification of both physical, temporal and mathematical, atemporal description: in fact, a unification being unavoidable for the description of the "dark phase of the universe" (in more detail in *Section VIII*) and the most general "quantum information conservation", being the subject of *Section IX*.

So, Noether's theorems (1918) are fundamental for the idea that the physical world is a particular case in the framework of Hilbert mathematics: namely, that of any nonzero distance between finiteness and infinity under the additional convention (elucidated above) that intuitionistic mathematics[29] can be reduced to dialectical mathematics. Speaking loosely, one may say that the physical dimension of action appears as a purely mathematical corollary by the consistent option for any group to be equivalently represented by the pair of a well-ordering (such as that implied by the one semigroup) and a certain conservation.

The logical inferability of the physical world from only mathematical premises results also in the direct link of the group of action (a physical quantity) and the conservation of quantum information (a physically dimensionless quantity able to be equally well interpreted both physically and mathematically). Unlike its particular case of energy conservation connecting two physical quantities (energy and time) and valid only to the light phase of the universe, during which mathematics and physics can be absolutely gapped from each other, quantum information conservation means the way in which the physical worlds appears omnipresently and omnitemporally from quantum information, which is "nothing" physically, i.e. as if "ex nihilo", but strictly observing the most fundamental law of its conservation[30].

The mythical "Big Bang" is only an illusory projection of that permanent violation of energy conservation, which generates the physical world always and everywhere by virtue of the mathematical necessity subjected to Hilbert mathematics. The "advantage" of the "Bing Bang" conjecture[31] is that the violation of energy conservation is collected at a single special and singular point of space-time: that at the beginning of spacetime and the universe so that energy conservation is alleged to be universally valid at any other point. This means that the Big Bang is due to the prejudice for energy conservation to be universally valid, in fact absolutely false.

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[29] For example, Corbett and Durt's paper (2010) relates intuitionistic mathematics to elementary particle physics by an "an intuitionistic model of single electron interference"; Weingartner (2010) means the "intuitionistic alternative" of basic logic to application in physics.

[30] The "Spontaneous creation of the universe from nothing" in the paper of He, Gao, Cai (2014) can obey quantum information conservation (as an example).

[31] The "Big Bang" theory dominates crucially in contemporary cosmogony and cosmology. Nonetheless, there exist many enough papers rejecting or at least questioning it (e.g. Das 2017; Guendelman, Herrera, Labrana, Nissimov, Pacheva 2015; Barbosa 2014; Robles-Pérez 2014; Belbruno 2013; Rashidi 2013; Stoica 2012; Bauer 2011; Finster, Hainzl 2010; Qiu 2010; Frampton 2009; Steinhardt, Turok 2007; Schmutzer 2007; 2000; 1999; Pecker 2005; Célérier, Schneider 1998; Pecker 1997; Chauvet 1996; Turner 1996; LaViolette 1995; Jha 1994; Arp, Flandern 1992; Goldwirth, Piran 1991; Carroll 1988; Allen 1976).

VI HILBERT MATHEMATICS BY NOETHER'S THEOREMS, OR NO PHYSICAL CONSERVATION IN GÖDEL MATHEMATICS

Meaning the conclusions of the last section, one can reinterpret back the relation of Hilbert mathematics to Gödel mathematics now starting from Noether's theorems and the variability of a parameter, which is the Planck constant in our universe (rather than a rather mathematical parameter of the distance between finiteness and infinity). One can illustrate the approach by the problem about the value of the curvature in great enough domains of the universe or about the magnitude of the gravitational constant.

If general relativity is a relevant theory of gravity, as contemporary physics suggests, the curvature of immense areas of the universe and the value of the gravitational constant are linked both mathematically and physically since gravity can be also interpreted to be a physical corollary from the curvature of pseudo-Riemannian space. Analogically, if the idea about the mathematical origin of the physical world is accepted as a kind of the "curvature of mathematics" (and represented by the variable parameter of the distance between finiteness and infinity) resulting in the existence of the physical world, the Planck constant corresponds to it.

So, the existence of the physical world itself refutes the suggestion that the "flat" Gödelian mathematics is the "real mathematics" of the world analogically to the way in which gravity or the gravitational constant after general relativity reject the hypothesis that Euclidean geometry is the "real geometry of the world". Though the expression the "real mathematical structure of the world" (whatever that structure be: whether "geometry" or "mathematics", or any other first-order logic studied by mathematics) seems to be metaphorical in the Cartesian organization of cognition or from the viewpoint of Gödel mathematics relevant to it, Hilbert mathematics accepts the same phrase literally.

Indeed (and following the Cartesian "slang"), the link between mathematics and reality can be only mediated by any physical or belonging to natural science theory, which uses one mathematical model or another, and only it can be tested experimentally whether it corresponds to reality or not rather than the corresponding mathematical model directly. So, Einstein's general relativity can be really tested experimentally, a pseudo-Riemannian space to be or not the "real geometry of the world" is only in a metaphorical sense by virtue of the fact that general relativity utilizes just it. That is the case also of any structure enumerated to belong to Gödel mathematics since it definitely needs a human arbiter, gifted by the divine "ability of free will" to decide for a natural science theory to utilize one mathematical model or not, who "would be erased, like a face drawn in sand at the edge of the sea"[32] by Hilbert mathematics, though.

