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# Analytical model of SPR phase shift against refractive index

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## Abstract

Surface plasmon resonance (SPR) is known for label-free, real-time and high resolution characteristics, rendering it a powerful biosensor. However, we face some problems when analyzing data. First of all, we cannot directly measure the refractive index change as a function of time, but we can measure the change of phase to time to know the mass change on the chip surface. Therefore, we need a transfer function ( $\Delta RI(\Delta\phi)$ ) to transfer the phase change to the refractive index change. Secondly, the relationship of the phase change and the refractive index change is in non-linear trends, so we cannot fit the result by single high power polynomial. Last but not least, SPR system works according to the Fresnel's equations. However, in traditional analyzing methods, the Fresnel's equations weren't applied. Hence, we propose an algorithm (**Fig.1**) to obtain the transfer function and simultaneously solve the non-linear issue and fit the Fresnel's equation. In this work, fitting the Fresnel model to the experimental data significantly declines the residual from the order of  $10^{-3}$  to  $10^{-5}$  (**Fig.3**).

Keywords: Phase sensitive surface plasmon resonance, Fresnel's equation, Polynomial fit.

## Introduction

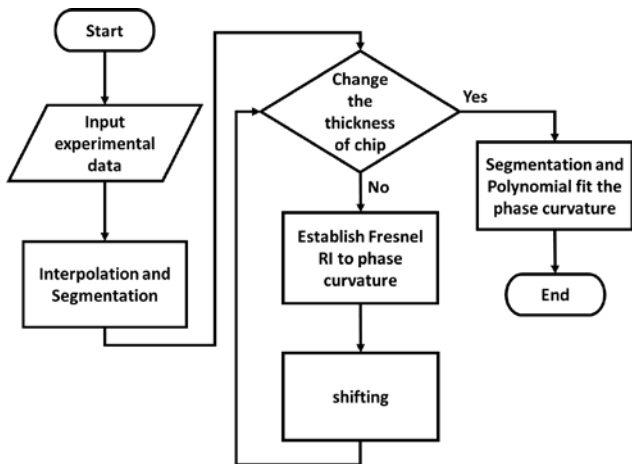
SPR has been applied in a wide variety of biosensing applications for its real-time, label-free and high sensitivity characteristics. Among the types of SPR, phase sensitive SPR (pSPR) provides superior sensitivity than intensity sensitive SPR owing to some factors. The main factor is that the phase responses of pSPR will pass through the sharp non-linear region near the SPR dip, which causes the signal to change abruptly [1], providing higher resolution and sensitivity. Moreover, in pSPR, the change in refractive index of the fluid can be measured. Thus, pSPR is an ideal measurement methodology. However, we can only obtain phase change as a function of time ( $\Delta\Phi(t)$ ) through pSPR experiments, a transfer function ( $\Delta RI(\Delta\Phi)$ ) to convert the phase change into the refractive index change ( $\Delta RI(t)$ ) is needed. To gain the transfer function, before measurement, several concentrations of known refractive index reference solutions (nominally 5 different concentrations solution) were applied. Through reference solutions, we can obtain the relationship between phase change and refractive index change. Traditionally, when analyzing, we approximate the data from reference solutions by single high-power polynomial. In this way, data analysis encounters some challenges due to the non-linear trends and errors caused by high-power polynomial fitting. Furthermore, the multi-layer Fresnel equation highly depends on the thickness of the gold-film and incident angle. Therefore, we aim to develop an algorithm to obtain the transfer function and solve the problems mentioned above.

Based on the method of segmented polynomial approximation, in our algorithm(**Fig.1**), we interpolate and segment our data from reference solutions. For each part of the data, we apply different second-power polynomial approximation to solve the problem of over fitting. The curvature of experimental data is thereby established. Second, curvatures were modeled by Fresnel's equation. Third, by means of the change of incident angle and the segmentation of the experimental data from reference solutions, the Fresnel model curvature will be shifted to fit experimental data curvature. Residuals between the two curvatures will be calculated and the minimum indicates the closest fitting result. Ultimately, through fitting result, we obtain a the transfer function of phase change to refractive index change.