However, the analogical expression about the "real mathematics of the world" is not metaphorical as to Hilbert mathematics since it identifies the physical world with a class of mathematical structures featured by any nonzero finite value of the "parameter of the distance between finiteness and infinity", and resulting into a minimal possible magnitude of physical action, which is the Planck constant in our universe. So the "problem about the real mathematics of the world" generally means the choice between Gödel mathematics and Hilbert mathematics,

[32] The famous end of Michel Foucault's "Le choses et mots" (1966).

on the one hand, and the choice of a certain Hilbert mathematics corresponding to the Planck constant among the entire class of Hilbert mathematics, on the other hand.

So, if the physical action is always zero as in Gödelian mathematics, to which Godel mathematics is identical in the meaning elucidated above, in the framework Hilbert mathematics, all physical quantities are trivially conserved being identically zero. The physical laws of conservation meant by Noether's theorems are essential to Hilbert mathematics only as to any non-Gödelian mathematics and thus to the certain mathematics being the "real mathematics of our universe" and featured by the Planck constant.

The most general law of conservation, that is the conservation of quantum information includes as also valid the particular case of energy conservation, but its essential physical sense to regulate the creation of physical world can be better visualized in the complementing case where energy conservation is not valid: consequently out of the scope of all contemporary physics and classical quantum mechanics sharing the universality of Pauli's "particle paradigm" and the Standard model relevant to it. Figuratively, it means the state "before" the mythical "Big Bang" and only mathematical laws rule the creation of the physical world occurring everywhere and always and resulting in the visible universe and its expansion rather than due to the alleged "Big Bang" ostensibly having taken place about 14 billion years ago.

In other words, this is the state where energy and time are not yet distinguishable from each other, but this is not the imaginary singular point at or within the Big Bang. This is the omnipresent and omnitemporal "dark phase of the universe", from which the known visible and light, local phase arises everywhere and always as if "ex nihilo". There exists the increasing quantity of physical action corresponding to the universal conservation of a purely mathematical quantity such as quantum information or respectively the qubit Hilbert space: it conserves only because it cannot but be inherently complete.

So, the dark phase of the universe is necessarily "caused" by a mathematical reason: thus "hypotheses non fingo" just as Newton said about gravitation, but now to the creation of the world (for which Newton needed the hypothesis of God, anyway). Then, the light phase of the universe arises from its dark phase again only by virtue of the mathematical necessity due to the inherent duality of the qubit Hilbert space. Even the "haven" of the Big Bang which might shelter God is not more necessary for science, and thus no place at all for God in physics[33]. The conclusion is:

The physical world appears necessarily and purely mathematically in any non-Gödelian mathematics that is without any physical cause, especially without any supernatural one such as "God".

[33] The problematic of the "Big Bang" and "God's Creation" (especially in Christianity) has been "entangled" since Lemaître's age up to the present day (e.g., Jahangir 2019; Pitts 2008; Flew 2003; Wallis 2003; Selman 2002; Alpher, Herman 2001; Faber 2001; Russell 2001; LaViolette 1995; Macintosh 1994; Craig 1992; Smith 1992; Drees 1990; Grünbaum 1990; Schroeder 1990). The collection of papers edited by Driessen and Suarez (1997) introduces the question of the existence of God to be linked to nonlocality.

VII LOCALITY AND NONLOCALITY IN BOTH PHYSICS AND MATHEMATICS

Noether's theorems (1918) can be literally related to the "Lagrangian viewpoint" to action since they are formulated to Lagrangians. Nonetheless, the above, purely algebraic interpretation means the Hamiltonian one since kinematic and dynamics variables are opposed to each other as independent of each other and even separated in two dual and anti-isometric semigroups, after which all dynamic variables can be interpreted physically as the description "backwards in time" in a way consistent with kinematic spacetime being "forwards in time". Of course, the absolute separability of the two semigroups from each other originates from the light and local "chauvinism" of physics until now totally neglecting and even not suspecting the dark and nonlocal phase of the universe: and thus, being quite conventional.

Applying the same absolute separability of the one Lie semigroup from its dual counterpart of conservation after the latter is interpreted to be already that of quantum information rather than that of energy, one should interpret quantum information to be kinematic, the "Fourrier counterpart" of spacetime after it is identified to be Minkowski space in special relativity, i.e after the formal substitution of the variable physically interpretable as "time" by that of "frequency" or in terms of Bohr's viewpoint to quantum mechanics: by the replacement of the description in terms of the macroscopic "apparatus" by its equivalent counterpart in terms of the investigated quantum entity.

Speaking loosely, quantum information conservation means the dual statement of the invariance of spacetime distance after Lorentz transformations physically interpretable as the direct conversion of "time" into "space" or vice versa therefore also rejecting the absolute separability of space and time featuring classical mechanics and physics, in which Lorentz invariance is not valid. Then, one can look at general relativity also otherwise: as able to describe all intermediate states in which Lorentz invariance appears gradually or mathematically said, smoothly by the corresponding limitation of the admissible infinite velocity of classical mechanics to the bound of the speed of light in a vacuum featuring special relativity as an initial postulate. This means that general relativity describes gravitation as the direct transformation between space and time under the condition of the invariance of spacetime distance as regulating how space and time may transform into each other however mapped only on the "wall of Plato's cave": that of locality.

Then, quantum information conservation regulates the correlating transformation of the counterpart of what is out of Plato's "cave" and represented as "shadows" on the "wall" at issue only visible by the "chained and pinned down people". That "wall" is the screen of locality and experimental *absolutely repeatable* verifiability required by any classical natural science claiming to be objective and shared also by special and general relativity, but not by quantum mechanics, on which Einstein paid attention sarcastically, by the pejorative epithet "spooky" or the sardonic metaphor of "God playing dice".

One can trace the origin of the absolute separability of the kinematic and dynamic semigroups from each other back: also still into the corresponding Gödelian or Gödel mathematics, in which kinematic is gapped from dynamics just as mathematics from physics or the material world by

itself. On other words, its postulate is that the two semigroups do not origin from any single group and thus cannot be unified by sharing the same element so that to be the least in the one and the greatest in the other therefore supplying any element of both semigroups with its inverse counterpart in the other semigroup and necessary for their joint structure to be a group.

Then, the Planck constant of the minimal possible action reflects the identity of the unit of the multiplicative group also available as the same unit of both successor functions though anti-isometric to each other so that the unit of the one Peano arithmetic can be multiplied by the unit of its dual anti-isometric twin: so one obtains again the unit of the Planck constant possessing the physical dimension of action.

On the contrary, the product of those two units is zero in Gödel mathematics in definition, or speaking otherwise, the two semigroups are "orthogonal" to each other so that the corresponding Gödelian mathematics is the single and special "flat" case of Hilbert mathematics not generating any physical world by itself. Now, one can translate the Hamiltonian viewpoint inherent for the algebraic interpretation into the proper Lagrangian language of Noether's theorems, after which the opposition of locality and nonlocality, distinguishing particularly classical quantum mechanics from quantum information mechanics, will be also translated into a "Lagrangian language" by means of Noether's theorems and quantum information conservation.

If one uses "Lagrangian" and "Hamiltonian" like "English" or "Bulgarian", i.e. as languages, the following formal mapping can describe the "translation" at issue. All infinitesimal neighborhoods are considered as the class of equivalence of localities and opposed to a single globality as two independent variables after passing from "Lagrangian" into "Hamiltonian" (as "languages"): respectively vice versa. Just the latter is what the "expression" by Noether's theorem needs.

Then, and also meaning the sketched above algebraic approach, one can reformulate the field of all rational numbers from its proper "Hamiltonian" (language), after which the additive and multiplicative groups are nor subordinated, but independent of each other just like the two kinds of variables in "native Hamiltonian", into an "alien kind of Langangian" where the one group (either of both) is alleged to be "local" and opposed to the other as being "global". In other words, any unit of the global Peano arithmetic ostensibly contains a local Peano arithmetic furthermore identifiable to be the same "within" any global unit. Since any global unit including a local Peano arithmetic is isomorphic to a qubit, the Lagrangian translation of the field of all rational numbers turns out to be the qubit Hilbert space provable to be complete in virtue of the theorems of the absence of hidden variables in quantum mechanics[34] (though literally referring to its counterpart: the separable complex Hilbert space).

Thus, Hilbert arithmetic being the dual twin of the qubit Hilbert space can be interpreted to restore the initial "original in Hamiltonian (language)" to its "translation in Lagrangian (language)" by the qubit Hilbert space. So, one may say that the completeness of both qubit Hilbert space and Hilbert arithmetic relies on the same completeness: that of the field all rational numbers and resulting into two dual anti-isometric Peano arithmetic rather than into a single one as after

[34] Kochen, Specker (1967); Neuman (1932)

the classical approach of Peano arithmetic and set theory inherently suffering from the Gödel dichotomy (the Gödel dichotomy is a "genetic pathology" of the classical approach to the foundations of mathematics and shared, though in different "symptomatologies", by the entire class of theories belonging to Gödel mathematics).

Following the same metaphor, one might say that the mathematical origin of the physical world belongs to the "genetic mutation" of Hilbert mathematics, in which the "pathology" is not more a pathology, but a new and very useful evolutionary advantage and thus normal for the new "species"[35]. In a maybe too abstract sense, Hegel's idea of dialectic logic means the same transformation of a "pathology" (the violation of the law of noncontradiction, respectively, "excluded middle") into a norm.

Then, and speaking Lagrangian, one can distinguish a fussy (or accordingly, probabilistic) area of "glocality", belonging whether to both locality and globality (as which nonlocality can be absolutely identified) or to the middle between them. What is quantum information out of that vague domain is the quantity of physical action obeying the Planck constant as to our universe. Quantum information conservation is able to describe uniformly both locality and globality, on the one hand, and glocality, on the other hand, and unlike energy conservation: which is valid only to the physical world of glocality, but being meaningless or even nonsense as to the proper mathematical world.