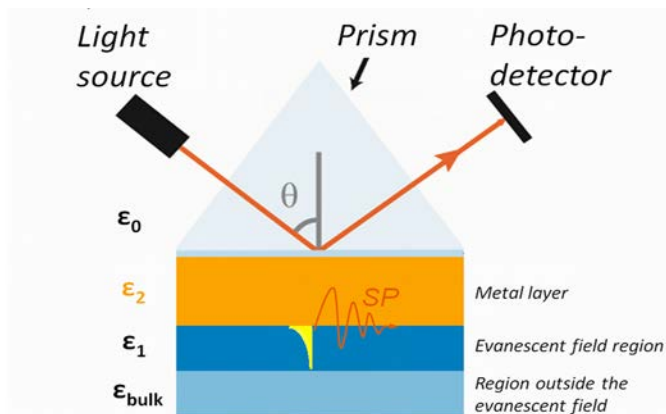
## Experimental

The pSPR system was built up on a phase modulation homodyne polarimetric interferometer. The SPR chips were composed of 48nm gold and 2nm of Chromium on BK-7 substrate, based on the Kretschmann configuration [2] (**Fig.2**). Since the light source of the SPR measurement system is an 850nm wavelength diode, the refractive index of gold and Chromium were modelled to be  $0.16+5.34i$  and  $3.24+3.49i$ . For calibration, the reference solution with known refractive index were 1.3342, 1.3346, 1.3349, 1.3354 and 1.3364, respectively. At the beginning, the surface anchored probe was modified on the surface of chip, and then the reference solutions were flowed through the chip. The sensorgram of the measurement would be achieved and introduced into the flow chart (**Fig.1**). Through interpolation and segmentation, the experimental data was expanded and fit by second-power polynomial. Additionally, through the change of incident angle and comparing the difference between experimental curvature and Fresnel model curvature, the minimum of the residuals will be picked out. As figure 3 shows, the residual for the experimental data

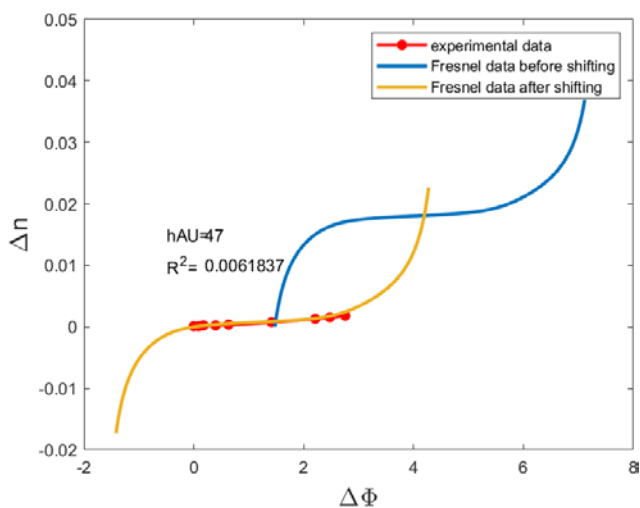
with and without segmentation significantly declines from the order of  $10^{-3}$  (**Fig.3(a)**) to  $10^{-5}$  (**Fig.3(b)**). In the last step, through segmenting and polynomial fitting the Fresnel model curvature, the final transfer function will be achieved.



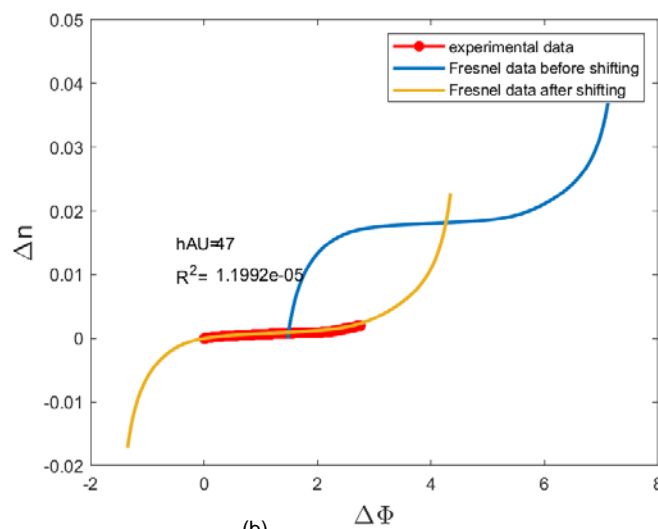
(Fig.1) Flow chart of the algorithm



(Fig.2) Kretschmann configuration[3]



(a)



(b)

(Fig.3) Fresnel model vs. experimental data (a) without segmentation and (b) with segmentation

### References

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