On the contrary, quantum information conservation is the only reason for the existence of the physical world by virtue of the nonzero interaction of glocality rather than "caused" by the mythical "Big Bang" trying to explain the physical world as if "by itself", but obeying energy conservation. Indeed, all in the physical world observes energy conservation, but not the physical world as a whole due to quantum information conservation "causing" for the physical world to appear "ex nihilo" omnipresently and omnitemporally.

The completeness of the separable complex Hilbert space being without any possible hidden variables, then transformed equivalently into the qubit Hilbert space, is the mathematical counterpart of quantum information conservation, in turn and by itself (in the sense of Noether's first theorem) generating the permanently increasing amount of physical action and visible as the expanding universe. Its source is not the "Big Bang" (in fact, an ostensibly scientific reading of the Creation in the Bible), but the completeness of the qubit Hilbert space in the final analysis, from which the expanding universe cannot but appear only by virtue of the mathematical necessity articulated in Noether's theorem[36].

[35] For example, many biological markers, e.g the absence of a tail, are pathological and more or less harmful to monkeys, but normal to humans.

[36] Said quite jokingly, Emmy Noether rather than God "created" the universe.

VIII THE DARK PHASE OF THE UNIVERSE, IN WHICH PHYSICS AND MATHEMATICS ARE "ENTANGLED"

Following the approach of the last section, the dark and light phases of the universe are opposed as locality and nonlocality (globality[37]) in both physical and mathematical meaning, therefore linking mathematics and physics in an inseparable whole, which can be called "entangled" more or less metaphorically for scientific common sense, but in fact, quite literally from the viewpoint of the present paper.

The terms of the "dark phase" versus the "light phase" of the universe suggests a smooth transition between the infinitesimal (in mathematics) opposition of locality versus nonlocality (both in physics) furthermore gapped by the speed of light in a vacuum so that the former is also literally the *light* phase being *elucidated* and *visible* unlike the prevailing and invisible nonlocal phase revealed only recently by its "dark mass" and "dark energy"[38].

The term "phase" is chosen also hinting at a "twilight area" of entanglement between them furthermore interpretable also in a proper mathematical sense after Hilbert mathematics as any finite nonzero distance between finiteness and infinity and implying a probabilistic transition between them illustratable for example by Einstein's sardonic metaphor of "God playing dice", the allegorical sense of which is now illuminated so: infinity (for "God") by means of probability (for "playing dice") is transformed into finiteness (for each individual "roll of the dice" what any single measurement of a quantum quantity is).

Certainly, only that "twilight area" is able to generate the fundamental physical quantity of action (as this is discussed above) following an only mathematical way, meant in Noether's theorems historically for the first time and rather in the context of classical mechanics and expressed in "Lagrangian" (language). Following them in a crucially generalized sense as in the present sense, one is to to unify thoroughly the light and dark phases by the ostensibly only intermediate twilight region between them: or following the same "light metaphor", one might say that "darkness" is "condensed twilight", and accordingly, "lightness" is "diluted twilight" proclaiming that all the universe is in the twilight entangled phase only conventionally distinguishable into its dark phase versus its light phase.

Gödel mathematics accepts that barely human convention after the Cartesian gap featuring Modernity so (too) seriously that it even dares not articulate it, thus transforming it into a "subconscious" and therefore irresistible imperative which any theory claiming to be mathematical is ought to obey. One can compare with the fifth postulate of Euclid (by the by, called by himself "postulate", i.e., "convention" rather than "axiom", i.e. "obviousness"): being expressly articulated, it culminated into non-Euclidean geometry, Riemann's "space curvature", and

[37] "Globality" as a term more relevant to the Standard model is a particular case of "nonlocality" relevant to entanglement and quantum information.

[38] The context of the dark phase of the universe is already explicit in the present paper, but it is implicitly, more or less available in many papers about dark mass and dark energy (e.g., Czachor 2021; Lee, Kim, Lee 2019; Saari 2015; Capozziello, Luongo 2013; Capozziello, Luongo Mancini 2013; Baushev 2010; Setare, Sadeghi, Amani 2009; Lee, Lee, Kim 2007; González-Díaz, Fernández 2006; Araujo 2005). The "tachyon trace" to dark energy is investigated (e.g.) by Setare, Sadeghi, and. Amani (2009).

Einstein's general relativity in the final analysis. On the contrary, if Euclid had not articulated obeying some unconscious and unconditional command (as in fact Gödel mathematics really does), all those did not take place maybe. Fortunately, that was not the case at least as to Euclid therefore aiding mathematics to overcome even the imperative of the Gödel dichotomy nowadays.

Then, the universal twilight and entangled phase of the universe is able therefore to unify physics and mathematics in an inseparable whole restricting energy conservation to be always valid only to the "light physics" obeying the principle of absolute repeatability of any experiment. One can illustrate this by the case of the Fleischmann and Pons (1989) claims of *cold fusion* that caused a sensation and was subsequently denied due to the impossibility of reproducing[39]. On the contrary, one can admit it if the requirement of the obligatory reproducibility of any experimental result is not granted, as physics including "dark phenomena" is forced to do, after which Einstein should "rather be a cobbler, or even an employee in a gaming house", in his own words (Einstein 1926), of course only as a joke now.

The metaphor of the universe as a computer (in fact much more than a metaphor since the universe is really a quantum computer and thus the metaphor is reduced to the visualization of a quantum computer as a computer such as any of ours) can represent the light phase of the universe as its "screen", and its dark phase as the "hardware" hidden, invisible (and "dark" in a sense) in the "box of the computer" and processing intangible "data" by an also intangible "algorithm" embedded in its "software". Indeed, the intangible data and algorithm processing are nonlocal being identically reproducible at any other spacetime point supplied by a relevant "iron". On the contrary, the computer as a material body is always local[40].

So one can generalize the intangible nonlocal "omnipresent and omnitemporal" "ware" of the metaphorical "computer" as mathematics, and its material and local "ware" as physics. Of course, both are absolutely separated from each other in any contemporary real computer, by the by, divided thoroughly even it its mathematical model as a Turing machine: the tape and the four or five possible elementary operation, which it can fulfill, on the one hand, versus the variable algorithm and data, on the other hand.

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[39] For example, Armstrong (1989); Hajdas, Kistryn, Lang, Sromicki, Jenny, P. Wachter (1989)Azbel, Sackler, Sackler (1990); Brudanin, V. B., V. M. Bystritsky, V. G. Egorov. S. G. Shamsutdinov, Shyshkin, Stolupin, Yutlandov (1990; 1990a); Bush, Eagleton (1990); Kim (1990); Rittner, Meulenberg (1990); Russell (1990); Turner (1990); Kühne (1991; 1991a); Mallove (1991); Rees, Donald, Dautovich, Linford, Thomassen 1991; Huizenga (1993); Kozima, Watanabe, Hiroe, Nomura, Ohta (1997); Nagel (1998); Labinger, Weininger (2005); Anyway, the statement and claim of Fleischmann and Pons created a new branch in science for searching for the conditions under which cold fusion might be possible (e,g. Kühne 1991; 1991a; Mallove 1991; Kozima, Watanabe, Hiroe, Nomura, Ohta 1997; Nagel 1998; Sutton 1999; Bockris, Chien, Hodko, Minevski, Beaudette 2000; Dragić, Marić, Vigier 2000; Novaković 2004; Kozima 2006 ; Hofmann 2009).

[40] Particularly, the opposition of natural humankind versus the artificial race of "computerkind" (or "AI") does not make sense in the dark phase of the universe, after the unification of physics (for "humankind") and mathematics (for "computerkind"). It is valid only as to the light phase of the universe just as that of mathematics and physics. Consequently, any fears or apprehensions about AI are relevant only to the light phase of the universe or to "light chauvinism", in fact, a generalization of "human chauvinism".

Anyway, one can at least imagine a future computer able to optimize relevantly its hardware for the problem that it resolves now, or vice versa: its software to fit maximally to its current hardware. This sounds utterly fantastic as to all contemporary computers, but not at all as to a quantum computer where the bound between its hardware and software is conventional, fussy and variable at the discretion of even the user. The mutual transformation of "hardware" and "software" of a quantum computer can well represent the complement of energy conservation (corresponding to the gap between hardware and software featuring any real computer) to quantum information conservation able to regulate the transformation at issue by natural laws.

Quantum information necessary to describe both entanglement and quantum computer links simultaneously hardware and software since both are qubits: indeed, complementary to each other in classical quantum mechanics, but entangled in general as to the theory of quantum information. The eventually entangled software and hardware are mutually transformable at the same time and very well represent the way in which physics and mathematics are "entangled" in the dark phase of the universe obeying only quantum information conservation, but not energy conservation.

Hilbert arithmetic (discussed in detail in a series of other papers: e.g. *Penchev 2021 August 24*) is not only a complete foundation of mathematics, bit also an instrument initially designed to describe mutual transformation of mathematics and physics since it exits in two dual copies: (1) as Hilbert arithmetic in a narrow sense consisting of two dual anti-isometric Peano arithmetics directly derivative from the field of all rational numbers, as this is inferred above; (2) the qubit Hilbert space interpretable as the version of the field of all rational numbers rewritten in "Lagrangian" (language) and demonstrated above as well.

Then, one has two alternative ways for the transition between its dual branches and thus between mathematics and physics. The one means any arithmetic unit as the corresponding empty qubit, i.e., the class of equivalence of all values of any certain qubits. The other translates between the descriptions of the same field of all rational numbers in both languages: (1) the proper "Hamiltonian" (language), from which the two dual Peano arithmetics of Hilbert arithmetic in a narrow sense are immediately inferable; (2) the newly introduced "Lagrangian" (language), in which the "expression" of the field of all rational numbers is the qubit Hilbert space.

If one equites secondarily the two approaches above as the same bridge between mathematics and physics, very interesting and instructive conclusions are possible, and first of all, the transition between: (1) a class of equivalence as a whole such as a set, and the same class as a collection of all its representatives such as all elements of the sets (furthermore able to be considered as the homomorphism of the class of all first-order logics to propositional logic, meant by set theory, and propositional logic if it is the shared single "zero-order" logic both being the same Boolean algebra[41]); (2) the two equivalent descriptions of the field of all rational numbers in "Hamiltonian" and "Lagrangian" (languages), and quantum measurement as the translation between them. An immediate corollary, perhaps shocking scientific common sense enslaved to the notion of absolutely separated physics and mathematics, is the option for physical (in fact, quantum

[41] In detail in: *Penchev 2023 January 3*.

mechanical) experiments about the foundations of mathematics just as the aforementioned problem, which the real mathematics of our universe is, is not more a metaphor.

IX QUANTUM INFORMATION CONSERVATION: THE ENERGY OF THE "BIG BANG" ACCESSIBLE TO HUMANKIND?

Quantum information conservation, its inference, corollaries, relations, and fundamental meaning for philosophy rather than only to physics and mathematics and their unity are mentioned above and discussed in detail in a series of other papers (e.g. Penchev 2020 October 5). The narrow subject in the same context in the present section is the reinterpretation of the colossal energy of the "Big Bang" alleged ostensibly to generate all the universe and cause its expansion.

The main conclusion from quantum information conservation is: no Big Bang at all! There has not ever been the Big Bang: it is only a serial product of the rich human imagination tending to fill with it in the immense cognitive gaps, i.e., due to humankind's ignorance though seemingly being science. In other words, the "Big Bang" only replaced "God's Creation" in the Bible, not rather more justified than the latter.

Science opposing religion to emancipate from it proclaimed and continuous to herald a new belief: the universal and absolute materiality of the world cognizable only experimentally or empirically by natural science obeying the repeatability and reproducibility by anyone anywhere and always as far as the conditions are described and copied exactly (ostensibly also being always possible). Even classical quantum mechanics was forced to generalize the latter dogma of scientificity, since the result of any single quantum measurement is fundamentally random therefore substituting it by the reproducibility of the probability (density or not) distribution of a long enough series of the results of the same experiment.

Nonetheless, classical quantum mechanics does not touch the former dogma of scientificity and even reconfirmed it by Pauli's "particle paradigm" of energy conservation culminating into the Standard model that managed to describe the 17 most fundamental elementary "particles", orthodoxically and literally following the "testament of the ancestors". However, gravitation of Einstein's general relativity, entanglement and quantum information, and crucially, "dark matter" and "dark energy" were Kelvin's "small clouds" (at that, three or more rather than the two original ones) remained on the horizon of the Standard model "Brave New World".

So, all anomalies not corresponding to that "Brave New World" due to not obeying energy conservation were deported in a special "concentration camp" called the "Big Bang" in the beginning of the universe, after which all the rest obeyed unequivocally energy conservation as all ideal residents of the Brave New World of the Standard model were obliged. However, the 2022 Nobel Prize for entanglement and quantum information opened the gates of the terrible camp and the released prisoners moved all over the Brave New World everywhere disseminating the "crime" of not obeying energy conservation, anyway observing the weaker quantum information conservation.

Not speaking eloquently and metaphorically, quantum information conservation implies the substitution of the mythical Big Bang by an omnipresent and omnitemporal medium of "creation ex nihilo" due to the completeness of the qubit Hilbert space in the final analysis and known a long

time ago as the theorems of the absence of hidden variables in quantum mechanics (Kochen, Specker 1967; Neumann 1932). The "light" and local phase originates gradually and smoothly from its "dark" phase everywhere and always therefore constituting a medium really. So, the creation "ex nihilo" is already accessible to humankind in principle, following the natural laws of that: those natural laws only are to be studied and then implemented in relevant technical devices.

So, an analogue of Einstein's famous "$E = mc^2$" seems to be forthcoming therefore hunting for a much greater counterpart of the "A- & H-bomb", to which the formula at issue is an emblem. It should be a corollary from the entanglement theory of quantum gravity just as the former formula follows more or less directly from special relativity. Since that theory at issue is not yet created though its contours can already be outlined, one might conjecture about its eventual future exact content:

It refers to the quantity of action rather than to energy. That action is due to the nonlocal interaction of the universe, but restricted to its local domain i.e. within the light cone. It should not depend on time or mass (energy), since, figuratively speaking, it means the state "before the Big Bang"[42], though now distributed as a medium at any point of spacetime, and thus before dividing spacetime from energy-momentum, before dividing space from time, or particularly, energy from time. All those quantities distinguished separately are irrelevant to that previous state. It can be described only by the quantities of action and information whether quantum or not and the three most fundamental constants: the speed of light in a vacuum, the gravitational constant, and the Planck constant.

It seems to be quite inaccessible to humankind, at least at first glance, as far all experience is empirical and experimental within spacetime, or figuratively speaking, "after the Big Bang" though now distributed in a continuous medium, all over the spacetime. There exists an initial paradox about the utilization of that action, possibly monstrous according to its magnitude, recollecting Baron Munchausen's efforts to pull himself out of the swamp by his hair. "God" being transcendent might assist just as anybody else if he or she stays on the solid shore of the swamp might really pull Baron Munchausen out.

Indeed, that action is the action of the "Creation", a privilege reserved for "God" rather for humankind at least until now. One might visualize that action of the creation, as a huge amount of energy as if infusing in the light cone each unit of time, in fact, to create it, i.e. to distinguish and separate by the bound of the speed of light in a vacuum from the real and nonlocal domain of

[42] The problematic of what had been "at, before, or beyond the Big Bang" can be traced back in a quite various series of papers (e.g. Doplicher, Morsella, Pinamonti 2020; Klinkhamer 2020; Zen Vasconcellos, Hadjimichef, Razeira, Volkmer 2020; Koslowski, Mercati, Sloan 2018; Das 2017; Tozzi, Peters 2016; Polishchuk 2015; Maceda, Madore, Manousselis, Zoupanos 2004; Stoica 2012; Sivaram, Kenath 2011; Sadeghi, Amani 2009; Varadarajan 2009; Bojowald 2008; 2007; Battisti, Montani 2007; Steinhardt, Turok 2007; Gasperini, Veneziano 2003; Durrer, Kunze, Sakellariadou 2002; Feinstein, Kunze, Vázquez-Mozo 2000; Veneziano 2000; 2000a; Biswas, Maharana, Pradhan 1999; Gasperini 1999; Oz 1999; Hwang 1998; Schucking 1992; Grünbaum 1990; Lauro, Schucking 1985; Akama, Terazawa 1983; Nardone 1983); also papers (e.g. Senovilla 1990; Rahman, Banerji 1985) how the singularity of the Big Bang to be avoided.

Minkowski space, under condition that the infusing energy for each unit of time originates from that area of nonlocality.

Those “efforts of Baron Munchausen” are now embodied in the greater and greater energies of the larger and larger hadron colliders such as that at CERN nowadays. This pathway is meaningless, a “dead end” just as Baron Munchausen’s pulls himself out since any energy, however vast it be, is “after the Big Bang” being just energy. The relevant approach to the creation seems to be rather that of information or rather quantum information, i.e., probabilistic and relied on quantum correlations in the final analysis since quantum information refers to action immediately not needing energy or time and able to mean inherently the state “before the Big Bang” including as a “medium”.

So, not too expensive experiments for entanglement and quantum information awarded by the 2022 Nobel Prize for physics (unlike more than the too expensive Large Hadron Collider and its eventual successors) are the only possible real approach of science to “God’s Creation” relied on the transformation of “pure information” into physical action observing quantum information conservation rather than greater and greater energies of colliders thus observing energy conservation. The *creation needs creativity:* that of the new dimension of quantum information rather than the dull persistence of ever greater energies by means of ever more costly accelerators.

X INSTEAD OF CONCLUSION: A NEW LOOK AT INFORMATION, QUANTUM INFORMATION, ACTION, ENERGY, TIME, AND TEMPERATURE

The idea about the physical world originating from Hilbert mathematics as a particular case in its framework by virtue of mathematical laws and their necessity is the main viewpoint of the present paper. It implies particularly a different understanding of basic physical quantities developing further the approach of classical quantum mechanics where they are defined by Hermitian operators. Involving them, only what is the change of a physical quantity is substituted in comparison with classical mechanics where it is a finite (experimentally) or also infinitesimal (theoretically) quantity. In other words, the change of a single value in classical mechanics is generalized to the change of a probability (density or not) distribution so that a Hermitiam operator obeying unitarity and energy conservation corresponds to the change of any quantity in classical quantum mechanics.

The physically dimensionful reference units determining which certain physical quantity is meant are the same in both cases, and they are absolutely gapped from the physically dimensionless measured results whether finite numbers or due to Hermitian operators. So, physics and mathematics continue to be reliably divided in classical quantum mechanics as well. However, the viewpoint of Hilbert mathematics, allowing for the violation of energy conservation in the framework of quantum information conservation, admits dimensionless mathematical quantities to generate dimensionful physical quantities however only by the mediation of action and thus accompanied by its corresponding conjugate counterpart complementing it to the quantity of action.

Then if one grants, for example conventionally, that the conjugate is constant, the change of dimensionless quantum information may generate the change of energy or any other physical

quantity embodying the aforementioned "creation ex nihilo" as far as quantum information being physically dimensionless can be interpreted to be "nothing". The change of quantum information at issue can be due to the change of the conjugate counterpart in turn, certainly under the condition of quantum energy conservation.

Thus, one can admit that energy can be directly transformed in time or vice versa as well as mutual transformations of any pair of conjugates. Quantum information conservation, though admits mutual transformation of physical quantities and thus violation of energy conservation, restricts it only to the pair of conjugate quantities, though. So, one can allow for e.g. time to transform into energy, but its transformation in space distance seems to be problematic therefore needing some elucidation since Lorentz invariance, respectively the invariance of spacetime distance, acknowledges it. Indeed, quantum information conservation can be considered to be conjugate to Lorentz invariance (or to that of spacetime distance) as above.

So, one can distinguish two fundamentally kinds of mutual transformations of physical qualities, i.e. different physical dimensions: (1) only local and which general relativity have started to discuss a long time ago e.g. as the violation of energy conservation for the option for energy to be directly altered in momentum in its framework, and even still in special relativity for Lorentz invariance and the newly introduced there space-like time; (2) between locality and nonlocality (globality), i.e. between conjugate quantities[43] granted to be complementary to each other in classical quantum mechanics now turning out to be mutually transformable after entanglement as long as they obey quantum information conservation.

So one can say, that the locally admissible mutual transformations of physical quantities obey "action conservation" as an only local counterpart of quantum information conservation, though action conservation cannot be valid nonlocally (globally) where only quantum information conservation regulating the quantitative link of locality and nonlocality is only valid. Nonetheless, there exists no obstacle for the transformation of time into energy under quantum information conservation, i.e. nonlocally, and then into momentum, i.e. only locally and under action conservation being locally valid, though the two transformations are complementary to each other and thus cannot be accomplished simultaneously.

There is a special quantity in physics, more precisely in thermodynamics[44]: that is temperature, which can assist for a generalized thermodynamic approach to quantum mechanics after it has overcome the restriction of classical quantum mechanics after the theory quantum information investigating the transition between locality and nonlocality due to entanglement. Any quantum entity unlike that in classical mechanics is both "particle" and "wave", the duality of which can be now interpreted as a unification of locality (and mechanics for "particle") and nonlocality (and Gibbs thermodynamics for "wave"), which means that it can be granted to consists of its possible states and their probability distributions whether density or not. So, the Gibbs quantity of temperature is relevant to it where the corresponding entropy is that of its states.

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[43] In the context of Noether's theorems, e.g., in the paper of Post (1980).

[44] Barvinsky and Kamenshchik (2006) discuss "thermodynamics via creation ex nihilo" relevant to the present paper.

Even more, quantum "temperature" can be analogically introduced to any quantity rather than only to energy, after which the definition of physical quantity in classical quantum mechanics can be interpreted as any change of the corresponding "temperature" featuring the quantity at issue. The peculiarity of that "temperature" consists in the fact that it is a relation (which is a ratio in the simplest case) between an exact value of any quantity according to its local "embodiment" and the corresponding probability (density) distribution of its conjugate counterpart. If the same approach is applied back to temperature usually defined by energy and entropy, this means the probability density distribution and its operator, Hermitian or not: both inadmissible in classical quantum mechanics due to implying the violation of energy conservation.

So, statistical thermodynamic, especially in its Einstein's version or interpretation[45] allowing for the mechanical consideration of the thermodynamic system as a whole rather than only that of its statistical units such as "atoms" or "molecules" after the Boltzmann approach, can be realized as a "quantum mechanics" under the only additional conduction for the limit of the Planck constant action: as well as vice versa, Then, the transition between locality and nonlocality being inherent for quantum information, but representable differently in relativity (by the bound of the speed of light in a vacuum weather in a zero or nonzero gravitational field), can be once again realized by means of thermodynamics, explicitly in Einstein's version of it only complementing it by the Planck constant.

The sketched approach to physical quantities aims to demonstrate their inferability of mathematics alone (as long as it is generalized to Hilbert mathematics as above) after granting the certain values of the three most fundamental physical constants (namely gravitational, Planck, and that of the light speed in a vacuum) featuring unambiguously our universe, only interpreting them mathematically, by the relations of locality and nonlocality. The speed of light in a vacuum is the exact boundary between them; the Planck constant is their minimally possible overlapping implying further their probabilistic fussiness and mutual diffusion; the gravitational constant means the relation of the reference units of locality and nonlocality.

All the three constants can be related equally well to the dark and light phases of the universe and thus, to quantum information conservation relevant to both phases unlike energy conservation valid only to the later. The compose physical dimensions of those three fundamental constants, seeming strange and artificial as they are defined by the distinctions of all physical quantities in the light and local phase, can be easily explained by the converging of: (1) space and time (as to the light constant); (2) any two conjugate quantities into the quantity of action (as to the Planck constant); (3) the former two inseparabilities in the most fundamental fusion of all physical quantities to be only mathematical in the final analysis (as to the gravitational constant allowing for the Planck units of time, space, and mass to be unambiguously determined by those three most fundamental physical constants).

One can add to them, the absolute zero temperature: the bound of "0°K" (or approximately −273.15 °C), sometimes identified as the "third principle of thermodynamics". It can be reinterpreted in the context marked in the present section as the maximally possible entropy

[45] For example, following Rudoi and Sukhanov (2000); Navarro (1998); or Klein (1967).

(conventionally and traditionally defined by temperature as a ratio of *energy* to entropy, but one can generalize it to any quantity by means of the former three constants). In other words, the absolute zero means the minimally possible information or quantum information[46] therefore directly corresponding to the Planck constant after quantum information conservation, in fact historically initially inferred as an *ad hoc* convention for the solution of the blackbody radiation problem.

[46] After the areas of locality and nonlocality are overlapped, and their probabilistic diffusion is unavoidable, the minimally possible information can be interpreted as mutual entropy of the same area granted to be both local and nonlocal and thus featureable by both local and nonlocal entropy and their mutual entropy as information can be defined.

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